ISSN 1392 – 1215 ELECTRONICS AND ELECTRICAL ENGINEERING 2011. No. 3(109)

ELEKTRONIKA IR ELEKTROTECHNIKA

T 190 ELECTRICAL ENGINEERING

ELEKTROS INŽINERIJA

The Joint Probability Density Function of the SSC Combiner Output Signal at two Time Instants in the Presence of Log-Normal Fading

D. S. Krstic, P. B. Nikolic, M. C. Stefanovic

Faculty of Electronic Engineering, University of Niš,
A. Medvedeva 14, Niš, Serbia, e-mail: dragana.krstic@elfak.ni.ac.rs

F. Destovic

Faculty of Educational Sciences,
Skenderija 72, 71000 Sarajevo, BiH

Introduction

In wireless communications, a variation of an instantaneous value of the received signal, i.e. fading of signal envelope is very common effect, due to the multipath propagation. Fading is one of the main causes of performance degradation of the receiver. The multipath fading is modeled by several distributions such as: Rayleigh, Rice, Nakagami-*m*, Weibull.

The link quality in terrestrial and satellite land-mobile systems is affected by slow variation of the mean signal level due to the shadowing from terrain, buildings and trees. The performance of communication system depends only on shadowing if the radio receiver is able to average out the fast multipath fading or if an efficient micro diversity system is used to eliminate the effects of multipath. Empirical measurements show that shadowing can be modeled by a log-normal distribution for various outdoor and indoor environments [1].

Various techniques for reducing fading effect and influence of shadow effect are used in wireless communication systems. Such techniques are diversity reception, dynamic channel allocation and power control. Upgrading transmission reliability and increasing channel capacity without increasing transmission power and bandwidth is the main goal of diversity techniques. Diversity technique is certainly one of the most frequently used methods for combating the deleterious effect of channel fading. Particular diversity methods and combining techniques are presented in [1].

Space diversity reception, based on using multiple antennas at the receiver, with two or more branches, is very efficient methods used for improving system's quality of service, so it provides efficient solution for reduction of signal level fluctuations in fading channels. Multiple received copies of signal could be combined on various ways. Among the most popular diversity techniques are: maximal ratio combining (MRC), equal gain combining (EGC), and generalized selection combining (GSC) [2 - 5], but their complexity of implementation is relatively high since they require a dedicated communication receiver for each diversity branch. On the other hand, among the simpler diversity combining schemes, the two most popular are selection combining (SC) and switch and stay combining (SSC).

Since the selection combining (SC) and switch and stay combining (SSC) do not require signal cophasing and fading envelope estimation, they are very often implemented in practice. Selection combining (SC) and switch and stay combining (SSC) types of diversity systems process only one of diversity branches, so they are less complicated. The SC is combining technique where the strongest signal is chosen among L branches of diversity system [1].

Switch and stay combining (SSC) is an attempt at simplifying the complexity of the system but with loss in performance. In this case, the receiver selects a particular antenna until its quality drops below a predetermined threshold. When this happens, the receiver switches to another antenna and stays with it for the next time slot, regardless of whether or not the channel quality of that antenna is above or below the predetermined threshold. The consideration of SSC systems in the literature has been restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two ([3] – [8]). This results in a reduction of complexity relative to the SC whereby simultaneous and continuous monitoring of both branches signals or SNRs is no longer necessary.

In [9] Alouini and Simon develop, analyze and optimize a simple form of dual-branch switch and stay combining (SSC). In [10] the moment generating function (MGF) of the signal power and the first-order derivative of the MGF, with respect to the switching threshold, at the output of dual-branch switch and stay selection diversity combiners are derived. These expressions are obtained for the general case of correlated fading and nonidentical diversity branches. They are valid for any common fading distributions, such as Rayleigh, Nakagami-m, Nakagami-q, and Rician. The MGF gives the performance of different digital modulation formats with SSC reception. For independent and identically distributed diversity branches, the optimal switching threshold in closed form is derived for three forms of the conditional error probability. A new look at the performance analysis of diversity systems over fading channels are given by Alouini in [11]. Diversity receiver performance in Nakagami fading are investigated in [12].

The probability density function (PDF) of the SSC combiner output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants in the presence of Rayleigh, Nakagami-m and Weibull fading are determined in [13-15], respectively.

The determination of the probability density of the combiner output signal can be used for the receiver performances determination [16], [17]. It is notable that level crossing rate, outage probability and average time of fade duration of the combiner output signal are very important system performances. These performances of the SSC combiner output signal at one time instat in the presence of Nakagami-m fading are determined in [18] and the results are shown graphically for different variance values, decision threshold values and fading parameters values.

In [19] the level crossing rate, the outage probability and the average time of fade duration of the SSC combiner output signal at one time instat in the presence of Rice fading are determined whereas switch and stay combining (SSC) is very popular combining model and the Rice fading is often present in wireless telecommunication systems with line of sight.

In this paper the probability density function of the SSC combiner output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants, in the presence of log-normal fading, will be determine in closed form. The joint probability density function of the SSC combiner output signal at two time instants is important when the decision is based on multiple samples.

The probability density function of the SSC combiner output signal at one time instant

The use of SSC combiner with great number of branches can minimize the bit error rate (BER) [20]. We consider here the SSC combiner with two inputs because the gain is the greatest when the SSC combiner with two inputs is used instead of one-channel system. When the number of branches is enlarged the gain becomes less. Therefore, it is the most economic using SSC combiner with two inputs.

The model of the SSC combiner with two inputs considering in this paper is shown in Fig. 1. The signals at the combiner input are r_1 and r_2, and r is the combiner output signal.

Let see how the SSC combiner with two inputs works. The probability of the event that the combiner first examines the signal at the first input is P_1, and for the second input is P_2. If the combiner examines first the signal at the first input and if the value of the signal at the first input is above the treshold, r_T, SSC combiner forwards this signal to the circuit for the decision. If the value of the signal at the first input is below the treshold r_T, SSC combiner forwards the signal from the other input to the circuit for the decision. If the SSC combiner first examines the signal from the second combiner input it works in the similar way.

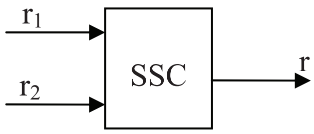

Fig. 1. Model of the SSC combiner with two inputs

The expression for the probability density of the combiner output signal will be determined first for the case: $r<r_T$. Based on the work algorithm of the SSC combiner in this case, the probability density is equal to the sum of two addends [15]. So, the probability density of the combiner output signal is, for $r<r_T$

$$p_r(r) = P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2}(r) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1}(r) . \quad (1)$$

In the case $r \geq r_T$ the expression for the probability density of the signal at the combiner output consists of four addends

$$\begin{aligned} p_r(r) &= P_1 \cdot p_{r_1}(r) + P_1 \cdot F_{r_1}(r_T) \cdot p_{r_2}(r) + \\ &+ P_2 \cdot p_{r_2}(r) + P_2 \cdot F_{r_2}(r_T) \cdot p_{r_1}(r), \end{aligned} \quad (2)$$

where r_T is the treshold of the decision, and the cumulative probability densities (CDFs) are given by

$$F_{r_i}(r_T) = \int_0^{r_T} p_{r_i}(x)\,dx , \quad i = 1,2 . \quad (3)$$

The probabilities P_1 and P_2 are:

$$P_1 = \frac{F_{r_2}(r_T)}{F_{r_1}(r_T) + F_{r_2}(r_T)} , \quad (4)$$

$$P_2 = \frac{F_{r_1}(r_T)}{F_{r_1}(r_T) + F_{r_2}(r_T)} . \quad (5)$$

The probability densities of the combiner input signals, r_1 and r_2, in the presence of log-normal fading, are

$$p_{r_i}(r_i) = \frac{1}{\sqrt{2\pi}\,\sigma_i\, r_i} e^{-\frac{(\ln r_i - \mu_i)^2}{2\sigma_i^2}} , \qquad r_i \geq 0 \ , \ i = 1,2 , \quad (6)$$

where μ_i is mean value and σ_i is standard deviation of log-normal fading.

The probability density of the combiner output signal, r, is, for $r<r_T$

$$\begin{aligned} p_r(r) &= P_1\left(\frac{1}{2} + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma_2 r} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} + \\ &+ P_2\left(\frac{1}{2} + erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\sigma_1 r} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}} \end{aligned} \quad (7)$$

and for $r \geq r_T$:

$$p_r(r) = P_1 \frac{1}{\sqrt{2\pi}\,\sigma_1\, r} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}} +$$

$$+ P_1\left(\frac{1}{2} + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\,\sigma_2\, r} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} +$$

$$+ P_2 \frac{1}{\sqrt{2\pi}\,\sigma_2\, r} e^{-\frac{(\ln r - \mu_2)^2}{2\sigma_2^2}} +$$

$$+ P_2\left(\frac{1}{2} + erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)\right)\frac{1}{\sqrt{2\pi}\,\sigma_1\, r} e^{-\frac{(\ln r - \mu_1)^2}{2\sigma_1^2}} \quad , \quad (8)$$

where *erfc(x)* is error function defined by [21, 22]

$$erf\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt \; .$$

The probabilities P_1 and P_2 are:

$$P_1 = \frac{\frac{1}{2} + erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)}{1 + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right) + erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \; , \quad (9)$$

$$P_2 = \frac{\frac{1}{2} + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right)}{1 + erf\left(\frac{\ln r_t - \mu_1}{\sigma_1\sqrt{2}}\right) + erf\left(\frac{\ln r_t - \mu_2}{\sigma_2\sqrt{2}}\right)} \; . \quad (10)$$

The joint probability density function of the SSC combiner output signal at two time instants

Now we consider the SSC combiner with two branches at two time instants. The model of the system is shown in Fig. 2. The signals at the inputs are r_{11} and r_{21} at first time moment and they are r_{12} and r_{22} at the second time moment. The output signals are r_1 and r_2. The indexes for the input signals are: first index is the number of the branch and the other signs time instant observed. For the output signal, the index represents the time instant observed.

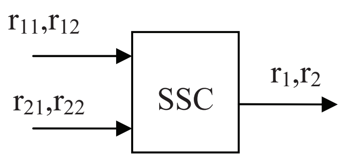

Fig. 2: Model of the SSC combiner with two inputs at two time instants

The probability of the event that combiner first examines the signal at the first input is P_1, and for the second input is P_2. Now we have four different cases. The first case is: $r_1<r_T$ and $r_2<r_T$. In this case all signals at the input are below r_T , i.e.: $r_{11}<r_T$, $r_{12}<r_T$, $r_{21}<r_T$ and $r_{22}<r_T$. Let the combiner first examines the signal r_{11}. Because $r_{11}<r_T$, it follows that $r_1=r_{21}$ and since $r_{22}<r_T$ it is $r_2=r_{12}$. The probability of this event is P_1. When SSC combiner first examines the signal r_{21}, then $r_1=r_{11}$ because $r_{21}<r_T$. Since $r_{12}<r_T$, then it is $r_2=r_{22}$. The probability of this event is P_2.

After previous, the joint probability density of the combiner output signals at two time instants, r_1 and r_2, for $r_1<r_T$ and $r_2<r_T$, is

$$p_{r_1r_2}(r_1,r_2) = P_1 \cdot \int_0^{r_T} dr_{11} \int_0^{r_T} dr_{22}\, p_{r_{11}r_{22}r_{21}r_{12}}(r_{11},r_{22},r_1,r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{21} \int_0^{r_T} dr_{12}\, p_{r_{21}r_{12}r_{11}r_{22}}(r_{21},r_{12},r_1,r_2) \, . \quad (11)$$

In the similar way we can derived the other joint probability density functions. For $r_1 \geq r_T$ and $r_2<r_T$ it is

$$p_{r_1r_2}(r_1,r_2) = P_1 \cdot \int_0^{r_T} dr_{12}\, p_{r_{12}r_{11}r_{22}}(r_{12},r_1,r_2) +$$

$$+ P_1 \cdot \int_0^{r_T} dr_{11} \int_0^{r_T} dr_{22}\, p_{r_{11}r_{22}r_{21}r_{12}}(r_{11},r_{22},r_1,r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{22}\, p_{r_{22}r_{21}r_{12}}(r_{22},r_1,r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{21} \int_0^{r_T} dr_{12}\, p_{r_{21}r_{12}r_{11}r_{22}}(r_{21},r_{12},r_1,r_2) \, , \quad (12)$$

for $r_1<r_T$ and $r_2 \geq r_T$ it is

$$p_{r_1r_2}(r_1,r_2) = P_1 \cdot \int_0^{r_T} dr_{11}\, p_{r_{11}r_{21}r_{22}}(r_{11},r_1,r_2) +$$

$$+ P_1 \cdot \int_0^{r_T} dr_{11} \int_0^{r_T} dr_{22}\, p_{r_{11}r_{22}r_{21}r_{12}}(r_{11},r_{22},r_1,r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{21}\, p_{r_{21}r_{11}r_{12}}(r_{21},r_1,r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{21} \int_0^{r_T} dr_{12}\, p_{r_{21}r_{12}r_{11}r_{22}}(r_{21},r_{12},r_1,r_2) \, , \quad (13)$$

and for $r_1 \geq r_T$ and $r_2 \geq r_T$ it is

$$p_{r_1r_2}(r_1,r_2) = P_1 \cdot p_{r_{11}r_{12}}(r_1,r_2) +$$

$$+ P_1 \cdot \int_0^{r_T} dr_{12}\, p_{r_{12}r_{11}r_{22}}(r_{12},r_1,r_2) + P_1 \cdot \int_0^{r_T} dr_{11}\, p_{r_{11}r_{21}r_{22}}(r_{11},r_1,r_2) +$$

$$+ P_1 \cdot \int_0^{r_T} dr_{11} \int_0^{r_T} dr_{22}\, p_{r_{11}r_{22}r_{21}r_{12}}(r_{11},r_{22},r_1,r_2) +$$

$$+ P_2 \cdot p_{r_{21}r_{22}}(r_1,r_2) + P_2 \cdot \int_0^{r_T} dr_{22}\, p_{r_{22}r_{21}r_{12}}(r_{22},r_1,r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{21}\, p_{r_{21}r_{11}r_{12}}(r_{21},r_1,r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{21} \int_0^{r_T} dr_{12}\, p_{r_{21}r_{12}r_{11}r_{22}}(r_{21},r_{12},r_1,r_2) \, . \quad (14)$$

The joint probability density function of correlated signals r_1 and r_2 with log-normal distribution and same σ is [23]

$$p_{r_1r_2}(r_1,r_2) = \frac{1}{\sqrt{2\pi}\,\sigma_1 r_1} e^{-\frac{(\ln r_1 - \mu_1)^2}{2\sigma_1^2}} \, .$$

$$\cdot \frac{1}{\sqrt{2\pi}\,\sigma_1\sqrt{1-\rho^2}\, r_2} e^{-\frac{\left(\ln r_2 - \left(\mu_2 + \rho\frac{\sigma_2}{\sigma_1}(\ln r_1 - \mu_1)\right)\right)^2}{2(1-\rho^2)\sigma_2^2}} \; . \quad (15)$$

We can derive the expression:

$$\frac{1}{\sqrt{2\pi}\,\sigma\, x_1} e^{-\frac{(\ln x_1-\mu)^2}{2\sigma^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu+\rho(\ln r_1-\mu))}{\sigma\sqrt{1-\rho^2}\sqrt{2}}\right)\right)^2 \tag{16}$$

from

$$\int_0^{r_T} \frac{1}{\sqrt{2\pi}\,\sigma_1\, r_1} e^{-\frac{(\ln r_1-\mu)^2}{2\sigma_1^2}} \frac{1}{\sqrt{2\pi}\,\sigma_1\sqrt{1-\rho^2}\, r_2} e^{-\frac{\left(\ln r_2-\left(\mu_2+\rho\frac{\sigma_2}{\sigma_1}(\ln r_1-\mu_1)\right)\right)^2}{2(1-\rho^2)\sigma_2^2}} dr_1 \tag{17}$$

where ρ is correlation factor between signals at two time instants.

Now, we can put (16) into (11)–(14) and obtain PDFs in closed form. For $r_1<r_T$ and $r_2<r_T$ it is

$$\begin{aligned}
p_{r_1r_2}(r_1,r_2) &= P_1 \frac{1}{\sqrt{2\pi}\,\sigma_1 r_2} e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1^2}} \cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_2\, r_1} e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+ \\
&+ P_2 \frac{1}{\sqrt{2\pi}\,\sigma_2 r_2} e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_1\, r_1} e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right).
\end{aligned} \tag{18}$$

For $r_1\geq r_T$ and $r_2<r_T$ it is

$$\begin{aligned}
p_{r_1r_2}(r_1,r_2) &= P_1 \frac{1}{\sqrt{2\pi}\,\sigma_2 r_2} e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2^2}} \cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_1\, r_1} e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right) \\
&+ P_1 \frac{1}{\sqrt{2\pi}\,\sigma_1 r_2} e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1^2}} \cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_2 r_1} e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+ \\
&+ P_2 \frac{1}{\sqrt{2\pi}\,\sigma_1 r_2} e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1^2}} \cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_2 r_1} e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2^2}} \cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+ \\
&+ P_2 \frac{1}{\sqrt{2\pi}\,\sigma_2 r_2} e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2^2}} \cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_1\, r_1} e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right),
\end{aligned} \tag{19}$$

for $r_1<r_T$ and $r_2\geq r_T$ it is

$$\begin{aligned}
p_{r_1r_2}(r_1,r_2) &= P_1\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)\right)\cdot \frac{1}{\sqrt{2\pi}\,\sigma_2^2\sqrt{1-\rho^2}\, r_1r_2} \\
&\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]} + \\
&+ P_1 \frac{1}{\sqrt{2\pi}\,\sigma_1 r_2} e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1^2}} \cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_2\, r_1} e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+ \\
&+ P_2\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)\right)\cdot \frac{1}{2\pi\sigma_1^2\sqrt{1-\rho^2}\, r_1r_2}\cdot \\
&\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)^2+\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)\right]} + \\
&+ P_2 \frac{1}{\sqrt{2\pi}\,\sigma_2 r_2} e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2^2}} \cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_2-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_1\, r_1} e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1^2}} \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right),
\end{aligned} \tag{20}$$

and for $r_1\geq r_T$ and $r_2\geq r_T$

$$\begin{aligned}
p_{r_1r_2}(r_1,r_2) &= P_1\cdot \frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\, r_1r_2}\cdot \\
&\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]} + \\
&+ P_1 \frac{1}{\sqrt{2\pi}\,\sigma_2 r_2} e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma_1 r_1} e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1^2}} \cdot \\
&\cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_1-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+ \\
&+ P_1\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_1}{\sigma_1\sqrt{2}}\right)\right)\cdot \frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\, r_1r_2} \\
&\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]} + \\
&+ P_1 \frac{1}{\sqrt{2\pi}\,\sigma_1\, r_2} e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1^2}} \cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_1+\rho(\ln r_2-\mu_1))}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right)\cdot \\
&\cdot \frac{1}{\sqrt{2\pi}\,\sigma_2 r_1} e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2^2}} \cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+ \\
&+ P_2 \frac{1}{2\pi\sigma_2{}^2\sqrt{1-\rho^2}\, r_1r_2}\cdot \\
&\cdot e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)^2+\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)^2-2\rho\left(\frac{\ln r_1-\mu_2}{\sigma_2}\right)\left(\frac{\ln r_2-\mu_2}{\sigma_2}\right)\right]} + \\
&+ P_2 \frac{1}{\sqrt{2\pi}\,\sigma_1\, r_2} e^{-\frac{(\ln r_2-\mu_1)^2}{2\sigma_1^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_2 r_1}\cdot e^{-\frac{(\ln r_1-\mu_2)^2}{2\sigma_2^2}} \cdot \\
&\cdot \left(\frac{1}{2}+erf\left(\frac{\ln r_t-(\mu_2+\rho(\ln r_1-\mu_2))}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)+ \\
&+ P_2\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\mu_2}{\sigma_2\sqrt{2}}\right)\right)\cdot \frac{1}{2\pi\sigma_1{}^2\sqrt{1-\rho^2}\, r_1r_2}\cdot
\end{aligned}$$

$$\cdot e^{-\frac{1}{2\left(1-\rho^{2}\right)}\left[\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)^2+\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{\ln r_1-\mu_1}{\sigma_1}\right)\left(\frac{\ln r_2-\mu_1}{\sigma_1}\right)\right]}+$$

$$+P_2\frac{1}{\sqrt{2\pi}\sigma_2 r_2}e^{-\frac{(\ln r_2-\mu_2)^2}{2\sigma_2^2}}\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\left(\mu_2+\rho\left(\ln r_2-\mu_2\right)\right)}{\sigma_2\sqrt{1-\rho^2}\sqrt{2}}\right)\right)$$

$$\cdot\frac{1}{\sqrt{2\pi}\sigma_1 r_1}e^{-\frac{(\ln r_1-\mu_1)^2}{2\sigma_1^2}}\cdot\left(\frac{1}{2}+erf\left(\frac{\ln r_t-\left(\mu_1+\rho\left(\ln r_1-\mu_1\right)\right)}{\sigma_1\sqrt{1-\rho^2}\sqrt{2}}\right)\right). \qquad (21)$$

The determination of the probability density function of the combiner output signal in closed form expression is very important for the receiver performances determination. We determined here these expressions and we presented them graphically in the next section.

The Numerical Results

The expressions for probability density function and joint probability density function of the combiner output signal are grafically presented using mathematical software "MatLab" [24]. Because of simplicity we supposed that the variances of both signals at the combiner input are equal.

In the case we observe one time instant, Fig. 3, the probability density function of the combiner output signal is determined versus input signal r and the treshold r_T, as a function of distribution parametars $\sigma=\sigma_1=\sigma$ and $\mu=\mu_1=\mu_2$.

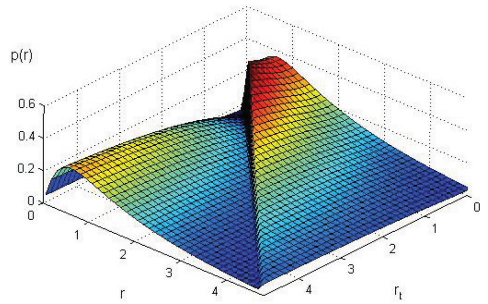

Fig. 3. Probability density function of the combiner output signal at one time instant: $\sigma=1$, $\mu=0.6$

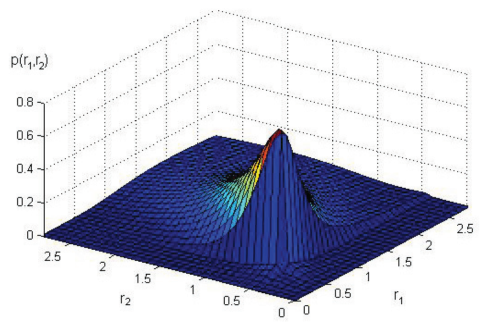

Fig. 4. The probability density function of the combiner output signal at two time instants: $\rho=0.8$, $r_t=0.5$, $\sigma=0.7$, $\mu=0.3$

When we observe two time instants, Figs. 4-6, the pdf is given versus input signals at two time instants, r_1 and r_2, for different values of distribution parametars σ and μ, the treshold r_T, and the correlation coefficient, ρ.

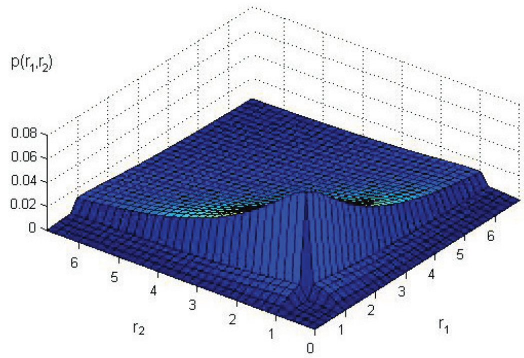

Fig. 5. Probability density function of the combiner output signal at two time instants: $\rho=0.1$, $r_t=1$, $\sigma=1$, $\mu=1$

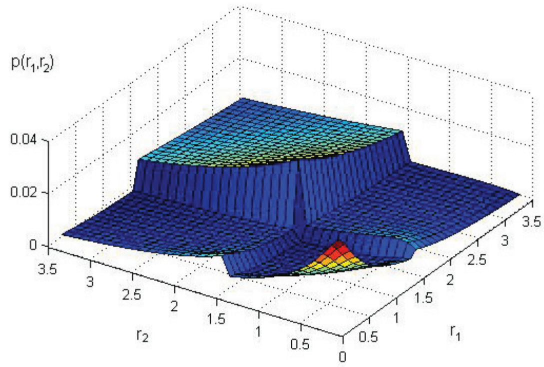

Fig. 6. Probability density function of the combiner output signal at two time instants: $\rho=0.8$, $r_t=1.5$, $\sigma=2$, $\mu=2$

Conclusions

The probability density function of dual brunches SSC combiner output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants are determined in closed form expression. The obtained results are shown graphically for different variance values and decision threshold values.

The bit error probability of digital telecommunication systems in the presence of log-normal fading can be calculated by the probability density function. This very important system performance can be significantly improved using the sampling at two time instants. The authors showed in an other work, based on the results obtained in this paper, that the error probability is significantly reduced if the decision making performed in two time instants. This fact shows that the results obtained in this study are very significant.

Acknowledgment

This work was supported by the Serbian Ministry for Science under the projects TR33035 and III 044006.

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Received 2010 07 15

D. S. Krstic, P. B. Nikolic, M. C. Stefanovic , F. Destovic. The Joint Probability Density Function of the SSC Combiner Output Signal at two Time Instants in the Presence of Log-Normal Fading // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 3(109). – P. 11–16.

In this paper the probability density function of the Switch and Stay Combiner (SSC) output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants, in the presence of log-normal fading, are determined in the closed form expressions. The results are shown graphically for different variance values and decision threshold values. If the digital telecommunication systems work on the manner described in this paper, the error probability will be significantly reduced. Ill. 6, bibl. 24 (in English; abstracts in English and Lithuanian).

D. S. Krstic, P. B. Nikolic, M. C. Stefanovic, F. Destovic. SSC daugintuvo bendrosios tikimybės tankio funkcija dviem laiko momentais // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 3(109). – P. 11–16.

Trumpiausia forma aprašyta SSC daugintuvo bendrosios tikimybės tankio funkcija dviem laiko momentais. Rezultatai pateikti grafiniu pavidalu esant įvairioms dispersijos ir ribinėms vertėms. Klaidos tikimybė sumažės, jei telekomunikacinė sistema veiks straipsnyje pateiktu principu. Il. 6, bibl. 24 (anglų kalba; santraukos anglų ir lietuvių k.).