A Wire Electrode inside Parallelepipedicaly-Shaped Ground Inhomogeneity: Comparison of Two Solutions

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***Abstract*—The results for the resistance of the wire electrode inside parallelepipedicaly-shaped domain surrounded with homogeneous ground obtained by one recently proposed procedure for modeling described structure are compared with recently published results realized applying one new hybrid boundary element method. The first approach is based on the approximating system wire-parallelepiped with one wire conductor off equivalent length and cross-section radius. The hybrid method is based on equivalent electrodes method and point matching method applied on matching values for potential of wire electrode and normal component of electric field on the boundary surface between conductive media. The quasi-stationary approach is applied.**

***Index Terms*—Grounding, moment methods, resistance, nonhomogeneous media.**

I. INTRODUCTION

The pillars realized in practice (parts of the overhead power network or the lightning protection system) necessarily include a corresponding grounding system. Such system is usually formed of a basic star-, rectangularly- or circularly-shaped electrode [1], [2] connected to an iron armature of the concrete pillar foundation (usually of the shape of parallelepiped [3]), which can be treated as a second part of the grounding system. Official publications as [1] usually neglect influence of the concrete foundation. One simple procedure for approximate modelling of the described influence was proposed in [4] and provides reduction of the analysed grounding system, which includes concrete foundation (ground inhomogeneity), into a problem of a grounding system in the homogeneous ground, which can be solved using well known procedures (in this case Method of Moments (MoM)). The approach is based on approximation of parallelepipedical concrete domain with circle cross-section cylinder and analysis of the current density field. Level of the described influence depends on concrete`s and ground`s electrical parameters, which can have different values depending on the ground`s structure, humidity, etc. The described procedure has been applied in [5]–[7].

In this paper, the resistance of the single wire electrode inside parallelepipedically-shaped domain surrounded by homogenous ground is determined and compared with available results obtained with hybrid boundary elements method (HBEM). This problem was often analysed [8], [9]. The HBEM was recently proposed in [10], based on the idea to substitute boundary segments with total charges placed in free space at the centres of boundary segments. This approach includes using of corresponding Green's function for electric scalar potential and point matching method (PMM) [11] for matching values of potential and boundary condition for normal components of the electric field. It has been applied for electrostatic as well as magnetostatic problems [12]. The results which are used for comparison in this paper are directly adopted from [13] where the method has been applied for analysis of quasi-stationary grounding systems with complex conductivity of multilayered media.

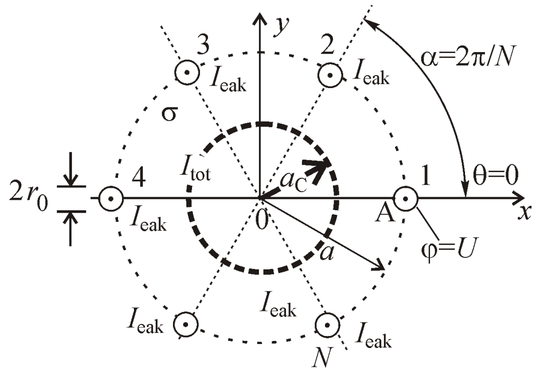

Fig. 1. Wire electrodes' system.

II. THE PROCEDURE FOR APPROXIMATING SINGLE WIRE CONDUCTOR INSIDE PARALLELEPIPED

The idea to reduce problem of a vertical conductor (which models armature's conductors system) inside the concrete foundation on the problem of wire electrode having equivalent length and cross-section radius placed in homogeneous ground, resulted with the procedure proposed and in details explained in [4]. The basics of this procedure (with additionally included description of the complex function theory application) are briefly presented in the text that follows.

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Firstly, using the complex function theory, the system of vertical conductors having a circular cross-section of radius r_0 and placed on the circle of radius a, can be replaced by a single wire vertical conductor of circular cross-section of radius a_C, Fig. 1 [14]. The complex function

$$\underline{w} = C_1 \ln \underline{z} + C_2 ,\ \underline{z} = r\, e^{j\theta} , \quad (1)$$

is applied on analysis of the cage system formed from N conductors placed in linear homogeneous media of electrical conductivity σ and having leakage current of density per unit length I_{leak}. Now, the complex potential is

$$\underline{w} = -(I_{leak} / 2\pi\sigma)\sum_{n=1}^{N} \ln(\underline{z} - \underline{z}_n) + C . \quad (2)$$

In previous expression are $\underline{z}_n = a\, e^{j(n-1)\alpha}$ and $\alpha = 2\pi/N$, while C is constant depending on the referent level of zero potential. Since is

$$\underline{z}^N - a^N = \prod_{n=1}^{N} (\underline{z} - \underline{z}_n) , \quad (3)$$

it obtaines

$$\underline{w} = -(I_{leak} / 2\pi\sigma)\ln(\underline{z}^N - a^N) + C . \quad (4)$$

The electrical scalar potental is

$$\phi = \mathrm{Re}(\underline{w}) = -\left[I_{leak} / (4\pi\sigma N)\right] \ln(r^{2N} + a^{2N} - 2(ra)^N \cos N\theta) + C. \quad (5)$$

Applying previous expression for determining the potential of the single conductor at the point A (Fig. 1) defined with $x = a + r_0, y = 0$, i.e. $r = a + r_0, \theta = 0$, obtains

$$U = \mathrm{Re}(\underline{w})\Bigg|_{\substack{x=a+r_0\\ y=0}} = -\left[I_{tot} / (2\pi\sigma N)\right] \ln((a + r_0)^N - R^N) + C. \quad (6)$$

In (6), with $I_{tot} = N I_{leak}$ the density per unit length of total leakage current from the cage system is labelled. Using condition $r_0 << 2a\pi/N$ one can write

$$(a + r_0)^N \approx a^N + N a^{N-1} r_0 \quad (7)$$

and consequently, the potential of the single cage conductors' system is approximately

$$U = \mathrm{Re}(\underline{w})\Bigg|_{\substack{x=a+r_0\\ y=0}} = -[I_{tot} / (2\pi\sigma N)]\ln(N a^{N-1} r_0) + C . \quad (8)$$

If instead of the cage the single cylindrical electrode of cross-section having radii a_C, and leakage current of density per unit length I_{tot} is observed, the following expression can be formed

$$U = -\left[I_{tot} / (2\pi\sigma)\right] \ln(a_C) + C. \quad (9)$$

Comparing expressions (8) and (9) for the equivalent radius of the conductors' system from Fig. 1 obtains [4]

$$a_C = a\sqrt[N]{(N r_0)/a}. \quad (10)$$

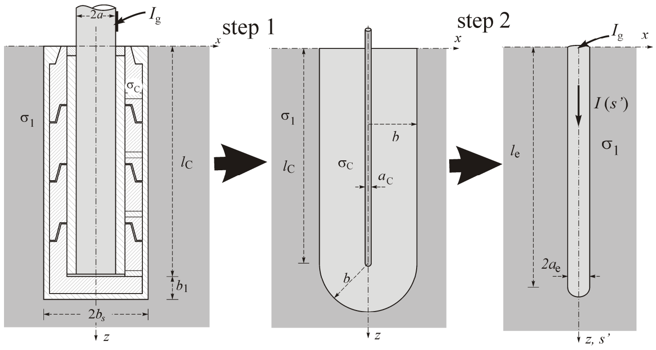

Fig. 2. Schematic illustration of modelling the vertical electrode system placed inside a concrete square cross-section foundation by an equivalent single vertical electrode placed in the homogeneous ground (Fig. 2 in [4]).

In the second step, Fig. 2, adopting that the vertical conductor has a semi-spherically shaped end, applying the equation of continuity and determining the electrical scalar potential in the surrounding ground, the resistance of the vertical electrode is obtained as ((5) in [4])

$$R_g = \frac{\ln\left(1 + \frac{l_C}{a_C}\right)}{2\pi\sigma_1 l_C} \left[\sigma_1/\sigma_C + (1 - \sigma_1/\sigma_C)\ln(1 + l_C/b)/\ln(1 + l_C/a_C)\right], (11)$$

where σ_1 and σ_C are electrical conductivities of the ground and the concrete, respectively, and the rest of parameters in (11) can be understood from Fig. 2. Applying the same procedure on a single electrode of equivalent length $l_e = K_e l_C$ and of cross-section radius $a_e = K_e a_C$, placed in a homogeneous ground of electrical conductivity σ_1, the electrode's resistance is obtained ((6) in [4])

$$R_{ge} = (1/K_e)(1/ 2\pi\sigma_1 l_C)\ln[1 + (l_C / a_C)], \quad (12)$$

and consequently, the constant K_e can be calculated using the expression ((7) in [4])

$$K_e^{-1} = \frac{\sigma_1}{\sigma_C} + \left(1 - \frac{\sigma_1}{\sigma_C}\right)\left[\ln\left(1 + \frac{l_C}{b}\right)\right]\Big/\ln\left(1 + \frac{l_C}{a_C}\right). \quad (13)$$

The general expression for calculating the electric scalar potential in the vicinity (point defined with field vector $\vec{r}$) of the single wire electrode of the length l_e from Fig. 1 is

$$\varphi(\vec{r}) = \frac{1}{4\pi\sigma_1}\frac{2I_g}{r} + \frac{1}{4\pi\sigma_1}\int_{s'=0}^{l_e} I(s')\frac{\partial}{\partial s'} K(\vec{r}, s')\, d s' . (14)$$

In previous expression, with $I(s')$ is labelled unknown longitudinal current distribution along conductor axis (s' coincide with z axis), $K(\vec{r},s') = (1/r_1) + (1/r_2)$ is the kernel, while r_1 and r_2 are distances from the conductor element, i.e. its image in the flat mirror from the point where the potential is determined, respectively. The antenna model is used and the longitudinal current is assumed in polynomial form having unknown coefficients I_m, $m = 0,1,...,M$ [15]

$$I(s') = \sum_{m=0}^{M} I_k \left(s'/l_e\right)^m . \quad (15)$$

Taking in consideration a fact that quasi-stationary regime is assumed, and after applying partial integration, the expression for calculating the electric scalar potential gets the form

$$\varphi(\vec{r}) = \frac{1}{4\pi\sigma_1} \sum_{m=0}^{M} I_m \left[\left(\frac{1}{r_1} + \frac{1}{r_2} \right) \Bigg|_{z = l_e} - \frac{m}{l_e} \int_{s'=0}^{l_e} \left(\frac{s'}{l_e} \right)^{m-1} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \mathrm{d}\, s' \right]. \quad (16)$$

Obviously, there are in total $M+1$ unknown coefficients. One equation can be obtained from the condition (Eq. (9c) in [4]):

$$\begin{cases} I(l_e) = a_e I_{leak}(l_e), \\ I_{leak}(s') = -\partial I(s')/\partial s' = -\sum_{m=1}^{M} I_m \left(m/l_e\right)\left(s'/l_e\right)^{m-1}, \end{cases} \quad (17)$$

where $I_{leak}(s')$ is leakage current density per unit length. The rest M equations are formed matching potential value (16) in the points at the conductors' surface [11]

$$\varphi(\vec{r}_n) = U, \ \vec{r}_n = s_n \hat{z}, \ s_n = m * l_e / (M+1), \ m = 1,...,M . \quad (18)$$

Since is $I(s'=0) = I_0 = I_g$, the electrode resistance is

$$R_g = U/I_g = U/I_0 . \quad (19)$$

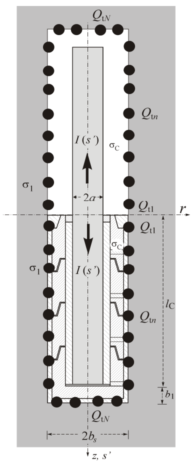

Fig. 3. Application of hybrid boundary elements method [13].

III. The Application of Hybrid Boundary Elements Method

The application of the hybrid boundary elements method (HBEM) is presented in Fig. 3. Unknown total charges distribution at the boundary surface total charges are modelled with point charges Q_{tn} $n = 1,...N$, placed at the points r_{tn} and z_{tn}, $n = 1,...N$. (In this case with $r = \sqrt{x^2 + y^2}$ is labelled radial coordinate of the cylindrical coordinate system).

The longitudinal current distribution is assumed in polynomial form (15). Potential of the system is expressed as [13]

$$\varphi(r,z) = \frac{1}{4\pi\sigma_2} \int_0^{l_C} I_{leak}(s') \left(\frac{1}{\sqrt{r^2 + (z-s')^2}} + \frac{1}{\sqrt{r^2 + (z+s')^2}} \right) \mathrm{d}\, s' + \sum_{n=1}^{N} \frac{Q_{tn}}{2\pi^2 \varepsilon_0} \left(\frac{K(\pi/2, k_{1n})}{\sqrt{(r+r_{tn})^2 + (z - z_{tn})^2}} + \frac{K(\pi/2, k_{2n})}{\sqrt{(r+r_{tn})^2 + (z + z_{tn})^2}} \right), \quad (20)$$

where are:

$$k_{1n}^2 = \frac{4 r r_{tn}}{(r + r_{tn})^2 + (z - z_{tn})^2}, \quad (21)$$

$$k_{2n}^2 = \frac{4 r r_{tn}}{(r + r_{tn})^2 + (z + z_{tn})^2} \quad (22)$$

and

$$K(\pi/2, k) = \int_0^{\pi/2} \left(1 - k^2 \sin^2 \alpha\right)^{-1/2} \mathrm{d}\,\alpha \quad (23)$$

is a complete elliptic integral of the first kind.

The system of charges and currents in Fig. 3 and potential expression (20) is formed using image theory in the flat mirror and symmetrical features of the structure from Fig. 3. Total M equations (the number of unknown coefficients in leakage current distribution (17)) are obtained matching potential value at the conductors' surface, and N equations (total number of point charges) is formed based on the relation between normal electric field component and surface charge density [13]. Since feeding current can be obtained as

$$I_g = \int_{l_C} I_{leak}(s')\,\mathrm{d}\, s' . \quad (24)$$

The electrode resistance can be calculated as

$$R_g = 1/G_g = U/I_g . \quad (25)$$

IV. Numerical Results

Based on the described models and procedures, the single resistance of wire electrode having length $l_C = 1$ m and cross-section radius $a_C = 1$ m placed in parallelepiped concrete cylinder of electrical conductivity σ_C and surrounded by ground of electrical conductivity $\sigma_1 = 0.001\text{S/m}$ is determined. The other parameters values are $b_s = b_1 = 0.3\,\text{m}$. The results obtained with procedure of approximating wire conductor inside parallelepiped (AWCP) are compared with those ones available from [13], which are calculated applying HBEM. Parameter M in (15) is M=6. Normalized values of the conductance obtained using both described procedures are presented in Table I. It can be noticed that matching between results obtained using described procedures increases with decreasing of electrical conductivity of parallelepiped. The graphs obtained based on the values from Table I are shown in Fig. 4.

TABLE I. NORMALIZED CONDUCTANCE OF THE WIRE ELECTRODE INSIDE PARALLELEPIPED.

σ_C(S/m)	$G_g/(4\pi\sigma_1 l_C)$	
	AWCP	HBEM
$5\bullet10^{-2}$	0.2420	0.3798
10^{-2}	0.2229	0.2864
$5\bullet10^{-3}$	0.2028	0.2412
10^{-3}	0.1180	0.1172
$5\bullet10^{-4}$	0.0774	0.0726
10^{-4}	0.0207	0.0181

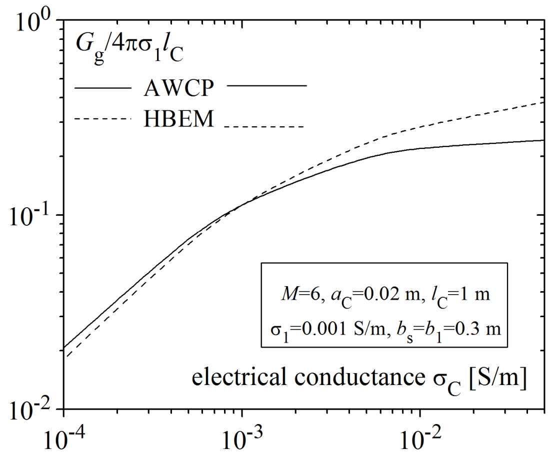

Fig. 4. Normalized values of the conductance of the single wire electrode inside parallelepiped obtained using AWCP and HBEM (see Table I).

V. CONCLUSIONS

The resistance of the single wire electrode inside parallelepipedically-shaped domain surrounded with homogeneous ground is determined in the paper. The results obtained applying one recently proposed procedure for approximating wire electrode inside parallelepiped with one equivalent single wire electrode placed in homogeneous ground are presented. In this way obtained values for the resistance of the observed system are compared with recently published results, realized applying hybrid boundary elements method. Agreement of the results improves with decreasing electrical conductivity (i.e. increasing electrical resistivity) of the parallelepipedically-shaped domain. The procedure of approximating wire conductor inside parallelepiped is more "engineering" oriented and simpler for application. Since this kind of a problem is very realistic model of the armature conductors inside concrete pillar foundation, one can conclude that for large values of the concretes' electrical resistivity, the procedure based on application of HBEM gives sufficiently correct results for the resistance of the observed structure.

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