

Comparative Analysis of (HF) Non-linear Circuits Modelled by Different Environments

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Abstract—Many electric circuits feature some type of non-linearity of their used devices. Non-linear resistors or inductors could be typical examples. Also, all semiconductor devices are in their nature non-linear ones. From the point of view of the circuit solution they are presented non-linear static and dynamical system described by differential equations. Particularly, in steady state, it could be system of non-linear algebraic equations.

Index Terms—Fictitious function, state variables, non-linear dependency.

I. INTRODUCTION

Many electric circuits feature some type of non-linearity of their used devices. Non-linear resistors or inductors could be typical examples. Also, all semiconductor devices are in their nature non-linear ones. From the point of view of the circuit solution they are presented by non-linear static and dynamical system described by differential equations. Particularly, in steady state, it could be system of non-linear algebraic equations.

II. NON-LINEAR STATIC SYSTEMS

Let's assume electric circuit in Fig. 1 considering steady state when capacitor current i_{CD} is equal zero.

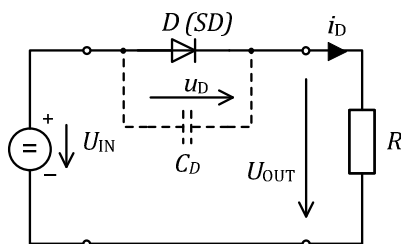


Fig. 1. Electric circuit with serial rectifier diode with non-linear static I - V characteristic.

General solution of the non-linear static system described in compact form as

$$\mathbf{x} = \mathbf{F}(\mathbf{x}). \quad (1)$$

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It is possible using iterative methods. Then one-step stationary iterative method yields

$$\mathbf{x}^{k+1} = \mathbf{F}(\mathbf{x}^k). \quad (2)$$

Numerical solution of non-linear algebraic equation system can be done by Newton-Raphson method using two members of *Taylor* expansion and *Jacobi* matrix

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}^{(0)}) + \mathbf{f}'(\mathbf{x}^{(0)}) \cdot (\mathbf{x} - \mathbf{x}^{(0)}) + \dots, \quad (3)$$

where $\mathbf{f}'(\mathbf{x})$ is *Jacobian* matrix of n -dimensional function at point $\mathbf{x}^{(0)}$ [1], [2], [5]

$$\mathbf{f}'(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_n}{\partial x_1} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}. \quad (4)$$

The simplest method of circuit solution is graphical one. According to electrical scheme in Fig. 1 and assuming diode can be modelled as a voltage controlled current source in the forward characteristics one can write

$$i_D = I_S [\exp(\theta \cdot u_D) - 1], \quad (5)$$

where $\theta = \frac{q_e}{MK\theta} \left[\frac{1}{V} \right]$, q_e – charge of electron $1.59 \cdot 10^{-19}$ [C]; M – idealist or emission factor (typically varies from 1.02 to 1.6); K – Boltzmann constant $1.38 \cdot 10^{-23}$ [J/K]; θ – PN junction temperature [K]; I_S – saturation current in reverse direction and u_D – voltage of diode.

Applying *Kirchhoff* law we obtain

$$RI_S [\exp(\theta u_D) - 1] + u_D - U = 0, \quad (6)$$

where $f(u_D) = U - RI_S [\exp(\theta u_D) - 1]$ and for one-step stationary iterative method

$$u_D^{k+1} = U - RI_S [\exp(\theta u_D^k) - 1], \quad (7)$$

where R – resistance of load resistor and U – input voltage of direct source ($= U_{IN}$).

The resulting solution for u_D quantity under steady-state condition is depicted in Fig. 2.

The Schottky diode model can be described [3]

$$i_{SD} = I_S \{ \exp[\theta (V_{bi} - IR_S)] - 1 \}, \quad (8)$$

where I_S - the saturation current; R_S - the series resistance; V_{bi} - the built in potential.

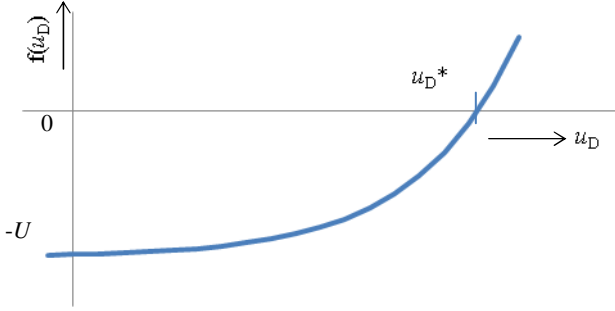


Fig. 2. Solution for u_D^* quantity determination under steady-state condition.

Note: By similar way circuit with *SiC PIN* diode, based on [1] can be modelled:

$$i_{SiCD} = I_S \cdot \exp(-\theta V_{bi}) [\exp(\theta u_D) - 1] \quad (9)$$

or

$$i_{SiCD} = I_S [(v - V_{bi})/k]^{1/(a+1)}, k = k_0/(a + 1). \quad (10)$$

III. NON-LINEAR DYNAMIC SYSTEMS

Many applications in technical practice which models are presented in following text should be described as non-linear differential equation (DE) systems. In compact form:

$$\begin{cases} \mathbf{F}[\dot{\mathbf{x}}(\mathbf{t}), \mathbf{x}(\mathbf{t}), \mathbf{t}] = \mathbf{0}, \\ \mathbf{y}(\mathbf{t}) = \mathbf{g}[\mathbf{x}(\mathbf{t}), \mathbf{t}]. \end{cases} \quad (11)$$

Such a system of ODE can be solved analytically and/or also numerically (e.g. by Euler explicit method) [1], [2]. If the matrix elements are non-stationary (e.g. time dependent ones) then system of equations cannot be solved by the methods using the matrix operation. As a solver for their solution the following numerical methods can serve, completed by fictitious exciting functions method making possible numerical solution of this DE system with non-stationary matrices [1], [2], [5]. It deals with Euler's- and Taylor expansion methods for consequent numerical solution in Matlab environment.

Simple electrical circuit given in Fig. 3 comprises a non-linear resistor (e.g. varistor – resistor depending on voltage, thermistor – resistor depending on temperature). In this case it is R^{non} with non-linear dependency on its current i_R

$$R^{non}(i_R) = R \cdot \exp(-i_R/K) \quad (12)$$

and consequently its voltage will be $u_R = R^{non}(i_R) i_R$.

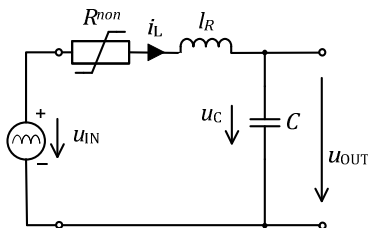


Fig. 3. Dynamical system with non-linear resistor R^{non} and its parasitic inductance l_R and serial capacitor C .

The following differential equations can be written for quantities of this simple electrical circuit

$$\begin{cases} \frac{di_L}{dt} = -\frac{R^{non}}{l_R} i_L - \frac{1}{l_R} u_C + \frac{1}{l_R} u_{IN}, \\ \frac{du_C}{dt} = \frac{1}{C} i_L. \end{cases} \quad (13)$$

Using fictitious exciting functions method and adapting system equations into matrix form yields

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} i_L \\ u_C \end{pmatrix} &= \begin{pmatrix} 0 & -1/l_R \\ 1/C & 0 \end{pmatrix} \begin{pmatrix} i_L \\ u_C \end{pmatrix} + \\ &+ \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_{IN}/l_R \\ R^{non} i_L/l_R \end{pmatrix} \end{aligned} \quad (14)$$

and it in discrete form using Euler explicit formula

$$\begin{aligned} \begin{pmatrix} i_L \\ u_C \end{pmatrix}_{n+1} &= \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + h \begin{pmatrix} 0 & -1/l_R \\ 1/C & 0 \end{pmatrix} \right] \begin{pmatrix} i_L \\ u_C \end{pmatrix}_n + \\ &+ h \begin{pmatrix} 1 & -r_L \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_{IN}/l_R \\ R^{non} i_L/l_R \end{pmatrix}_n. \end{aligned} \quad (15)$$

where h – integration step.

Another example of R - L - C circuit, with non-linear inductor L^{non} , is shown in Fig. 4.

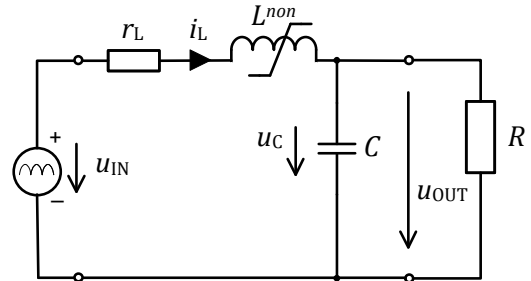


Fig. 4. Dynamical system with non-linear inductor L^{non} and its parasitic resistance r_L .

The following differential equations can be written

$$\begin{cases} \frac{di_L}{dt} = -\frac{r_L}{L^{non}} i_L - \frac{1}{L^{non}} u_C + \frac{1}{L^{non}} u_{IN}, \\ \frac{du_C}{dt} = \frac{1}{C} i_L - \frac{1}{RC} u_C. \end{cases} \quad (16)$$

The system equations into matrix discrete form using Euler explicit formula will be

$$\begin{aligned} \begin{pmatrix} i_L \\ u_C \end{pmatrix}_{n+1} &= \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + h \begin{pmatrix} 0 & 0 \\ 1/C & -1/RC \end{pmatrix} \right] \begin{pmatrix} i_L \\ u_C \end{pmatrix}_n + \\ &+ h \begin{pmatrix} 1 & -r_L & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_{IN}/L^{non} \\ i_L/L^{non} \\ u_C/L^{non} \end{pmatrix}_n \end{aligned} \quad (17)$$

The non-linear dynamical system with serial rectifier diode is presented in Fig. 5.

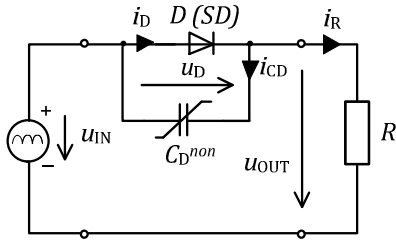


Fig. 5. Dynamical system with serial rectifier diode D with respect of its non-linear dynamical model.

Dynamical model of diode is presented by Q - V characteristic and non-linear capacitor C_D^{non} model as follow:

$$q_D = q_0 [\exp(\theta u_D) - 1], \quad (18)$$

$$C_D^{non} = C_{D0} \exp(\theta u_D), \quad (19)$$

where $C_{D0} = q_0 \theta$ and q – charge of capacitor (C)

Applying Kirchhoff law for resistor current i_R , diode current i_D and capacitor current i_{CD} we obtain

$$i_{CD} = C_{D0} \frac{du_D}{dt} = i_R - i_D, \quad (20)$$

thus the resulting differential equation will be

$$\frac{du_D}{dt} = \frac{u_{IN} - u_D}{R[C_{D0} \exp(\theta u_D)]} - I_S \frac{[\exp(\theta u_D) - 1]}{[C_{D0} \exp(\theta u_D)]} \quad (21)$$

Rather complex electrical circuit with serial rectifier diode is shown in Fig. 6.

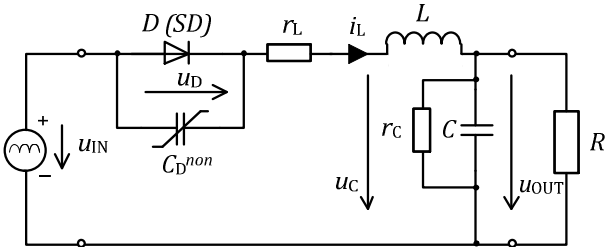


Fig. 6. Output part of SMPS (DC/DC) converter with isolating HF transformer and non-linear rectifier diode and inductor.

Based on above given approach the non-linear model of the circuit can be created and in discrete form using Euler explicit formula with $R_T = (R + r_T)$ we have

$$\begin{pmatrix} i_L \\ u_C \\ u_D \end{pmatrix}_{n+1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + h \begin{bmatrix} -r_L/L & -1/L & 0 \\ 1/C & 1/R_T C & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} i_L \\ u_C \\ u_D \end{pmatrix}_n + h \begin{pmatrix} 1/L & -1/L & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} u_{IN} \\ u_{CD} \\ 1/C_D^{non} \cdot i_L \\ 1/C_D^{non} \cdot i_D \end{pmatrix}_n. \quad (22)$$

IV. SIMULATION AND EXPERIMENTAL VERIFICATION

Simulation experiments have been done with non-linear inductor which inductance depends on its current. It has been measured using static biased method with bifilar windings of the inductor. One of them serves for providing

of desired magnetic field strength and the other for precise inductance measuring, Fig. 7.

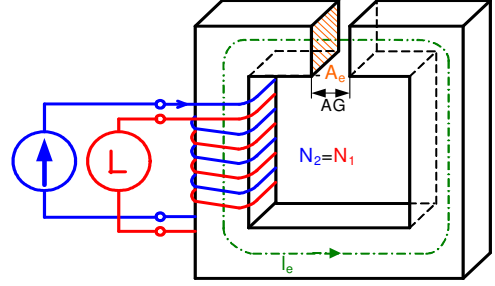


Fig. 7. Static biased measuring of non-linear inductance.

Parameters of the measured inductor [10]: $L = 32.5 \mu\text{H}$; $S_{Fe} = 31 \text{ cm}^2$; $N_1 = N_2 = 6.8$; magnetic material 3F3; air gap $\delta = 28.5 \mu\text{m}$; $L_{sat} = 0$.

Values of the inductance L^{non} have been calculated from measured data as follow

$$L_k^{non} = \frac{u_{L(k)} - u_{L(k-1)}}{i_{L(k)} - i_{L(k-1)}} \Delta t. \quad (23)$$

Calculated $L - i_L$ characteristic from measured data is given in graphic form in Fig. 8(a), the pre-calculated and simulated by different non-linear models [5] from data sheet in Fig. 8(b).

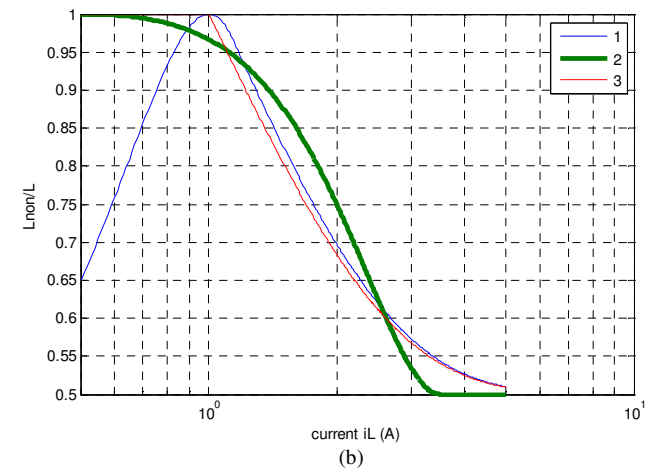
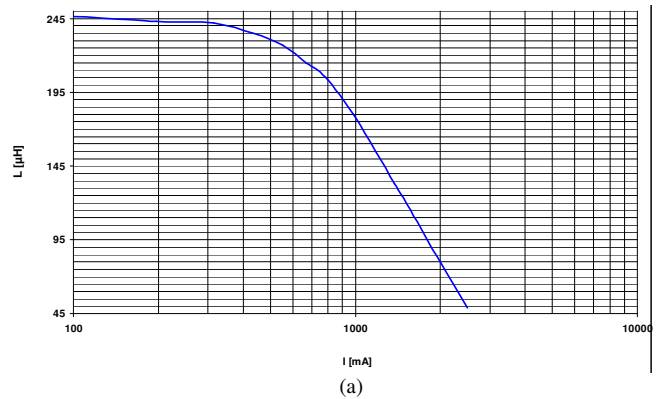


Fig. 8. Non-linear dependency of the inductor inductance - measured (a) and pre-calculated by different models (b).

Simulation results for the electric circuit given by Fig. 4 are shown using both Matlab and OrCAD-PSpice programming environment in Fig. 9.

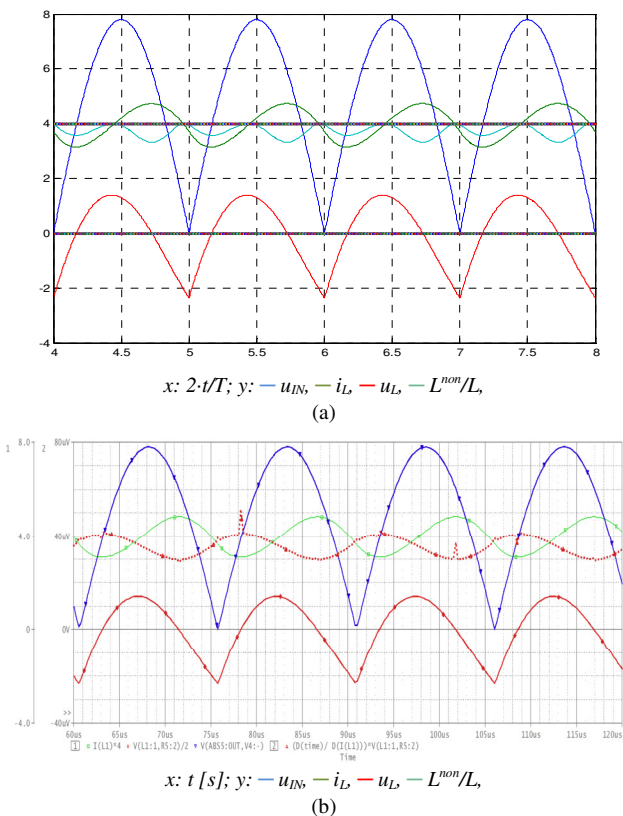


Fig. 9. Simulated quantities of the circuit in Fig. 4 under rectified AC supply and MatLab (a); simulated quantities of the circuit in Fig. 4 under rectified AC supply and PSpice environment (b).

Parameters of simulations: $L = 32.5 \mu\text{H}$; $C = 45 \text{ nF}$; $f = 132 \text{ kHz}$; $u_{IN} = 6 \text{ V} \sin(\omega t)$, $r_L = 0.001\omega L$; $r_C = 1000R$; $R = 33 \Omega$.

V. CONCLUSIONS

The non-linear static and dynamical systems described by differential equations were presented with non-linear components. Particularly, in steady state, it could be system of non-linear algebraic equations using fictitious exciting functions method. The analysis of electrical circuit with real non-linear inductor was presented provided by different modelling environments – Matlab and PSpice. The inductance of inductor has been measured and next used for simulation. Simulation results of both modelling environments have proved very good agreement. The comparison of measured proved simulation results will be given in future work.

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