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Switched Quasi-Logarithmic Quantizer with Golomb–Rice Coding

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***Abstract*—This paper proposes a model of switched quasi-logarithmic quantizer for speech signal based on G.711 standard with usage of Golomb-Rice (GR) coding. In order to achieve better performances a method with switched quantizer is applied. Variance range is split into quantizers and for each of them a separate quantizer is designed, i.e. the support region is determined. Optimization of the support region and choice of the parameter μ is done in order to obtain a quantizer that obeys G.712 standard and gives minimal average bit rate. Every quantizer within the variance range has own model with a two-stage coder. Two stages are introduced with purpose to reduce the bit rate, whereby GR code plays its role as Variable Length Code (VLC). The first stage uses a GR coder for coding segments of the quantizer's support region, whereas the second stage applies the coding method with fixed code lengths for coding cells within a segment. GR has simpler and cheaper hardware realization than other VLC codes, Huffman's for instance, with very satisfying results regarding quality of quantized signal.**

***Index Terms*—Quantization; speech processing; speech coding; signal to noise ratio; Golomb-Rice coding.**

I. INTRODUCTION

The G.711 standard (G.711 quantizer) defines fixed length coding that provides high quality of reconstructed signal for fixed bit rates [1]. The G.711.0 standard defines the G.711 quantizer with variable length codes (VLC) and assumes the usage of one of many coding techniques whereby the choice is made upon the input signal's characteristics [2]. This standard does not examine switched quantizers, and therefore our motivation is to design a switched quantizer with some VLC. Golomb-Rice (GR) coding method [3], [4], as well as Huffman's, belongs to the group of VLC techniques. Some VLC scalar quantizers with Huffman's code and similar ones are analysed in [5]–[9], whereby [8] studies VLC for lossless compression. GR coding is simpler than Huffman, although the latter is by custom used for smaller code books. We examine here a code book with N = 256 levels, thus GR code is better solution for our model.

The G.712 standard gives a lower limit for the signal to quantization noise ratio (SQNR) of the transmitted speech signal depending on input variances [10], [11]. Paper [12] proposes fixed quasi-logarithmic quantizer with GR coding. In paper [13] switched quantizers are examined and adaptive quantizers in [14]. Our motivation is to make an analysis of different switched quantizers with μ-law of compression and GR coding, as well as to determine switched quantizer which satisfies G.712 standard and has the lowest possible average bit rate.

II. SWITCHED QUASI–LOGARITHMIC QUANTIZER WITH GOLOMB–RICE CODING

A. Switched Quasi–Logarithmic Quantizer

As analysed in [13] the switched quantizer provides better performances than the quantizer designed according to the G.711 standard. The idea of introducing switched quantizer for speech signal transmission brings division of observed variance range L from −20 dB to 20 dB into Ng equally wide quantizers. Signal's variance σ^2 is defined as $\sigma^2[dB]=10\log\left(\sigma^2/\sigma_{ref}{}^2\right)$, whereby σ_{ref}^2 = 1. Each quantizer is labelled with index γ, whereby γ = 1, ..., Ng. With known signal's variance σ^2 the matching quantizer γ is unambiguously determined. For every quantizer within the L range we try to find the parameters that lead to the achievement of optimal results. Additional information about the quantizer, for a frame of size M, is transmitted to the receiver via $\log_2 N_g$ bits per frame with M samples, where M = 120,180,240.

Compression characteristic of proposed model has a piecewise linear approximation of the μ-law characteristic, where μ + 1 marks a ratio between the biggest and the smallest quant for non-linear logarithmic quantizer [10], whereby applies

$$\mu = 2^{\alpha} - 1, \tag{1}$$

where $\alpha \in N$. The compression function with μ-law of compression [10] is

$$c(x) = x_{\max}{}^{\gamma}\,\frac{\ln\left(1+\mu|x|/x_{\max}{}^{\gamma}\right)}{\ln(1+\mu)}\operatorname{sgn}(x). \tag{2}$$

It is very important to notice that parameters μ and Ng are

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given for the entire range L, whereas support region is different for every quantizer and therefore we introduce the notation x_{max}^{γ}. The support region $[-x_{max}^{\gamma}, x_{max}^{\gamma}]$ of this symmetric quantizer is divided into $2l = 16$ segments whose thresholds x_i are defined as

$$x_i^{\gamma} = \left[\left(\mu+1\right)^{i/L} - 1\right]\frac{x_{\max}^{\gamma}}{\mu}, \tag{3}$$

where $i = 0,...,l.$ The symmetry of proposed quantizer allows us to develop further examination only on segments from positive part of support region. Inside the segment quantization is uniform and the step size of quantization Δ_i^{γ} is

$$\Delta_i^{\gamma} = \frac{x_{i+1} - x_i}{t} = 2^{i\alpha/l}\left(2^{\alpha/l} - 1\right)\frac{x_{\max}^{\gamma}}{t\mu}, \tag{4}$$

$i = 0,...,l-1.$ Step sizes of consecutive segments have the following ratio

$$\frac{\Delta_{i+1}^{\gamma}}{\Delta_i^{\gamma}} = 2^{\alpha/l} = \sqrt[l]{\mu+1}. \tag{5}$$

Decision levels $x_{i,j}^{\gamma}$ which are equally distanced within i-th segment can be calculated in this way

$$x_{i,j}^{\gamma} = x_i + j\Delta_i^{\gamma}, \tag{6}$$

where $i = 0,...,l-1,$ $j = 1,...,t-1.$ The borderline cases are defined as well:

$$\begin{cases} x_{i,0}^{\gamma} = x_i^{\gamma}, \\ x_{i,t}^{\gamma} = x_{i+1}^{\gamma}. \end{cases} \tag{7}$$

All the samples between $x_{i,j-1}^{\gamma}$ and $x_{i,j}^{\gamma}$ are represented with the representation level $y_{i,j}^{\gamma}$

$$y_{i,j}^{\gamma} = x_i^{\gamma} + \frac{(2j-1)}{2}\Delta_i^{\gamma}, \tag{8}$$

where $i = 0,...,l-1,$ $j = 1,...,t.$ Parameters that have an influence to performances are: the parameter of the μ-compression law [10], the number of switched quantizers Ng, as well as the support regions x_{max}^{γ}. Our goal is actually to determine each x_{max}^{γ}, for $\gamma = 1, ..., Ng$ and with the given values of μ and Ng, within the range L.

All support regions x_{max}^{γ} will influence quantizer's performances, among which for us the most important are: SQNR and the average bit rate $\overline{R}$. The design of the quantizer often requires simultaneous fulfilment of two opposing criteria: maximal quality of signal (SQNR) and minimal average bit rate. Hereby we decided to develop a quantizer whose performances fulfil the G.712 standard [11] along with as small as possible $\overline{R}$ when using the VLC coding. Our goal is to minimize the average bit rate, since the difference between the achieved bit rate and the one proposed by G.711 standard presents an important saving in signal transfer. The lower limit of SQNR is conditioned by G.712 standard, and therefore we try to find the value of x_{max}^{γ} which satisfies this standard and gives minimal possible value of $\overline{R}$.

B. Two-Stage Coder

Having in mind that the support region is split into $2l = 16$ segments and each segment is split into t=16 cells, we need to find an efficient way to code this information. The G.711 coder as shown in Fig. 1 codes as follows: the whole region is symmetrically split into two parts, positive and negative; therefore the position of the segment is coded with one bit ("0" or "1"). Then each of l=8 segments is coded with 3 bits (from "000" to "111") and each of t=16 cells is coded with 4 bits (from "0000" to "1111"). Accordingly, G.711 coder always requires 8 bits, classifying it to the group of coders with fixed length code words [1]. Therefore the average bit rate is always equal to 8 bits per sample.

Our goal is to find a solution that requires lower bit rate than G.711 quantizer's. Therefore we propose the coder depicted in Fig. 2 which is implemented as a two stages coder. In first stage we split whole support region into $2l = 16$ segments and try to code them with code words of variable length. The segments are labeled from left to right on the axis one after the another with numeral i=1,…,2l where i=1 marks the furthest segment to the left on the axis, i.e. the segment bounding $-x_{max}^{\gamma}$, and i=16 marks the furthest segment to the right on axis, i.e. segment next to the x_{max}^{γ}. In this way with a proper technique we could achieve better bit usage than the G.711 standard. Here we use a Golomb-Rice coder where values of n from 0 to 15 are coded with code words of various lengths, as seen in Table I. In order to have the most efficient results we must try to designate shorter codes to the segments with higher probability of appearance and longer codes to the segments with lower probability of appearance. Therefore we developed a rule for this designation, as shown in the last column of Table I. In this way we conclude the first stage of proposed coder. In second stage we code each of t=16 cells naturally with 4 bits (from "0000" to "1111").

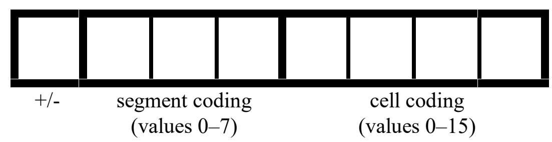

Fig. 1. G.711 speech signal coding scheme.

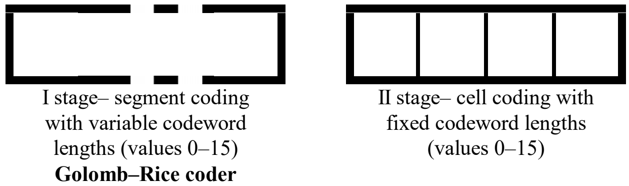

Fig. 2. Proposed two-stage speech signal coding scheme.

C. The Golomb–Rice Coding Technique

The Golomb-Rice (GR) coding technique uses code words of variable length and was created by combining Golomb's (Solomon W. Golomb) and Rice's (Robert F. Rice) coding techniques [3], [4]. Golomb's coding assumes that number n is decomposed with divisor m into integer quotient q and a remainder r, whereby stays

$$n = qm + r\ ,\ n, m, q, r \in N. \tag{9}$$

Golomb-Rice coding has an additional requirement:

$$m = 2^k\ ,\ k \in N_0. \tag{10}$$

The number n is coded as follows: first the quotient q is coded unary, i.e. we write an array of q ones, then we put a zero in order to provide correct decoding, and in the end the remainder r is binary coded on k bits. Therefore, depending on the parameter k, the GR coding can be done in various ways. In our research we examined all variants and came to the conclusion that the best results for this model are achieved for k=1. In Table I is given an overview of GR code for k=1, n=0,…,15 where Si represents code and li its length, whereas i marks the coder's segment which is going to be coded with this particular code [3], [4].

Decoding is done in the following way: we count ones before the first zero, i.e. decimal point. The number of counted ones is quotient q. Then we identify k bits after the zero and they represent binary record of remainder r. Since k, q and r are known, the number is calculated with formulae (9) and (10). Starting from the (k+1)-th bit after decimal point we continue counting ones up to the next zero and the decoding process continues [3], [4].

TABLE I. GOLOMB–RICE CODE, K = 1, M = 2; DESIGNATING SEGMENTS I= 1,…, 2L WITH PROPER CODES N= 0,…,15.

n	Si	li	i
0	00	2	9
1	01	2	8
2	100	3	10
3	101	3	7
4	1100	4	11
5	1101	4	6
6	11100	5	12
7	11101	5	5
8	111100	6	13
9	111101	6	4
10	1111100	7	14
11	1111101	7	3
12	11111100	8	15
13	11111101	8	2
14	111111100	9	16
15	111111101	9	1

III. PERFORMANCES AND NUMERICAL RESULTS

We assume that quantizer's input speech signal can be described with the Laplacian probability density function

$$p(x,\sigma) = \frac{1}{\sqrt{2}\sigma}\exp\left(-\frac{|x|\sqrt{2}}{\sigma}\right). \tag{11}$$

The measure of signal's quality is distortion. It is consisted from granular D_g and overload D_{ov} distortion whose sum is total distortion $D=D_g+D_{ov}$. Signal to Quantization Noise Ratio (SQNR) is defined as

$$\text{SQNR}\ (\sigma) = 10\log\ \left(\sigma^2 / D\right). \tag{12}$$

Granular distortion D_g is the distortion for finite cell widths, i.e. within the support region [$-x_{\max}{}^{\gamma}$, $x_{\max}{}^{\gamma}$]. The overload distortion D_{ov} measures out of that region, i.e. on ($-\infty$, $-x_{\max}{}^{\gamma}$) ∪ ($x_{\max}{}^{\gamma}$, ∞). Thus, granular distortion for piecewise linear quantization and high bit rate (asymptotic quantization) is [10]

$$D_g(\sigma) = \int_{-x_{\max}{}^{\gamma}}^{x_{\max}{}^{\gamma}} \frac{\Delta^2(x,\sigma)\,p(x,\sigma)}{12}dx, \tag{13}$$

where $\Delta(x,\sigma)$ represents cell width defined in (4). We can further derive it as in [10]

$$D_g(\sigma) = \frac{1}{12}\sum_{i=0}^{l-1}\Delta_i^2(\sigma)P_i(\sigma), \tag{14}$$

whereas P_i is probability of i-th segment

$$P_i(\sigma) = \int_{x_i}^{x_{i+1}} p(x,\sigma)dx =$$
$$= \exp\left(-\frac{\sqrt{2}x_i}{\sigma}\right) - \exp\left(-\frac{\sqrt{2}x_{i+1}}{\sigma}\right). \tag{15}$$

On the other side, overload distortion is calculated as [10]

$$D_{ov}(\sigma) = 2\int_{x_{\max}{}^{\gamma}}^{\infty}\left(x - y_{l-1,t}\right)^2 p(x,\sigma)dx, \tag{16}$$

whereas $y_{l-1,t}$ is defined according to (8). Therefore, the exact expression is

$$D_{ov} = \exp\left(-\frac{\sqrt{2}x_{\max}{}^{\gamma}}{\sigma}\right)\times$$
$$\times\left(\left(x_{\max}{}^{\gamma} - y_{l-1,t} + \frac{\sigma}{\sqrt{2}}\right)^2 + \left(\frac{\sigma}{\sqrt{2}}\right)^2\right). \tag{17}$$

A. *Signal to Quantization Noise Ratio – SQNR*

On the range L, whereby $\sigma_s{}^2 = 10^{\sigma_s{}^2[dB]/10}$, we calculate the average SQNR in a large number of points p as following

$$\text{SQNR}_{\text{av}} = \frac{1}{p}\sum_{s=1}^{p}\text{SQNR}(\sigma_s). \tag{18}$$

The values for SQNR_{av} are shown in Table II. For the fields marked with slash (/) it is not possible to design a quantizer which respects G.712 standard with current parameters Ng and μ.

TABLE II. AVERAGE SIGNAL TO QUANTIZATION NOISE RATIO SQNR [dB] CALCULATED DEPENDING ON THE NUMBER OF QUANTIZERS Ng AND PARAMETER μ.

$SQNR_{av}$	μ=15	μ=31	μ=63	μ=127	μ=255
Ng=1	/	/	/	34.1900	33.8617
Ng=2	/	/	34.3254	33.8864	33.5398
Ng=4	34.4067	34.0171	33.6413	33.3424	33.0904
Ng=8	33.4359	33.2147	32.9884	32.8127	32.6519
Ng=16	32.8550	32.6967	32.5699	32.4753	32.3789
Ng=32	32.5454	32.4156	32.3428	32.2838	32.2305

B. Bit Rate

Since the SQNR depends on signal variance and not on code word lengths, the crucial parameter for the code choice will be the average bit rate. Hereby we try to get with the described two-stage coding method a lower average bit rate than the G.711 standard's coder, which has a constant value of 8 bit/sample. The first stage's average bit rate for a particular standard deviation σ is equal to

$$\overline{R}_I(\sigma) = 2\sum_{i=1}^{l} l_i P_i(\sigma), \tag{19}$$

whereas l_i is the code word's length, defined in Table I, and P_i is defined in (15). We calculate the average bit rate of the first stage on the variance range L in a large number of points p as

$$\overline{R}_I = \frac{1}{p}\sum_{s=1}^{p} \overline{R}_I(\sigma_s). \tag{20}$$

The bit rate of second stage is constant

$$R_{II} = \log_2(t). \tag{21}$$

The total average bit rate of the proposed quasi-logarithmic quantizer, with additional information about quantizer, is:

$$\overline{R} = \overline{R}_I + R_{II} + \frac{\log_2 N_g}{M}. \tag{22}$$

Values for $\overline{R}$ are shown in Table III, where the frame size is M=240. The values under the double line are lower than 8 bit/sample and belong to the area of our special interest in this research.

TABLE III. AVERAGE BIT RATE $\overline{R}$ [bit/sample] CALCULATED DEPENDING ON THE NUMBER OF QUANTIZERS Ng AND PARAMETER μ.

$\overline{R}$	μ=15	μ=31	μ=63	μ=127	μ=255
Ng=1	/	/	/	11.0585	9.2762
Ng=2	/	/	11.8040	9.8311	8.5547
Ng=4	13.2640	10.5641	8.9969	8.0123	7.3480
Ng=8	10.5745	8.9155	7.9433	7.3106	6.8717
Ng=16	9.5135	8.2445	7.5124	7.0208	6.6739
Ng=32	9.0534	7.9729	7.3206	6.8923	6.5869

C. Numerical Results

As can be seen in Table III, the minimal $\overline{R}$ is obtained for Ng=32 and μ=255. These parameters are the best choice for our model and will be used to compute the performances. In this case a saving of bit rate equal to 1.4331 bit/sample is made, whereby the $SQNR_{av}$ is 32.2305dB.

SQNR(σ) is calculated on the entire variance range L and shown in Fig. 3. Further, we can get support regions as a set of values $x_{max}[\gamma]$ = [103 102 100 98 98 97 95 92 88 89 96 103 110 117 124 139 160 185 214 247 285 329 380 439 507 586 677 782 903 1043 1204 1390], for γ=1,...,32. Segments are coded as described in Table I. whereby code lengths are $l[i]$= [2 3 4 5 6 7 8 9], i=1,…,8. For the variance values σ_1^2[dB]=0.625dB and σ_2^2[dB]= −19.375dB the most important quantizer's parameters are given in Table IV.

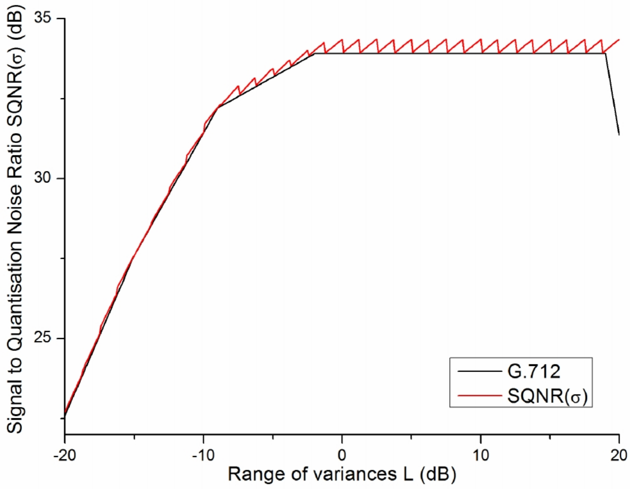

Fig. 3. SQNR(σ) on the variance range L.

TABLE IV. NUMERICAL RESULTS FOR PARTICULAR VALUES OF SIGNAL VARIANCE σ^2[dB].

	σ_1^2[dB]=0.625dB, γ=17			σ_2^2[dB]= −19.375dB, γ=1		
i	x_i	Δ_i	P_i	x_i	Δ_i	P_i
1	0.6275	0.0392	0.2810	0.4039	0.0252	0.4975
2	18.824	0.0784	0.1770	12.118	0.0505	0.0025
3	43.922	0.1569	0.0404	28.275	0.1010	0
4	94.118	0.3137	0.0015	60.588	0.2020	0
5	194.510	0.6275	0	125.216	0.4039	0
6	395.294	12.549	0	254.471	0.8078	0
7	796.863	25.098	0	512.980	16.157	0
8	160	50.196	0	103	32.314	0
	$SQNR_{av}$ = 34.1437 dB			$SQNR_{av}$ = 23.3097 dB		
	$\overline{R}$ = 6.5458 bit/sample			$\overline{R}$ = 6.0257 bit/sample		

In comparison with non-adaptive fixed quantizer with GR code described in [12] where the quantizer fulfilling G.712 standard for μ=255 has an average bit rate 7.9195 bit/sample, this model has a bit rate saving of around 1.3 bit/sample. Comparing with an adaptive quantizer [14] where the bit rate is $R = \log_2 N + \log_2 N_g / M$ our model's average bit rate (22) is significantly lower.

IV. CONCLUSIONS

We calculated the performances for switched quantizers that satisfy G.712 standard and for different values of μ: 15, 31, 63, 127, 255. Opposite to the expectation that better performances would have been obtained for the lowest value of parameter μ, actually the lowest average bit rate is reached with the usage of the switched quantizer for maximal value μ=255. Comparing to the non-adaptive logarithmic G.711 quantizer, the proposed quantizer achieves an average bit rate's decrease of 1.43 bit/sample.

The recommended VLC Golomb-Rice code is simpler than Huffman's code, which is another advantage of proposed solution. We are looking forward to seeing this quantization method being applied for coding and compression of other signal types.

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