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Performance Analysis of Amplify-and-Forward Relay System in Hoyt Fading Channels with Interference at Relay

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Introduction

Diversity at the receiver is a well-known promising avenue for improving mean signal strength and reducing signal level fluctuations in fading channels. Performance improvement using diversity reception is considered in [1, 2, 5, 6, 13–15]. The effect of cooperative diversity on the system performance is analyzed in [3, 4] and [16]. In paper [7] diversity techniques operating over shadowed fading channel was presented, but influence of interference was not analyzed.

Relay communications as a means to improve the range and link reliability has recently rekindled enormous interest in the context of user-cooperative communications. Relay processing can be classified as either amplify-and-forward (AF) or decode-and-forward (DF). There are two types of AF relaying schemes considering two different power constraints at the relay: fixed-gain and channel state information (CSI)-based.

Hoyt distribution is commonly used to describe the short-term signal variation of certain wireless communication systems subject to fading [9] and that distribution is normally observed on satellite links subject to strong ionospheric scintillation. Specifically, the Hoyt channel model has been applied in satellite-based cellular communications to characterize more severe fading conditions than those modelled by Rayleigh [9]. Although considerable attention has been paid to outage probability analysis, few published results for Hoyt fading channels are found in the literature, mainly due to reasons of mathematical tractability. Recently, exact closed-form results for the outage probability of interferencefree Hoyt fading channels were published in [10]. It is shown that this model is applicable for describing the statistics of the fading envelope of real-world mobile radio channels.

Few works that have studied the impact of interference on the AF and DF relaying performance have assumed interference either at the relay(s) or the destination(s). Nevertheless, co-channel interference (CCI) is an important issue. Consideration of CCI is necessary because of the aggressive reuse of frequency channels for high spectrum utilization in cellular systems. It has a very long history for investigating the performances of wireless systems in the presence of CCI. It was shown in [8] that the interference can cause a severe performance degradation. Recently, in [11] authors studied the outage probability and the average bit error rate (BER) of the CSI-assisted AF protocol with interference at the relay in Rayleigh fading channel.

Performance analysis of a amplify and forward relay system, with co-channel interference at relay in Hoyt fading environment is presented in this paper. We will consider a single interferer scenario at relay and derive the outage probability and the average BER expressions. We will consider 4-QAM modulation format, but the results may be easily extended for any other modulation scheme.

System Model

In this paper we consider a communication between source *S* and destination *D* using relay *R* where *S* does not have a direct link to *D* [11]. All nodes are equipped with a single antenna. The communication in the system is divided into two orthogonal time intervals. In the first time interval, *S* sends its symbol s_0 to *R* which is supposed to operate in an interference limited environment. Received signal, in the presence of single interference at relay *R*, can be written as

$$y_r = \sqrt{P_s}\, h_{sr} s_0 + \sqrt{P_j}\, h_j s_j + n_r , \tag{1}$$

where h_{sr} is the complex channel between S and R with average fading power Ω_{sr}, P_s is the transmit power, h_j is the channel from interference to R with average fading power Ω_j independent of h_{sr}, P_j is interference average power and n_r is the AWGN at R with variance σ_r^2. All links are assumed to be subject to Hoyt fading. Transmitted symbols s_0 and interfering symbols s_j are assumed to have zero mean and unit variance.

Relay R amplifies signal y_r with gain G which is, in the presence of interference, equal to [8]

$$G = \sqrt{\frac{P_r}{P_s \mid h_{sr} \mid^2 + P_j \mid h_j \mid^2 + \sigma_r^2}} \,. \tag{2}$$

In the second time interval R forwards y_r to D. The received signal at D is

$$y_d = h_{rd} G \left(\sqrt{P_s} h_{sr} s_0 + \sqrt{P_j} h_j s_j + n_r \right) + n_d \,, \tag{3}$$

where h_{rd} is the complex channel between R and D with average fading power Ω_{rd}, independent of h_{sr} and h_j , and n_d is the AWGN at D with variance σ_d^2.

Signal-to-interference plus noise ratio (SINR) of the decision variable can be written as

$$\gamma_{eq} = \frac{P_s \mid h_{sr} \mid^2 \mid h_{rd} \mid^2}{\mid h_{rd} \mid^2 \left(P_j \mid h_j \mid^2 + \sigma_r^2 \right) + \sigma_d^2 / G^2} \,. \tag{4}$$

Since R is interference limit (the effect of n_r is negligible), Eq. (4) becomes [11]

$$\gamma_{eq1} = \frac{\gamma_1 \gamma_2}{\gamma_{INF} (\gamma_2 + 1) + \gamma_1} \,, \tag{5}$$

where $\gamma_1 = P_s \mid h_{sr} \mid^2$, $\gamma_2 = \frac{P_r}{\sigma_d^2} \mid h_{rd} \mid^2$, and $\gamma_{INF} = P_j \mid h_j \mid^2$.

Performance analysis

In this section we analyze important system performance measures such as the outage probability and average BER.

The outage probability, P_{out} is defined as the probability that γ_{eq1} drops below an acceptable threshold γ_{th}

$$P_{out} = \Pr(\gamma_{eq1} < \gamma_{th}) = F_{\gamma_{eq1}}(\gamma_{th}) \,, \tag{6}$$

where $\Pr(\cdot)$ denotes probability and $F_{\gamma_{eq1}}(x)$ is the cumulative distribution function (cdf) of γ_{eq1} which is [11]

$$\begin{aligned} F_{\gamma_{eq1}}(\gamma_{th}) = 1 - \int_0^\infty \int_0^\infty \Pr\left(\gamma_1 > \frac{\gamma_{th}(w + \gamma_{th} + 1) z}{w} \right) \times \\ \times f_{\gamma_2}(w + \gamma_{th}) f_{\gamma_{INF}}(z) dw dz, \end{aligned} \tag{7}$$

where $f_{\gamma_2}(x)$ and $f_{\gamma_{INF}}(x)$ are probability density functions (pdf) of γ_2 and γ_{INF}, respectively. Since h_{sr}, h_{rd}, and h_j are Hoyt random variables, γ_1, γ_2 and γ_{INF} are random variables with the following pdfs:

$$f_{\gamma_1}(x) = \frac{(1 + q_1^2)}{2 q_1 \bar{\gamma}_1} e^{-\frac{(1+q_1^2)^2 \cdot x}{4 q_1^2 \bar{\gamma}_1}} I_0 \left(\frac{(1 - q_1^4) \cdot x}{4 q_1^2 \bar{\gamma}_1} \right), \tag{8}$$

$$f_{\gamma_2}(x) = \frac{(1 + q_2^2)}{2 q_2 \bar{\gamma}_2} e^{-\frac{(1+q_2^2)^2 \cdot x}{4 q_2^2 \bar{\gamma}_2}} I_0 \left(\frac{(1 - q_2^4) \cdot x}{4 q_2^2 \bar{\gamma}_2} \right), \tag{9}$$

$$f_{\gamma_{INF}}(x) = \frac{(1 + q_{INF}^2)}{2 q_{INF} \bar{\gamma}_{INF}} e^{-\frac{(1+q_{INF}^2)^2 \cdot x}{4 q_{INF}^2 \bar{\gamma}_{INF}}} I_0 \left(\frac{(1 - q_{INF}^4) \cdot x}{4 q_{INF}^2 \bar{\gamma}_{INF}} \right), \tag{10}$$

where $\bar{\gamma}_1 = P_s \Omega_{sr}$, $\bar{\gamma}_2 = P_r \Omega_{rd} / \sigma_d^2$ and $\bar{\gamma}_{INF} = P_j \Omega_j$, while q_1, q_2 and q_{INF} are fading parameters. The cdf of γ_1 is

$$F_{\gamma_1}(x) = \int_0^x f_{\gamma_1}(y) dy \cdot \tag{11}$$

The cdf $F_{\gamma_1}(x)$ may be written as [10]:

$$\begin{aligned} F_{\gamma_1}(x) = \mathrm{Q}\left(\alpha(q_1) \sqrt{\frac{x}{\bar{\gamma}_1}}, \beta(q_1) \sqrt{\frac{x}{\bar{\gamma}_1}} \right) - \\ - \mathrm{Q}\left(\beta(q_1) \sqrt{\frac{x}{\bar{\gamma}_1}}, \alpha(q_1) \sqrt{\frac{x}{\bar{\gamma}_1}} \right), \end{aligned} \tag{12}$$

where $\mathrm{Q}(x,y)$ is the Marcum Q function and

$$\begin{cases} \alpha(q) = \dfrac{\sqrt{1 - q^4}}{2q} \sqrt{\dfrac{1 + q}{1 - q}}, \\ \beta(q) = \dfrac{\sqrt{1 - q^4}}{2q} \sqrt{\dfrac{1 - q}{1 + q}}. \end{cases} \tag{13}$$

After substituting (7)-(13) in (6), the outage probability is

$$\begin{aligned} P_{out} = 1 - \int_0^\infty \int_0^\infty \left(1 - F_{\gamma_1} \left(\frac{\gamma_{th}(w + \gamma_{th} + 1) z}{w} \right) \right) \times \\ \times \frac{(1 + q_2^2)}{2 q_2 \bar{\gamma}_2} e^{-\frac{(1+q_2^2)^2 (w+\gamma_{th})}{4 q^2 \bar{\gamma}_2}} I_0 \left(\frac{(1 - q_2^4)(w + \gamma_{th})}{4 q_2^2 \bar{\gamma}_2} \right) \times \\ \times \frac{(1 + q_{INF}^2)}{2 q_{INF} \bar{\gamma}_{INF}} e^{-\frac{(1+q^2)^2 \cdot z}{4 q^2 \bar{\gamma}_{INF}}} I_0 \left(\frac{(1 - q_{INF}^4) \cdot z}{4 q_{INF}^2 \bar{\gamma}_{INF}} \right) dw dz. \end{aligned} \tag{14}$$

The average BER, derived using 4-QAM modulation format, is equal [12]

$$P_b \approx E\left[\mathrm{Q}\left(\sqrt{\gamma_{eq1}} \right) \right] , \tag{15}$$

where $\mathrm{Q}(\cdot)$ is the Gaussian Q-function. Because of easier mathematical operations, γ_{eq1} will be replaced by

$$\gamma_{eq2} = \min\left(\frac{\gamma_1}{\gamma_{INF}}, \gamma_2 \right), \tag{16}$$

as in [8]. Using γ_{eq2}, the average BER may be written as

$$P_b \approx \frac{1}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{eq2}}(x^2) \exp(-x^2 / 2) dx \,, \tag{17}$$

where $F_{\gamma_{eq2}}(x)$ is the cdf of γ_{eq2}, which is equal to [11]

$$F_{\gamma_{eq2}}(x)=1-C_{\frac{\gamma_1}{\gamma_{INF}}}(x)C_{\gamma_2}(x)\,, \tag{18}$$

where $C_{\frac{\gamma_1}{\gamma_{INF}}}(x)$ and $C_{\gamma_2}(x)$ are complementary cdfs of γ_1/γ_{INF} and γ_2, respectively. The pdf of γ_1/γ_{INF} is

$$f_{\frac{\gamma_1}{\gamma_{INF}}}(x)=\int_0^\infty z\cdot f_{\gamma_1}(z\cdot x)f_{\gamma_{INF}}(z)dz\,. \tag{19}$$

The complementary cdfs of γ_1/γ_{INF} and γ_2 are:

$$C_{\frac{\gamma_1}{\gamma_{INF}}}(x)=1-\int_0^x f_{\frac{\gamma_1}{\gamma_{INF}}}(x)dx\,, \tag{20}$$

$$C_{\gamma_2}(x)=1-\int_0^x f_{\gamma_2}(x)dx\,. \tag{21}$$

After substituting (18), (20), and (21) in (17), by numeric integration we get the average BER.

Numerical results

Fig. 1, Fig. 2 and Fig. 3 show results from (14), with assumption $q_1=q_2=q_{INF}=q$. The strength of the interference is studied using signal-to-interference ratio (SIR) $\rho=\overline{\gamma}_1/\overline{\gamma}_{INF}$.

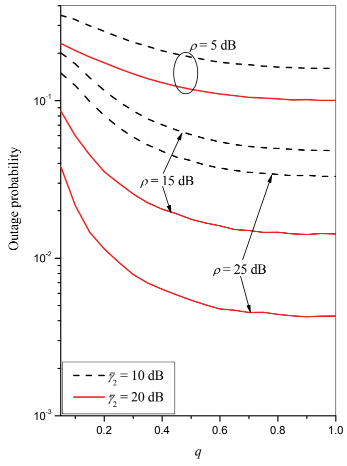

Fig. 1. Outage probability as a function of q for γ_{th} = – 5 dB

Fig. 1. shows the P_{out} as a function of q, with ρ and $\overline{\gamma}_2$ as a parameters. It can be seen that q has stronger influence on the P_{out} for higher values of ρ. Also, the P_{out} changes more rapidly for smaller values of q. Signal-to-noise ratio (SNR) $\overline{\gamma}_2$ at R has the same influence on the P_{out} for any considered q.

The P_{out} as function of γ_{th} is shown in Fig. 2. The outage probability threshold γ_{th} has stronger impact on the P_{out} for lower values of γ_{th} and for higher values of q.

Fig. 3 shows the P_{out} as a function of $\overline{\gamma}_2$, with q and ρ as parameters. It can be seen that there is the outage probability threshold for higher values of $\overline{\gamma}_2$ because of the influence of interference.

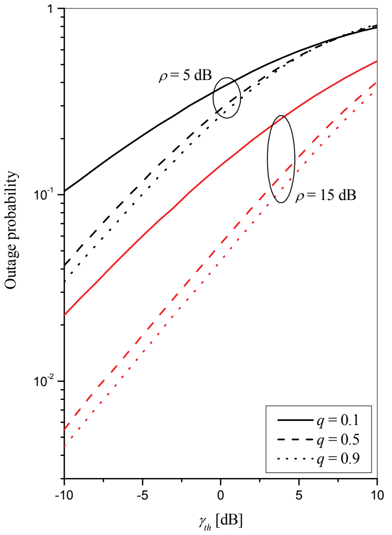

Fig. 2. Outage probability as a function of γ_{th} for $\overline{\gamma}_2$ = 20 dB

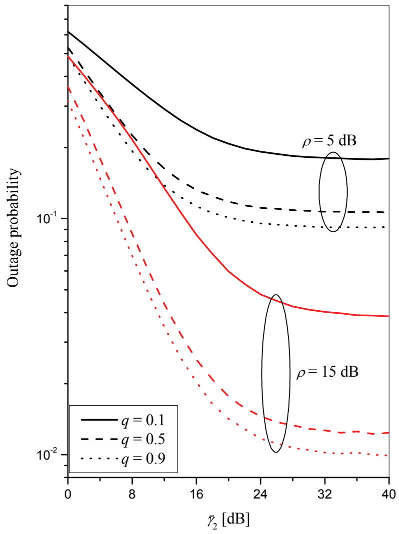

Fig. 3. Outage probability as a function of $\overline{\gamma}_2$ for γ_{th} = – 5 dB

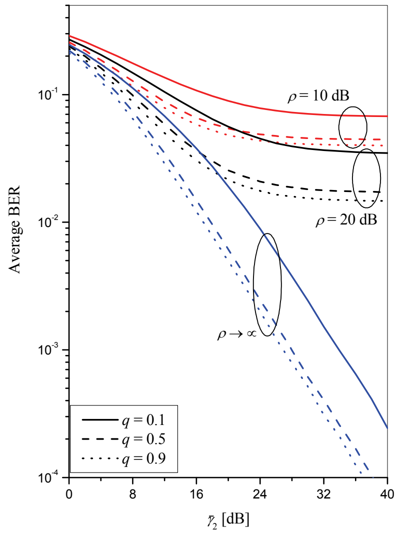

Fig. 4. Average BER as a function of $\overline{\gamma}_2$

The average BER as a function of $\bar{\gamma}_2$, with q and ρ as parameters, is shown in Fig. 4. Due to impact of interference there is BER floor for higher values of $\bar{\gamma}_2$. Considering case of no interference, it can be seen that the interference may cause a significant performance loss for higher values of SNR.

Conclusions

Performance of an amplify-and-forward relay system with co-channel interference at relay in Hoyt fading environment are presented in this paper. We consider 4-QAM modulation format. The results show that the outage probability is more influenced by the Hoyt fading parameter q for higher SIR. In the presence of interference there is a BER floor for higher SNR. Acceptable threshold γ_{th} has stronger impact on the outage probability for lower values of γ_{th}.

Acknowledgements

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References

1. **Petrović I., Stefanović M., Anastasov J., Panić S., Djordjević G., Milić D.** Outage probability of SSC receivers in correlated Rayleigh fading channel with multiple cochannel interferers // Electronics and Electrical Engineering. – Kaunas: Technologija, 2010. – No. 10(106). – P. 67–70.
2. **Aleksić D., Sekulović N., Stefanović M.** Outage Probability of System with Selection Combining over Correlated Weibull Fading Channels in the Presence of Rayleigh Cochannel Interference // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 2(90). – P. 7–10.
3. **Cvetković A., Stefanović M., Sekulović N., Mekić E., Manić D.** Dual–Hop System Analysis in Nakagami–*m* Environment // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 2(108). – P. 47–50.
4. **Cvetković A., Djordjević G., Stefanović M.** Performance of interference–limited dual–hop non–regenerative relays over Rayleigh fading channels // IET Communications, 2011. – Vol. 5. – No. 2. – P. 135–140.
5. **Raja A., Martverk P., Lossmann E.** Exact Analysis of PSK Systems in the Presence of Nakagami Fading // Electronics and Electrical Engineering. – Kaunas: Technologija, 2003. – No. 3(45). – P. 26–30.
6. **Milosevic B., Spalevic P., Petrovic M., Vuckovic D., Milosavljevic S.** Statistics of Macro SC Diversity System with Two Micro EGC Diversity Systems and Fast Fading // Electronic and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 8(88). – P. 55–58.
7. **Stefanović H., Stefanović V., Cvetković A., Anastasov J., Stefanović D.** Some Statistical Characteristics of a New Shadowed Rician Fading Channel Model // Proc. The Fourth International Conference on Wireless and Mobile Communications, ICWMC'2008. – Athens, Greece, 2008. – P. 235–240.
8. **Krikidis I., Thompson S., Mclaughlin S., Goertz N.** Max–min relay selection for legacy amplify–and–forward systems with interference // IEEE Transactions on Wireless Communications, 2009. – Vol. 8. – No. 6. – P. 3016–3027.
9. **Youssef N., Elbahri W., Patzold M., and Elasmi S.** On the crossing statistics of phase processes and random FM noise in Nakagami–*q* mobile fading channels // IEEE Transactions on Wireless Communications, 2005. – Vol. 4. – P. 24–29.
10. **Paris J. F.** Nakagami–*q* (Hoyt) distribution function with applications // Electronics Letters, 2009. – Vol. 45. – No. 4. – P. 210–211.
11. **Suraweera H., Garg H., Nallanathan A.** Performance analysis of two hop amplify–and–forward systems with interference at the relay // IEEE Communications Letters, 2010. – Vol. 14. – No. 8. – P. 692–694.
12. **Proakis J.** Digital Communications. – New York: McGraw–Hill, 2000.
13. **Anastasov J., Stefanović M., Panić S., Mosić A.** Generalised approach for performance analysis of SIR–based multiple selection diversity over constant correlated alpha – µ fading channels // European Transactions on Telecommunications, 2010. – Vol. 21. – No. 7. – P. 655–662.
14. **Nikolić B., Djordjević G.** Performance of SC and SSC Receivers in Hoyt Channel in the Presence of Imperfect Reference Signal Extraction // Proc. 9th International Conference on Applied Electromagnetics, ΠΕС'09. – Niš, Serbia, 2009.
15. **Stefanović M., Milović D., Mitić A., Jakovljević M.** Performance analysis of system with selection combining over correlated Weibull fading channels in the presence of cochannel interference // International Journal AEU, In Press. DOI: 10.1016/j.aeue.2007.09.006.
16. **Cvetković A., Milić D., Anastasov J.** Outage Probability of Dual–Hop Non–regenerative Relaying System over Weibull Fading Channels // Proceedings of XLIII International Scientific Conference on Information, Communication and Energy Systems and Technologies, ICEST'2008. – Niš, Serbia, 2008. – Vol. 1. – P. 147–150.

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Performance of an amplify-and-forward relay system, with co-channel interference at relay in Hoyt fading environment is presented. The outage probability and the average bit error rate of the system are determined. The influence of the interference, as well as the influence of other system's parameters on the system's performance is considered. Ill. 4, bibl. 16 (in English; abstracts in English and Lithuanian).

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Pateikiamos stiprinančių ir tiesioginių relinių perdavimo sistemų su tarpkanale interferencija Hoito slopinimo kanale charakteristikos. Nustatyta sistemos prastovos tikimybė ir vidutinis bitų klaidų lygis. Nagrinėta interferencijos bei kitų sistemos parametrų įtaka sistemos našumui. Il. 4, bibl. 16 (anglų kalba; santraukos anglų ir lietuvių k.).