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Simulation of GNSS Signal with Noise and Distance, Speed Probability Distribution Calculation

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Introduction

Calibration or verification of measurement equipment needs standard measurement which must be directly traceable, have sufficient precision and accuracy. GNSS systems use electromagnetic signals and timing synchronized with atomic oscillator. It makes GNSS signals object for time, frequency measurements through which distance or pseudorange and speed can be derived. Main problem with GNSS usage for measurements is variable precision and accuracy. This problem can be addressed by using statistics on measurement data after signal post-processing. It is suitable for stationary measurements, when measurement time is not strictly limited, conditions do not change rapidly and sufficient statistical data can be gathered. Dynamic measurements may not have sufficient or reliable enough statistical data for measurement precision estimation. Then measurement precision must be downgraded for the worst possible measurement or additional data from GNSS signal must be used.

Article discusses possibility to use noise parameters from GNSS signal to evaluate precision of time delay and frequency measurement. It is done by calculating demodulated signal, its maxima and maxima distribution. It allows measurement uncertainty calculation for all satellites independently.

Method used for signal simulation is using signal calculation in frequency, delay domain. It allows faster calculation and has lower memory requirement than direct signal generation and demodulation.

Signal that was obtained is used for delay and frequency distribution calculation using additive noise. Satellite communications can use Rayleigh, Rice, lognormal, Nakagami noise or fading models. Calculations in this article are using noise with Rice distribution.

GNSS signal simulation

GNSS signal demodulation for GPS and GALILEO is discussed in detail by Borre et al. [1]. The same authors give demodulation formula for GPS which can be written as Fourier transform of demodulated signal

$$U(\omega,t)=\frac{2}{t_s}\int_0^{t_s}u(t_x)C(\omega,t)e^{-i\omega t_x}\,dt_x\ , \qquad (1)$$

where C is modulating code (C/A code for GPS), with values 1 or -1; u is signal, t_s is signal length. It represents phase-shift-keying (phase shift 180°). Formula is written so, that modulated sine signal amplitude before and after demodulation match. Data transmission is not taken into account as it has no significant effects in areas covered by this article. Data transmission preferably has to be added in some applications. Demodulation formula can be written for discrete signal with n samples

$$U(\omega,t)=\frac{2}{n}\sum_{k=1}^{n}u_kC(\omega,t)e^{-i\omega\frac{k-1}{f_s}}\ . \qquad (2)$$

Demodulation is closely related to the Fourier transform of signal. The only difference is signal delay, which is dependant on frequency, but can have initial delay. It makes demodulated signal 2D signal. Frequency is directly related to velocity as phase is related to time delay or distance or pseudorange so later these related terms are used to describe the same signal feature. Demodulation can be represented (Fig. 1).

Signal demodulation is done by trying different frequencies, different delays and searching for maximal amplitude (Fig. 2).

Signal can be split into variable length sections (Fig. 2.). Demodulated signal can be calculated directly from them. Fourier transform of signal section with constant frequency and time delay is

$$U = Cu\left[\frac{\sin(\pi(t_2 - t_1)(f + f_0))}{\pi(f + f_0)} i \cdot e^{-i\pi(t_1+t_2)(f+f_0)} - \frac{\sin(\pi(t_2 - t_1)(f - f_0))}{\pi(f - f_0)} i \cdot e^{-i\pi(t_1+t_2)(f-f_0)}\right], \quad (3)$$

where u is signal amplitude, C is modulating code, t_1 and t_2 time (Fig. 2), f_0 is satellite frequency, f receiver or demodulated frequency. Special case is when $f=f_0$, frequency of signal and modulating code match, then this formula becomes

$$U = Cu\left[\frac{\sin(2\pi f_0(t_2 - t_1))}{2\pi f_0} i \cdot e^{-i2\pi f_0(t_1+t_2)} - i(t_2 - t_1)\right]. \quad (4)$$

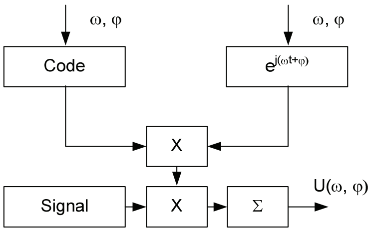

Fig. 1. Signal demodulation

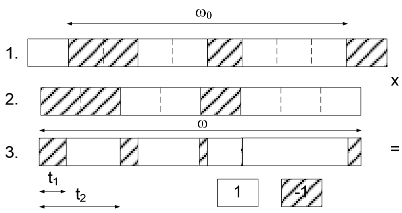

Fig. 2. Signal demodulation (1 – signal code, 2 – demodulating code C, 3 – demodulated signal code)

Demodulated signal then is a sum of spectra of all sections

$$U_\Sigma = \frac{1}{T}\sum_{i=1}^{N} U_i . \quad (5)$$

Signal after demodulation using this simulation method can be represented (Fig. 3). This method allows calculation of demodulated signal in frequency-delay domain without the need of Fourier transform and signal in time domain. This decreases mathematical operation count and memory requirements. It can also be applied in multipath interference correction or any other signal decomposition where signal reconstruction is needed.

Frequency changes are represented as speed. Relativistic Doppler Effect for speed is used

$$v = c\frac{(f_0/f)^2 - 1}{(f_0/f)^2 + 1}, \quad (6)$$

where c is speed of light in vacuum. When satellite movement in vacuum and receiver movement in air must be accounted for, (6) formula must be modified. Time delay is represented as distance

$$x = tc/n, \quad (7)$$

where t is time delay, n is refraction index for electromagnetic waves in air (1.0003 is used).

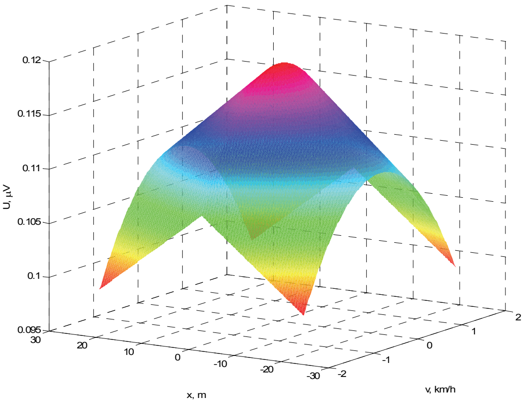

Fig. 3. Peak of simulated GPS signal (100ms)

In calculation 100ms signal (Fig. 3) is used. Signal level is defined as a lowest possible (-158.5dBW). Signal simulation in delay, frequency representation has advantages in speed of calculation and can be used for signal evaluation, multi-path interference correction, signal decomposition, etc.

Method can be used when frequency is constant and is not changing rapidly. In case of variable frequency, this method can be adjusted either using more sections with different frequency or using chirp interpolation. Interpolation using chirp sections may not have the same advantages as constant frequency calculation in speed of calculation.

GPS Signal with noise

Signal is modeled as a signal with additive noise. Noise is close to Gaussian distribution and is influenced by surroundings, atmosphere. More details are provided by Milosevic et al. [2]. It can be measured using direct calculation, envelope determination. Possible envelope methods are discussed in more detail by Augutis et al. [3]. After Fourier transformation or demodulation, amplitude distribution becomes well known Rice distribution. Distribution is derived and discussed in more detail by Hamish D. Meikle [4]. Signal noise cumulative distribution can be represented

$$F_i(U) = 1 - Q_1\left(\frac{S_i}{\sigma_i}, \frac{U}{\sigma_i}\right), \quad (8)$$

where S is signal amplitude, σ_i is noise standard deviation. Marcum Q function is defined as

$$Q_m(a,b) = \frac{1}{a^{m-1}}\int_b^\infty x^m e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax)dx, \quad (9)$$

where I is modified Bessel function.

Noise level after demodulation changes and standard deviation of demodulated signal is:

$$\sigma = \sigma_1 \sqrt{\frac{2}{n}}, \tag{10}$$

$$\sigma = \sigma_1 \sqrt{\frac{2}{fs \cdot t_s}}, \tag{11}$$

where n is sample number; f_s sampling time; t_s is demodulated signal length; σ_1 is initial noise standard deviation.

GNSS receivers can use several peak points for faster maxima finding. Method for calculating maxima measurement distribution uses only one maximal value. This method can only be applied if noise at each frequency and delay sample is statistically unrelated. Inappropriate filtering can make noise at close samples related and in such case this method must be modified.

When there is known sampled signal with noise, distribution of which is known, it is possible to calculate maxima distribution. Probability that amplitude r is below certain level R in multiple n samples can be calculated using

$$P(r < R) = \prod_{i=1}^{n} F_i(R). \tag{12}$$

It is possible to calculate distribution of signal maximal value. In continuous case $n \to \infty$.

Probability that value of first sample (maxima or position of maximal value) is more than limit value R_1 and all other samples are below this limit can be found using

$$P_1 = (1 - F_1(R_1)) \prod_{i=2}^{n} F_i(R_1). \tag{13}$$

It can be represented graphically (Fig. 4).

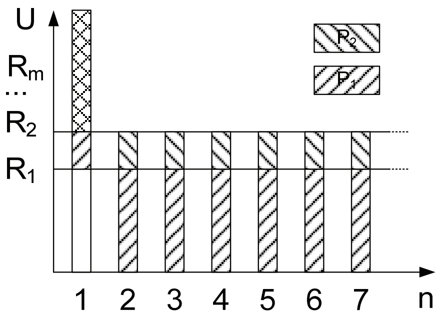

Fig. 4. Maxima distribution calculation

To find that maxima is above limit R_2 and all other are below this limit and exclude probability covered by P_1

$$P_2 = (1 - F_1(R_2)) \cdot \left(\prod_{i=2}^{n} F_i(R_2) - \prod_{i=2}^{n} F_i(R_1) \right). \tag{14}$$

Combining both probabilities can have

$$p_k = (1 - F_k(R_1)) \prod_{\substack{i=1 \\ i \neq k}}^{n} F_i(R_1) + \sum_{j=2}^{m} \left[(1 - F_k(R_j)) \cdot \left(\prod_{\substack{i=1 \\ i \neq k}}^{n} F_i(R_j) - \prod_{\substack{i=1 \\ i \neq k}}^{n} F_i(R_{j-1}) \right) \right], \tag{15}$$

where n is sample number and m is amplitude number used in calculation, R are amplitude values and k is sample that is used for calculation.

In calculation previous signal (Fig. 3) is used. Signal has additive (thermal, etc.) noise. Results when this method of calculation is applied to 2D signal are provided in (Fig. 5).

It can be seen that both frequency and delay can be used for measurement. To evaluate possibility of measurements using delay probabilities, 2D distribution is summed across all frequencies (Fig. 6).

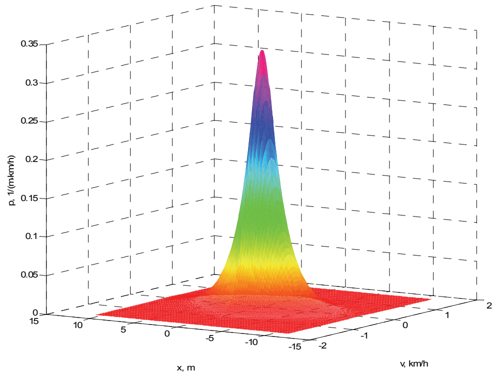

Fig. 5. Maxima distribution

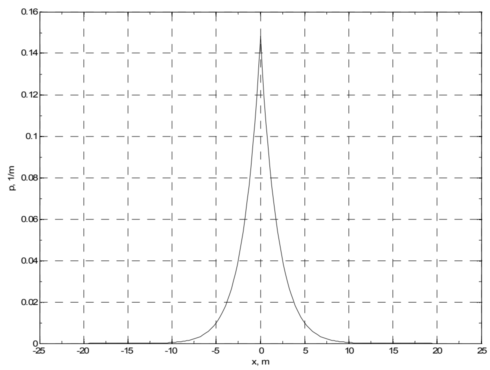

Fig. 6. Distance distribution

It can be seen that delay distribution is non Gaussian. Uncertainty (p=0.95) is calculated using directly this distribution. When noise is present, relationship between signal noise ratio (S/N) and distance or pseudorange measurement uncertainty is in (Fig. 7).

Uncertainty has slightly uneven line that is comparable to calculation error. GNSS measurement uncertainty calculations using entropy and tolerances are provided by Skeivalas [5] where Gaussian distribution is used. Maxima distribution calculation method provides a way to improve uncertainty estimation using real measurement data and distribution derived from it. To evaluate possibility of measurements using frequency, 2D distribution is summed across all delays (Fig. 8).

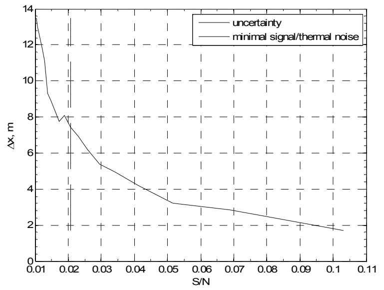

Fig. 7. Signal noise (S/N) ratio and distance uncertainty relationship

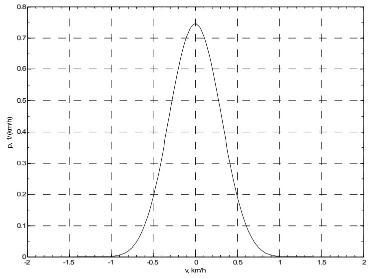

Fig. 8. Frequency or speed distribution

It can be seen, that probability is close to Gaussian (R^2=0.999896). The same calculation is done to get signal noise ratio and speed measurement uncertainty relationship (Fig. 9).

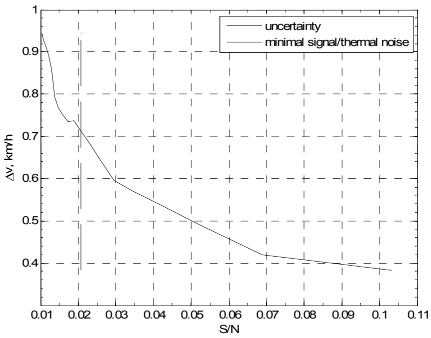

Fig. 9. Signal noise (S/N) ratio and speed uncertainty relationship

Modeling or simulation of signal using noise parameters and signal level from measurement can reveal both measurement distribution and uncertainty. It can be done during measurement and it makes distributions available for all satellites.

Conclusions

Method used for signal simulation allows fast demodulated signal which is in frequency-delay domain calculation and does not need signal in time domain, lowering both mathematical operation count and memory requirements. It can also be applied in multipath interference correction or any other signal decomposition.

Method allowing maximal value distribution calculation from signal with known noise level is presented. It is used for speed and distance or pseudorange distribution calculation. Relationships between signal noise level and uncertainty can be calculated and used for fast uncertainty estimation.

Uncertainty and distributions of distance or pseudorange and speed can be obtained for each satellite separately and later used for calculations.

Calculated distance distribution is non Gaussian. Statistics using distance measurements should account for distribution shape as usage of standard Gaussian statistics may downgrade measurement precision.

Speed distribution is close to Gaussian. It makes Gaussian statistics applicable.

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Calibration or verification of measurement equipment needs standard measurement which must have sufficient precision and accuracy. GNSS usually has variable precision and accuracy with each measurement. In this article signal generation directly in frequency-delay representation is presented which can be used for decomposition or multipath interference correction. Demodulated signal maximal value distributions for frequency and delay are found from signal and known noise. These distributions allow to calculate measurement uncertainties for each satellite separately. Ill. 9, bibl. 5 (in English; abstracts in English and Lithuanian).

J. Barzdziukas. Palydovinės navigacijos signalų modeliavimas su triukšmais ir atstumo, greičio tikimybės skirstinių skaičiavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 6(122). – P. 133–136.

Matavimo įrenginiams kalibruoti ir tikrinti reikalingi matavimai turi pasižymėti nedidele paklaida bei sklaida. Palydovinių navigacijos sistemų paklaida ir sklaida dažniausiai kinta su kiekvienu matavimu. Šiame straipsnyje pateikiamas signalo generavimas tiesiogiai dažninėje ir vėlinimo išraiškoje ir gali būti naudojamas dekompozicijai ar daugelio kelių signalo interferencijos korekcijai. Randami demoduliuoto signalo maksimumo dažnio ir vėlinimo skirstiniai pagal signalą ir žinomą triukšmą. Gauti skirstiniai leidžia apskaičiuoti matuojamų dydžių neapibrėžtis kiekvienam palydovui atskirai. Il. 9, bibl. 5 (anglų kalba; santraukos anglų ir lietuvių k.).