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Characterization of Three - Dimensional Rough Fractal Surfaces from Backscattered Radar Data

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***Abstract*—This paper discusses the scattering of electromagnetic (EM) waves from three – dimensional (3D) rough fractal surfaces using the Kirchhoff approximation. In particular, it introduces a novel method to characterize 3D rough fractal surfaces from spectral information of the backscattered EM wave in Synthetic Aperture Radar (SAR) applications. It represents an important extension of a previous recent paper by the same research group from 2D fractal surfaces to 3D fractal surfaces (the latter representing real – life SAR radar scenes). More specifically, in the present simulation scenarios it is assumed that the radar emits a burst of radar pulses of increasing carrier frequency [therefore a 'stepped – frequency' (SF) SAR radar]. By calculating the backscattered EM wave from 3D fractal surfaces as a function of the above radar frequencies (therefore, a 'spectral method'), it is found here that the slope between the main backscattered lobe and its adjacent side lobes increases with increasing surface fractal dimension (i.e. with increasing surface roughness). In this way a characterization of 3D fractal rough surfaces from backscattered SAR radar data is achieved, as explained in detail in this paper.**

***Index Terms*—Three-dimensional fractal surface; surface fractal dimension; scattering of electromagnetic waves; Kirchhoff approximation; synthetic aperture radar (SAR); backscattering coefficient.**

I. INTRODUCTION

In the last decades there has been a growing interest in the scattering of optical, electromagnetic and acoustical waves from rough surfaces. A very interesting and very popular approach to represent rough surfaces encountered in real – life has been proposed by Mandelbrot and other scientists, who presented the concept of fractals [1], [2]. Scattering of EM waves from fractal rough surfaces has been examined in the past in [3]–[7], where 2D self – similar fractal surfaces have been considered, the corresponding surface 'fractal dimension' has been introduced and the Kirchhoff approximation for EM wave scattering was used. In contrast to our recent publication [3], where monostatic scattering is considered and the main idea of the present paper is introduced for the simpler case of 2D fractal surfaces, bistatic scattering was considered in all other cases of Refs. [4]–[7]. Furthermore, a variant of 2D self – similar fractal surface representation, namely a 'one-dimensional bandlimited Weierstrass function', was introduced in [8], which will be generalized in this paper for 3D fractal surface. Furthermore, in those years (1994), a 'two-dimensional bandlimited Weierstrass function' (i.e. a 3D surface) was also proposed by Lin et. al. [9], in order to study the bistatic scattering of EM waves from 3D fractal surfaces. However, in this paper we adopt a much simpler representation of 3D fractal surface, introduced in 2012 by Zaleski [10]. Moreover, the mathematical formulation for EM scattering from either 2D or 3D rough surfaces, and the 'Kirchhoff approximation' to that in particular, used in the majority of the above references, was initially presented by Beckmann and Spizzichino in [11], also used in this paper for our backscattering spectral considerations. Furthermore, the Kirchhoff approximation mentioned above is initially valid under the assumption that the wavelength of the incident EM wave is much smaller than the local radius of curvature of the surface roughness, i.e. surface roughness is small as compared to the wavelength of the incident EM wave radiation [11]. Finally, further references concerning the problem considered in this paper are provided at the References Section [12]–[20].

In Section II of this paper, the problem geometry, as well as a short presentation of the mathematical formulation for EM wave scattering from rough surface using the Kirchhoff approximation, are presented. Furthermore, in Section III our simulation results in the spectral domain (i.e. backscattering coefficient as a function of radar frequency) are presenting, ultimately yielding to 3D rough surface characterization from backscattered SAR radar data. Finally, conclusions are presented in Section IV and future research goals in Section V of this paper.

II. PROBLEM GEOMETRY AND MATHEMATICAL FORMULATION

The geometry of the 3D scattering problem is shown in Fig. 1. Here, an incident EM plane wave illuminates a three dimensional (3D) rough fractal surface $f(x,y)$, where the illuminated area of the surface (due to limited radar antenna beamwidth) is given by $A = 4L_xL_y$,with $-L_x < x < L_x$, $-L_y < y < L_y$.

The angle of incidence of the EM wave is θ_1 with respect to the vertical z axis, while the elevation and azimuth angles of scattering of the EM wave are θ_2 and θ_3 with respect to the vertical z axis and horizontal x axis, respectively (see Fig. 1). The incident and scattering wave vectors are denoted by k_i and k_s, respectively. Regarding the modeling of the 3D rough fractal surface, we introduce here a

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bandlimited Weierstrass function of two variables, (1) below, as a straightforward extension of similar Weierstrass functions provided in the past by Jaggard [8] (function of one variable) and by Zaleski [10] (function of two variables).

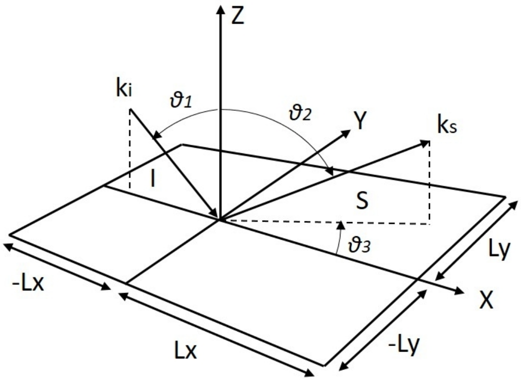


Fig. 1. Geometry of rough surface scattering problem, in which an incident plane EM wave illuminates a three dimensional fractal rough surface patch of size ($2L_x \times 2L_y$) at an angle of incidence equal to θ_1.

Therefore, the equation proposed here, which describes the modified two-dimensional bandlimited fractal Weierstrass function for modeling 3D rough fractal surfaces is given by

$$f(x,y) = \sigma C \sum_{n=0}^{N-1} b^{(D-3)n} \sin(K_1 b_1^n x + \varphi_{n1}) \times \times \cos(K_2 b_2^n y + \varphi_{n2}), \quad (1)$$

where σ is the standard deviation (rms height), C is the amplitude control factor, N is the number of tones, b_1 and b_2 ($b_1 > 1$, $b_2 > 1$) are the spatial - frequency scaling parameters, b is a constant ($b > 1$), D ($2 < D < 3$) is the roughness fractal dimension, K_1 and K_2 are the fundamental spatial wave numbers in the direction of x and y respectively, and φ_{n1}, φ_{n2} are arbitrary phases with uniform distribution over the interval [$-\pi$, π]. The amplitude control factor C is calculated below, so that the above function has always standard deviation (rms height) equal to σ. This calculation yields the following expression for C (see, e.g. [5])

$$C = 2\left[\frac{1-b^{2(D-3)}}{1-b^{2(D-3)N}}\right]^{\frac{1}{2}}, \quad (2)$$

To illustrate the fractal function of (1) for different values of fractal dimension D, some representative simulations of that are shown in Fig. 2. As the fractal dimension increases through the values D = [2.10, 2.50, 2.90] of Fig. 2, the roughness of the 3D fractal surfaces increases, as well.

For the plane EM wave incidence of Fig. 1, the so – called 'scattering coefficient' γ at an arbitrary scattering direction (θ_2, θ_3), which represents the scattered electric field at that direction normalized by the scattered electric field at the so – called 'specular direction', i.e. at direction ($\theta_3 = 0$, $\theta_2 = \theta_1$) is given by [11]

$$\gamma = \frac{E_{SC}}{E_{SC0}} = \frac{F(\theta_1,\theta_2,\theta_3)}{A} \times \int_{-L_x}^{L_x}\int_{-L_y}^{L_y} \exp[ikv_x x + ikv_y y + ikv_z f(x,y)]dxdy + \frac{e(Lx,Ly)}{A}. \quad (3)$$

where:

$$F(\theta_1,\theta_2,\theta_3) = \frac{1+\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos\theta_3}{\cos\theta_1(\cos\theta_1+\cos\theta_2)}, \quad (4)$$

$$A = 4L_x L_y, \quad (5)$$

$$v_x = \sin\theta_1 - \sin\theta_2\cos\theta_3, \quad (6)$$

$$v_y = -\sin\theta_2\sin\theta_3, \quad (7)$$

$$v_z = -(\cos\theta_1 + \cos\theta_2), \quad (8)$$

and $e(L_x, L_y)$ represents the so called 'edge effect'.

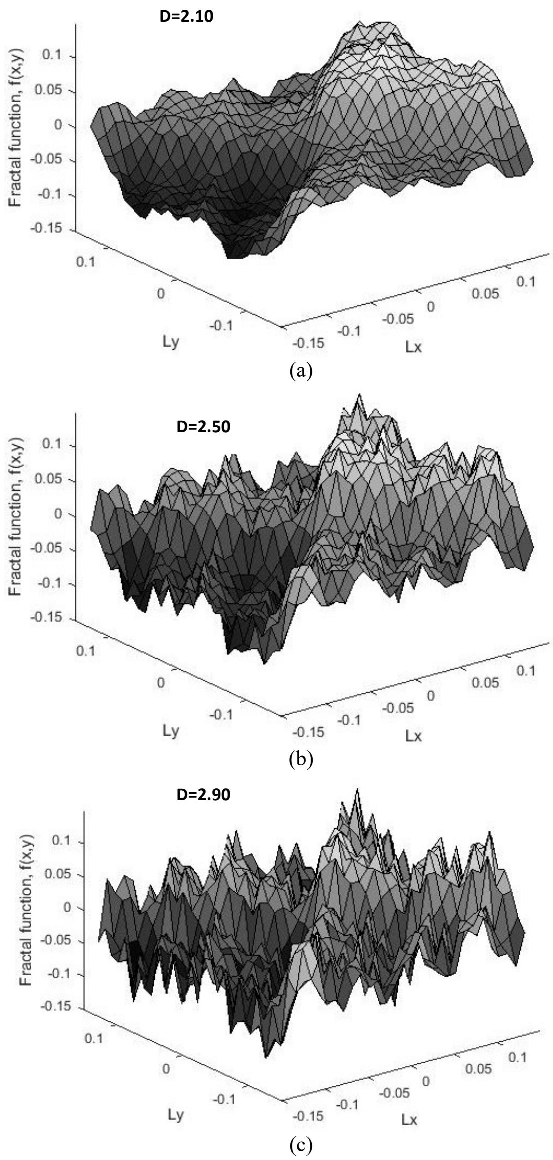
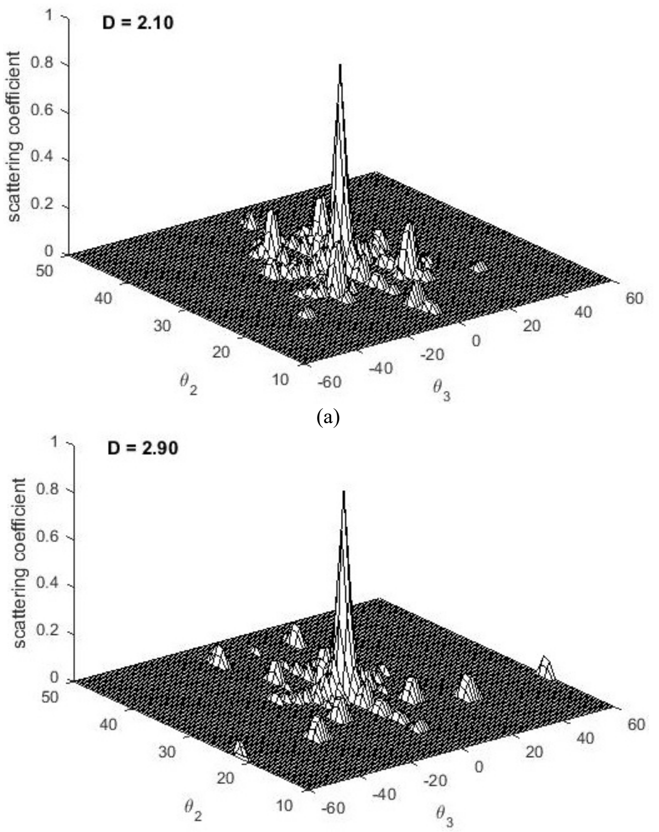


Fig. 2. Sample plots of fractal function f(x,y), (1), for different values of fractal dimension D.

Note that in (3) above, k is the wavenumber of the incident EM wave ($k = 2\pi f/c$, where f is the frequency of the incident EM wave). Furthermore, note that at the right – hand – side (RHS) of (3), the first term provides the most significant contribution to the scattering process, while the second term $e(X, Y)$ is the so – called 'edge effect', which can be neglected when $A >> \lambda^2$ [11], as assumed in this paper.

Figure 3 gives representative examples of scattering patterns from 3D fractal surfaces for different values of fractal dimension D, as numerically calculated by our research group by using (3)–(8) above.



(a)

(b)

Fig. 3. Bistatic scattering patterns for the scattering coefficient γ (magnitude) as a function of the scattering angles θ_2 and θ_3 of Fig. 1, from fractal rough surfaces with increasing value of fractal dimension, D = [2.10, 2.90]. Here the angle of incidence is $\theta_1 = 30^o$.

It appears that the scattering patterns of Fig. 3 are angularly directed mostly towards the specular direction for low fractal dimensions, while they are more angularly diffused for high fractal dimensions (this was a clear observation of ours for a very large number of computer simulations).

Finally note that in the special case of backscattering ($\theta_3 = 0$, $\theta_2 = -\theta_1$) considered in the next Section, the following equations are derived for the 'backscattering coefficient' γ_{bsc}

$$\gamma_{bsc} = \frac{1}{4L_x L_y \cos^2\theta_1} \int_{-L_x}^{L_y} \int_{-L_y}^{L_y} e^{i2k\varphi(x,y)} dxdy, \tag{9}$$

where

$$\varphi(x,y) = x\sin\theta_1 - f(x,y)\cos\theta_1. \tag{10}$$

III. Simulation Results

In this Section, computer simulations are performed concerning the calculation of the 'backscattering coefficient' γ_{bsc}, (9)–(10) above, as a function of fractal dimension D of the rough surface, for a variety of frequencies of a 'stepped – frequency' (SF) SAR radar, i.e. a spectral method for rough surface characterization. Here, the 3D fractal surface is simulated by the band-limited fractal function of (1), while the 'surface roughness' is controlled by the fractal dimension D, as already illustrated in Fig. 2. Namely, the backscattering coefficient $|\gamma(k)|$ was calculated from (9) and (10) for a variety of frequencies $f_m = f_0+(m-1)\,\Delta f$, where $m=1,2,\ldots,M$, M is the number of frequencies of the SF SAR radar, f_o is the carrier frequency, $\Delta f = B/M$ is the frequency step and B is the bandwidth of the radar, as also mentioned in [3], [12]. In these simulations we used, in (1), for simplicity, $K_1 = K_2 = K$ ($K = 2\pi/\Lambda_0$, where Λ_0 is the surface fundamental wavelength). The parameters used to perform the simulations are summarized in Table I, where it was used here $L_x = L_y = L$, $2L >> \Lambda_0$ and $k\sigma < 1$, where $k = 2\pi/\lambda$ is the radar wavenumber [3], [5].

TABLE I. SIMULATION PARAMETERS.

No	Description	Symbol	Value
1	spatial frequency scaling parameters	$b=b_1$	*1.8122*
		b_2	*2.7183*
2	number of tones	N	*6*
3	number of frequency steps	M	*100*
4	radar bandwidth	B	*2.0 GHz*
5	frequency step	Δf	B/M = *20 MHz*
6	carrier radar frequency	f_0	*10 GHz*
7	radar wavelength	λ	c/f_0
8	surface rms height	σ	0.05λ
9	fundamental surface wavelength	Λ_0	10λ
10	fundamental surface wavenumbers	$K_1=K_2$	$2\pi/\Lambda_0$
11	rough surface illuminated length (patch size)	$4L_xL_y$	$80\lambda \times 80\lambda$
12	incident angle	θ_1	30^o

In all our simulations incidence angle $\theta_1 = 30^o$ and backscattered angle $\theta_2 = -\theta_1$, $\theta_3 = 0$. Figure 4 illustrates plots of $|\gamma(k)|$ from simulation runs for different values of fractal dimension D ('surface roughness'), i.e. D = [2.05, 2.55, 2.85] correspondingly for each figure. From this figure, we can conclude that as the value of the fractal dimension D increases, i.e. as the 'roughness' of the fractal surface increases, the emerging slope between the main lobe of function $|\gamma(k)|$ and the side lobes in the 'spectral domain' also increases.

Table II summarizes the relation between the fractal dimension D and the calculated slope between the main and first side lobes. Each slope is equal to $|\Delta\gamma| / |\Delta k|$, where $\Delta\gamma$ represents the amplitude difference between the peak of the main lobe and the peak of the first side lobe, and Δk defines the variation of the wavenumber peaks.

TABLE II. FRACTAL DIMENSION D AND SLOPE CALCULATIONS.

D	**Left slope calculations**	**Right slope calculations**
2.10	0.1×10^{-3}	0.1×10^{-3}
2.20	0.1×10^{-3}	0.1×10^{-3}
2.30	0.1×10^{-3}	0.7×10^{-3}
2.40	0.6×10^{-3}	1.0×10^{-3}
2.50	0.9×10^{-3}	1.5×10^{-3}
2.60	1.4×10^{-3}	2.2×10^{-3}
2.70	1.8×10^{-3}	2.4×10^{-3}
2.80	2.2×10^{-3}	3.0×10^{-3}
2.90	2.6×10^{-3}	3.4×10^{-3}

An important remark regarding the derivation of the simulation results of Fig. 4 is that the radar bandwidth must be sufficiently large, so that the phenomenon of the slope increase with increasing surface fractal dimension is observed. It was found here that the bandwidth must be at least 15 % of the radar carrier frequency, so that this phenomenon is observed. However, the possible problem of small available bandwidth in actual radar measurements can be counter – balanced by increased 'patch size of observation' $2L$ (see e.g. [3], [5], [7]).

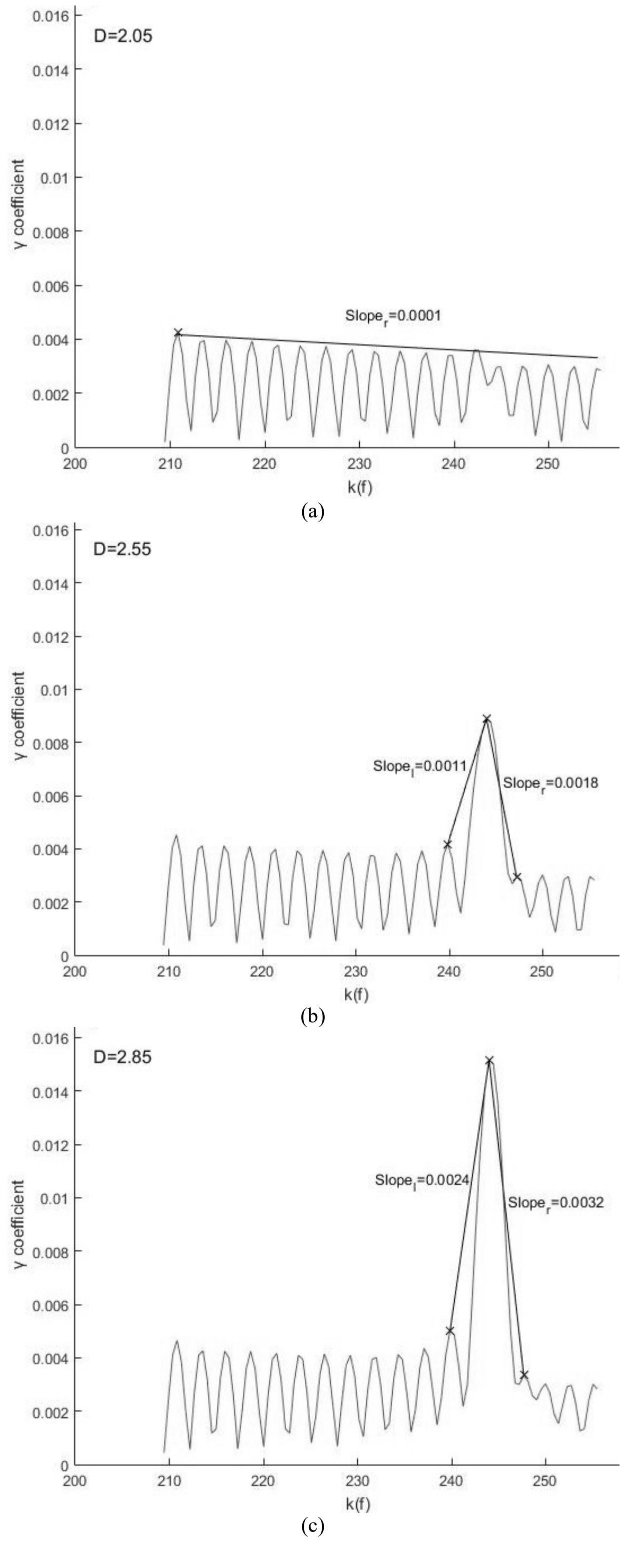


Fig. 4. Magnitude of the backscattering coefficient $|\gamma(k)|$ as a function of the wavenumber k, for values of fractal dimension D = [2.05, 2.55, 2.85], respectively.

To illustrate the relation between the surface fractal dimension D and the slopes of the scattering coefficient $|\gamma(k)|$, a calculation of the scattering coefficient $|\gamma(k)|$ was performed sequentially for different values of fractal dimension D for values beginning from 2.05 up to value 2.95, with a step equal to 0.05.

All other parameters used in these simulations remained the same, as shown in Table I. The approach is the same with that followed by Kotopoulis *et. al*. [3], where *the left and right slope* calculations of $|\gamma(k)|$, for each value of D, were *averaged* for creating one average slope calculation of $|\gamma(k)|$ for each value of D. To verify the proposed method, uniformly distributed random phase variables φ_{n1}, φ_{n2} were used in (1) in the interval $[-\pi, \pi]$ in order to create test fractal surfaces. In particular, we assume here that the scatterer's variations are negligible within the duration of a radar burst, each one of radar burst consisting of M radar pulses with increasing radar frequency. The calculation results, after 10 simulations for each D value, are presented in Fig. 5, below.

Furthermore, by 'inverting' the data (slope values and fractal dimension) provided in Fig. 5, the plots of Fig. 6 are obtained, where in this case the surface fractal dimension D is plotted as a function of the 'slope calculations' of the scattering coefficient $|\gamma(k)|$, similarly to the method used in [3]. The above simulations show that the roughness of the 3D fractal surface can be characterized by the mean slope between the main lobe of function $|\gamma(k)|$ and the first two side-lobes, adjacent to the main lobe (see Fig. 4–Fig. 6). In particular, from Fig. 6 it can be seen that there exists a curve fitting the simulated data, when the fractal dimension D is provided by the following

$$D = a \times slope^{b} + c, \tag{11}$$

where the constants are a = 49.9, b = 0.71 and c = 2.08.

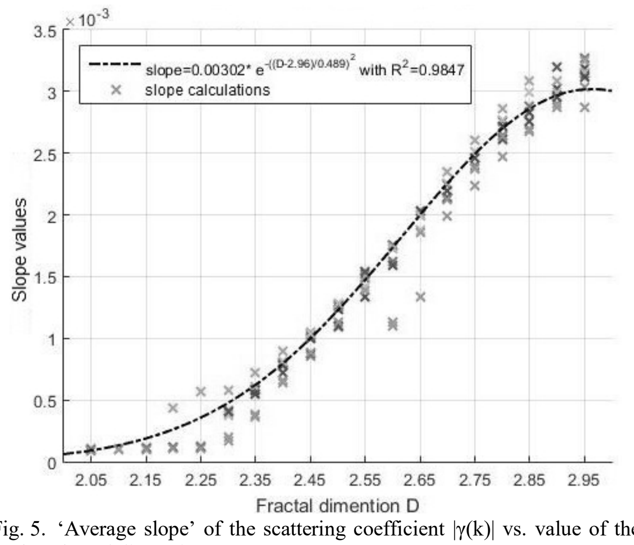


Fig. 5. 'Average slope' of the scattering coefficient |γ(k)| vs. value of the surface fractal dimension D.

In addition, for measuring the fit of the data of Fig. 6 to the equation provided by (11) above, the 'R-square criterion for curve fitting' was used, which was calculated as R^2 = 0.9638 (R^2 = 1 corresponds to the 'perfect curve fitting'). Moreover, the prediction bounds for the fitted curve were plotted, where the probability of occurrence is 90 %.

For visual representation of the fractal dimension D which was used for the simulations, the calculated slope $|\Delta\gamma|/|\Delta k|$ and finally the calculated D_{calc} value, the reader is referred to Table III and Table IV, below. In particular, Table III shows the fractal dimension D that was used in the simulations vs. the slope that was calculated from these and the estimated fractal dimension D_{calc}. These results show a good estimation of the rough surface fractal dimension D (i.e. of the surface roughness), except, for values of D less than 2.25, in which case the fractal dimension D is predicted with rather lower accuracy.

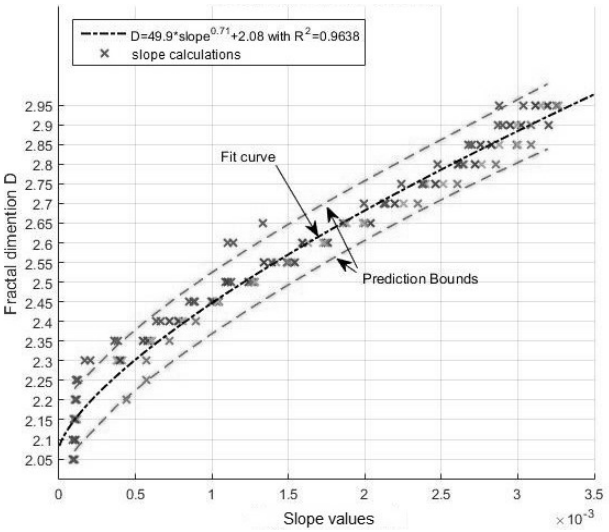


Fig. 6. Value of surface fractal dimension D vs. 'slope calculation' of the scattering coefficient $|\gamma(k)|$.

Finally, Table IV shows the 'prediction intervals' for the calculated fractal dimension D_{calc} estimation from the 'spectral slopes' of the backscattering coefficient. As mentioned also above, the values of 'prediction intervals' shown in Table IV correspond to 90 % 'probability of occurrence'.

TABLE III. ESTIMATION OF D_{CALC}, $D_{calc} = a \times slope^b + c$.

D	***Slope***	***Dcalc***
2.05	0.1×10^{-3}	2.15
2.15	0.1×10^{-3}	2.15
2.25	0.1×10^{-3}	2.16
2.35	0.6×10^{-3}	2.33
2.45	1.0×10^{-3}	2.45
2.55	1.5×10^{-3}	2.57
2.65	1.9×10^{-3}	2.66
2.75	2.4×10^{-3}	2.76
2.85	2.8×10^{-3}	2.85
2.95	3.2×10^{-3}	2.95

TABLE IV. D_{CALC} PREDICTION INTERVAL USING PREDICTION BOUNDS.

D_{calc} ***lower***	***D***	D_{calc} ***upper***
2.05	2.05	2.23
2.08	2.15	2.23
2.08	2.25	2.24
2.25	2.35	2.40
2.37	2.45	2.52
2.49	2.55	2.65
2.58	2.65	2.73
2.68	2.75	2.84
2.77	2.85	2.93
2.78	2.95	2.99

All the above results show that the proposed method in this paper is robust enough regarding the characterization of a 3D rough fractal surface from backscattered radar data.

IV. CONCLUSIONS

In this paper, a method for 3D rough fractal surface characterization from backscattered radar data is presented. Namely, in this paper we are using the Kirchhoff approximation [11] in order to obtain simulation results for the backscattered radar signal amplitude as a function of radar frequency, therefore a 'spectral method' [3]. Regarding the modeling of the rough fractal surface, here a recently introduced novel 3D Weierstrass Mandelbrot fractal function is used [10], modified by our research group in order to be appropriate for the current investigation.

Based on the simulation results, provided in Section III above, we observed that the average slope between the main lobe and the adjacent left and right side lobes increases with increasing fractal dimension (roughness) of the rough fractal surface, as in Fig. 5 above. Therefore, a characterization of the rough surface (estimation of its fractal dimension) from the spectral backscattered radar data described above is possible. In this way, similar results obtained recently by our research group for a 2D modeling of fractal rough surface [3], [13] were obtained here for the more realistic 3D representation of fractal rough surfaces.

In order that the above rough surface characterization is possible, enough radar bandwidth is needed, as it can be seen in Table I. The latter effect may be counterbalanced, to some extent, by increased surface 'patch size' (i.e. by increased radar beamwidth or radar altitude of surface observation).

V. FUTURE RESEARCH

In our future research we intend to concentrate in the following aspects: (i) effect of SNR in our proposed method of rough surface characterization. Note that this aspect has already been investigated in a very successful manner in the case of 2D rough surface modelling [13]. (ii) sea state characterization by using *measured* SAR radar data, e.g. for a 'stepped-frequency' (SF) SAR radar waveform, as that used in Refs. [12], [14].

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