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The Temperature Propagation near the Peltier Cooling Elements

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Introduction

The regulation of the human body temperature becomes more relevant problem for climate conditions cardinal exchanges. When environmental temperature increases or decreases too much, human has no possibility to achieve his thermal comfort by himself, additional help is necessary for this reason. In our case we offer additional temperature regulation system of the human body integrated into wearing clothes [1]. We can summarize this system as a system human – clothing – environment. The cooling elements based on the thermoelectric effect are used for this system development, they called Peltier elements [2, 3]. The basic advantages of this elements are – reliability, safety for human, „green“ technology, precision, long exploitation time, absence of mechanical parts, possibility to use the same system for heating too.

So suppose the value of the eliminated heat flow from the human body is Q_{Σ}. We can reach this value with one powerful Peltier element, or with some quantity of those elements with less power. Obviously, the human will feel enough comfortable only in this case, when whole his body simultaneously is cooled [4]. But that isn’t very rational solution the entire body to lay with Peltier elements. Let’s try to find how we need to distribute well–balanced Peltier elements PE for effectiveness of the human body cooling. We analyze the case when body is covered by clothes and don’t has direct contact with external surroundings. In this case thermal resistance of the surface layer of the human body is considerably less than resistance of the system air–clothes–ambience. So we can carry that the heat from the body is transferred only by Peltier cooling element.

Temperature propagation near one Peltier element

We suppose that we cool the human back, which surface we can approximate as plane. Besides for this problem solution we accept the heat is flowing only in particular upper layer of the human body, and whole rest part of body which is under upper tissue is maintaining a constant temperature T_{p}.

Suppose we have heat (cold) source, its form is disc and its outside radius is r_0. Source temperature differences with temperature T_{p} in value T_0. Let’s keep the layer of the body surface, in which heat flow is circulating, is solid in form of the parallelepiped rectangular. Its height is h, and it has no limit in the different directions.

Suppose the thermal resistivity of the skin per unit length is R_{q0} [(K·m)/W] and thermal conductivity of the skin per unit length is G_{q0} [W/(K·m)]. Let‘s say both of the parameters does not depend on coordinates. It means the skin is homogeneous in longitudinal and transverse directions. Let us solve problem of the temperature distribution on a surface of the skin based on these assumptions by applying an analogy to long line theory with only difference that a heat from such disc type source in homogeneous environment propagates uniformly in all directions. It means that thermal resistivity of the unit volume of the skin in longitudinal direction and conductivity of the skin in transverse direction depends on a distance from a source. Cylindrical coordinate system will be used. Origin point of the model will be on the line which is also the axis of the Peltier element and in the point where the line intersects with a bottom layer of the surface body. Whole volume of the surface layer will be segmented to elementary volumes, each of these occupy the same angle α. Elementary volume of the surface layer is shown in Fig. 1.

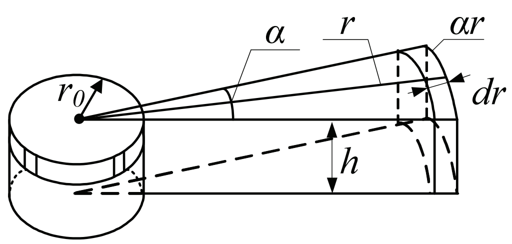

Fig. 1. The basic volume of the surface layer in the surroundings of the Peltier element PE

Heat in the elementary volume length of which is dr travels through an area S in longitudinal direction

$$S = \alpha \cdot r \cdot h\,. \tag{1}$$

So, the longitudinal thermal resistivity $\mathrm{d}R_\mathrm{q}$ could be expressed

$$\mathrm{d}R_\mathrm{q} = R_{\mathrm{q}0}\frac{\mathrm{d}r}{\alpha r h}\,. \tag{2}$$

Heat in a transverse direction travels through an area $\mathrm{d}S$ and distance h

$$\mathrm{d}S = \alpha \cdot r \cdot \mathrm{d}r\,. \tag{3}$$

Thermal conductivity of the elementary volume $\mathrm{d}G_\mathrm{q}$ in a transverse direction is

$$\mathrm{d}G_\mathrm{q} = G_{\mathrm{q}0}\frac{\alpha r\mathrm{d}r}{h}\,. \tag{4}$$

Equivalent electric circuit of the thermal elementary circuit under consideration is shown in Fig. 2.

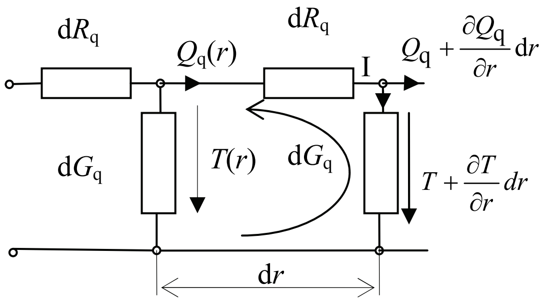

Fig. 2. The part of the thermal circuit, situated in interval $[r, r{+}\mathrm{d}r]$ and inclusive angle α

Applying Kichhoff's current law to the node I of the circuit shown in Fig.2 yields

$$Q_\mathrm{q} - (Q_\mathrm{q} + \frac{\partial Q_\mathrm{q}}{\partial r}\mathrm{d}r) - \mathrm{d}G_\mathrm{q}(T_\mathrm{q} + \frac{\partial T_\mathrm{q}}{\partial r}\mathrm{d}r) = 0. \tag{5}$$

The equation could be overwritten if the values of the second smallness order are ignored by formula

$$\frac{\partial Q_\mathrm{q}}{\partial r}\mathrm{d}r = -\mathrm{d}G_\mathrm{q}T_\mathrm{q}. \tag{6}$$

The Kichhoff's voltage law for a loop shown in figure 2 is

$$T_\mathrm{q} - (T_\mathrm{q} + \frac{\partial T_\mathrm{q}}{\partial r}\mathrm{d}r) - \mathrm{d}R_\mathrm{q}Q_\mathrm{q} = 0. \tag{7}$$

After simplification it will be

$$\frac{\partial T_\mathrm{q}}{\partial r}\mathrm{d}r = -\mathrm{d}R_\mathrm{q}Q_\mathrm{q}. \tag{8}$$

After the (8) equation differentiation, application of the (2) equality and after ignoring second smallness order values we get

$$\frac{\partial^2 T_\mathrm{q}}{\partial r^2} = -\left[\frac{\partial}{\partial r}\cdot\left(\frac{1}{\alpha h r}\right)\right]Q_\mathrm{q} + \left(\frac{1}{\alpha h r}\right)\cdot\frac{\partial Q_\mathrm{q}}{\partial r} = \\ = \frac{R_{\mathrm{q}0}}{\alpha h}\cdot\frac{1}{r^2}\cdot Q_\mathrm{q} - \left(\frac{R_{\mathrm{q}0}}{\alpha h r}\right)\cdot\frac{\partial Q_\mathrm{q}}{\partial r}. \tag{9}$$

Let's write $\partial Q_\mathrm{q}/\partial r$ from equation (6) and Q_q from equation (8) by taking in consideration (2) and (4).

Angle variable α after some mathematical actions is canceled out. So we have just one variable r in the obtained equation. Further, we apply total but not partial derivatives and obtain equation

$$\frac{\mathrm{d}^2 T_\mathrm{q}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}T_\mathrm{q}}{\mathrm{d}r} - \frac{R_{\mathrm{q}0}G_{\mathrm{q}0}}{h^2}T_\mathrm{q} = 0\,. \tag{10}$$

Characteristic equation of this differential equation is

$$k^2 + \frac{1}{r}k - \left(R_{\mathrm{q}0}G_{\mathrm{q}0}\right)\cdot\frac{1}{h^2} = 0. \tag{11}$$

The equation has two roots. One of the roots is positive and the other is negative. Positive root is related with the reflected wave which is not present for the case because the area under consideration is homogeneous until the infinity. Therefore the real process is related with just a negative root

$$k = -\frac{1}{2r} - \sqrt{\frac{1}{4r^2} + \frac{M^2}{h^2}}, \tag{12}$$

where

$$M = \sqrt{R_{\mathrm{q}0}G_{\mathrm{q}0}}\,. \tag{13}$$

Solution of the (10)-th equation could be expressed by an exponential function

$$T_\mathrm{q} = A\mathrm{e}^{k\cdot r} = A\mathrm{e}^{-\frac{1}{2}-\sqrt{\frac{1}{4}+M^2\cdot\left(\frac{r}{h}\right)^2}}, \tag{14}$$

where A is the constant of integration. Value of A could be established by taking in consideration the fact that temperature changes by the exponential law just beyond the limit of the Peltier element. The limit is defined by a circle $r{=}r_0$, where the temperature is $T\big|_{r=r_0} = T_0$. Under consideration are just the values $r \ge r_0$ in a range outside the circle $r{=}r_0$. Substituting $A = T_0 \cdot \mathrm{e}$ we obtain

$$T_\mathrm{q} = T_0 \cdot \mathrm{e}^{-\frac{1}{2}\cdot\left[\sqrt{1+4M^2\cdot(\frac{r-r_0}{h})^2}-1\right]}, \tag{15}$$

where $T_\mathrm{q}(r)$ and T_0 in this equation is the difference of the appropriate point temperature with an internal temperature of the body T_p.

Temperature distribution in the case of two Peltier elements is present

The case is shown in Fig. 3.

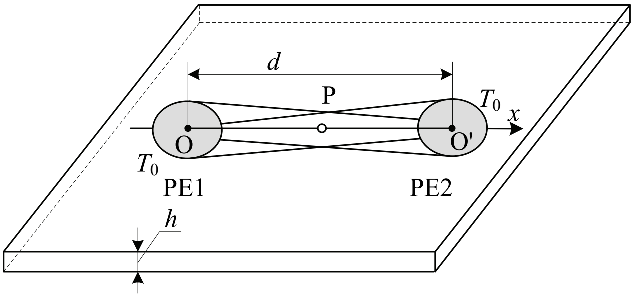

Fig. 3. An interaction between two Peltier elements

In a case when the two thermal energy sources are present temperature distribution in the area around them could be calculated by applying superposition principle. However the fact that each of the elements interacts to another and one creates the uneven temperature of the other source must be taken into account. Therefore reflected waves appear in the case. Besides the local system of axes of each PE are not concentric so to obtain the distribution of the temperature by analytical expressions is a hard task. We are mostly considered about the temperature in the point P, which is in the middle of the line which connects the centers of PE1 and PE2, which is in a middle of the line which connects point O and point O‘ in a case temperatures of the both elements are the same and are equal to T_0. Axis x of the global system of axes will be superposed with the line crossing point O and point O'. Suppose the distance from a point O to a point O' is d. Also d is a distance from center of the one PE to center of the other PE. We will get an expression of temperature distribution in the line OO‘ as a superposition of two waves which travels from PE1 to PE2 in opposite directions

$$T = A\mathrm{e}^{-\gamma(x+d/2)} + B\mathrm{e}^{-\gamma[-(x-d/2)]}, \tag{16}$$

where A and B are the constants of integration. The constants define transmitted and reflected waves to one and to the opposite directions. Function $\gamma\ (x)$ in the equation (16) could be expressed

$$\gamma(x) = -\frac{1}{2}\left(\sqrt{4M^2\frac{x^2}{h^2}+1}-1\right), \tag{17}$$

where $M = \sqrt{R_{q0}G_{q0}}$.

Constants of integrations are obtained from the boundary conditions: $T=T_0$, when $x=\pm d/2$. By applying the values and by taking in consideration $\gamma(x)$ is twin function we will get:

$$\begin{cases} A + B\mathrm{e}^{-\gamma(d)} = T_0, \\ A\mathrm{e}^{-\gamma(d)} + B = T_0. \end{cases} \tag{18}$$

By applying some mathematical operations i.e. the second equation difference from the first we will get equality $A=B$. After the summing of these equations we will get

$$A = B = \frac{T_0}{1+\mathrm{e}^{-\gamma(d)}} = \frac{T_0}{\mathrm{e}^{-\gamma(d)/2}(\mathrm{e}^{\gamma(d)/2}+\mathrm{e}^{-\gamma(d)/2})} = \\ = \frac{T_0}{2\mathrm{e}^{-\gamma(d)/2}\mathrm{ch}[\gamma(d)/2]}. \tag{19}$$

Temperature dependence on a coordinate x in the line OO‘ could be expressed

$$T(x) = \frac{T_0}{2\mathrm{e}^{-\gamma(d)/2}\mathrm{ch}[\gamma(d)/2]}[\mathrm{e}^{-\gamma(x+d/2)} + \\ + \mathrm{e}^{-\gamma[-(x-d/2)]}]. \tag{20}$$

Temperature T_P at a point P will be

$$T_P = \frac{K\cdot T_0}{\mathrm{ch}[\gamma(d)/2]}, \quad K = \frac{\mathrm{e}^{-\gamma(d/2)}}{\mathrm{e}^{-\gamma(d)/2}}. \tag{21}$$

The multiplier K represents T_P value‘s difference from a case temperature travels in one coordinate of Cartesian coordinate system.

Temperature distribution in a case when several regularly placed Peltier elements is present

Under consideration is the case of Peltier elements placed in squares as shown in a Fig. 4. Minimal temperature in a square T_D will be at a point D. D is a point where diagonals of a square intersect. Length of the diagonal will be $l\sqrt{2}$ in a case the length of side of the square is l. In a case just two PE are present and distance between them is $l\sqrt{2}$ temperature in a point D would be expressed by equation (21) where $d^2=2l^2$.

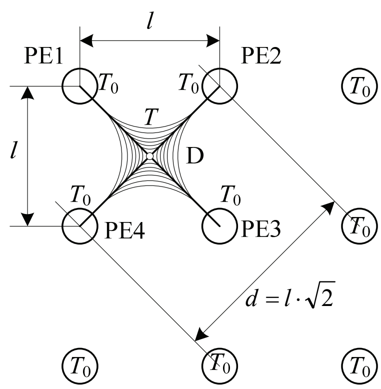

Fig. 4. Distribution of the Peltier elements on the cooling surface

However some parts of the thermal flow travel to the other PE through the point D, too. We must evaluate the reflections at other PE when we use the circuit theory for temperature T_D calculation. It can be made by modeling. The temperature T_D which can be obtained by expression (21) is the estimation of temperature difference maximum between the point D and PE. The real temperature difference between D and PE is lower. Temperature difference in all the other points of any square area is lower then T_D. We can make sure temperature in the point D is the highest by drawing the lines of the same temperature as it is shown in Fig. 4.

Assurance of the desirable distance with permissible temperature change

Placement the Peltier elements must assure desirable temperature in the problematic points. If the PE placement manner is chosen as shown in Fig. 4 the problematic point is point D. Let suppose the temperature difference is T_D. Value of Δr could be increased by decreasing the value of M. We can make sure from the equation (12) that $M = 1$ in the case resistivity of the body surface is uniform. Value of the M could be decreased by decreasing the value of longitudinal thermal resistivity R_{q0}. The goal could be reached by applying some textile materials adherent to a body. Total longitudinal thermal resistivity of the system body – fabric could be decreased to a desirable level by applying the materials with a sufficiently small resistivity R_{qa0}. Total thermal resistance could be obtained considering fabric and body surface layer in longitudinal direction resistances are connected in parallel. Let's designate body thermal resistivity in longitudinal direction R_{qk0}. Suppose thickness of the fabric is l_a. Let's assume that h is total fabric and body surface layer thickness. Therefore part of the fabric in a total body surface layer – fabric volume will be d/h. Fabric thermal resistance in the length r and in a segment α will be

$$R_{qa} = R_{qa0}\frac{r}{\alpha r d} = R_{qa0}\frac{1}{\alpha d}. \quad (22)$$

The thermal resistance of the same body element will be

$$R_{qk} = R_{qa0}\frac{r}{\alpha r(h-d)} = R_{qk0}\frac{1}{\alpha(h-d)}. \quad (23)$$

Total thermal resistance will be obtained by parallel connection of the resistances

$$R_b = \frac{R_{qa}R_{qk}}{R_{qa}+R_{qk}} = \frac{1}{\alpha}\frac{R_{qa0}R_{qk0}}{R_{qa0}(h-d)+R_{qk0}d}. \quad (24)$$

Conclusions

1. An efficiency of the human body cooling (heating) system directly depends on distribution of the cooling elements;
2. The propagation of the temperature near Peltier elements we can get by using analogy of the long lines;
3. The temperature around one Peltier element varies exponentially;
4. Temperature distribution in the line connecting two cooling elements is descriptive by circuit function;
5. We can identify the biggest temperature differences near Peltier elements using received expression. These Peltier elements are arranged by grid step.

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Human organism must keep thermal comfort in any environmental condition, but when ambient temperature reaches dangerous level, the supplementary cooling or heating system is necessary. This system we are integrating into wearing clothes for saving comfortability. The effectiveness of that system depends on rational arrangement of the cooling elements, in our case – it is Peltier element. Temperature propagation around one Peltier element in the human skin upper layer by using analogy of the long lines is analyzed in this paper. Also there are presented analysis for two cooling elements and for several Peltier elements arranged by quadrate. Analytical expressions of the temperature propagation are presented. Ill. 4, bibl. 4 (in English; abstracts in English and Lithuanian).

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Žmogaus organizmui būtinas šiluminis komfortas esant bet kokioms aplinkos sąlygoms, tačiau aplinkos temperatūrai pasiekus pavojingą lygį, reikalinga papildoma vėsinimo arba šildymo sistema. Komfortui išlaikyti ši sistema integruojama į dėvimą aprangą. Tokios sistemos efektyvumas priklauso nuo vėsinimo elementų, nagrinėjamu atveju – Peltier elementų – racionalaus išdėstymo. Šiame darbe, taikant ilgosios linijos analogiją, išanalizuotas temperatūros pasiskirstymas žmogaus odos paviršiuje aplink vieną Peltier elementą, esant dviems vėsinimo elementams ir esant keliems, Peltier elementams, išdėstytiems kvadratu. Gautos temperatūros pasiskirstymo analizinės išraiškos. Il. 4, bibl. 4 (anglų kalba; santraukos anglų ir lietuvių k.).