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Statistical Analysis of Wind Turbine's Output Power

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Introduction

During the last years, the concerns related to the growth of the dependence on the fossil fuels and environmental issues have increased the interest for renewable energy. Unlike other renewable energy sources, wind energy has the highest growth among other sources.

Even with the major benefits, the wind energy brings some challenges in electrical power systems operation, related to the wind generation capacity unpredictability, intermittence and high variability. The stability of the electrical power systems is based on the reliable power generation that is permanently balanced by the load. The variability of power from wind turbine has several negative effects on the power systems operation. The electrical power output of a wind turbine depends on many factors, including the wind profile associated to the placement's site and the operational parameters of the wind turbine generator. The variability of the generated power increases due to the variability of the wind, but it can be estimated and limited through a suitable choice of the operational parameters. The influence of the wind power generation on the power systems operation can be controlled by reducing the standard deviation of the wind turbines' output power [1]. Therefore, the power systems must be extended with those wind turbines that best match to the wind speed profile and to the power system units' structure.

Selection of the suitable wind turbine was discussed differently in various papers. Two important aspects must be considered in the choice of the wind turbines, namely the energy production and the power system's stability. Many authors focus their research to select the most suitable wind turbine generators that maximize the annual energy production, as well as the capacity factor [2,3]. Capacity factor is defined as the first statistical moment of the normalized output power. The second moment of the output power variable is the variance that relates the deviation of the output power to the mean, reflecting the variability of the wind turbines' generated power. Therefore, the mean of the output power may be used to estimate the average energy production, while the variance of the output power may be used to relate the degree of the intermittent behaviour of the output power.

In the paper there are developed two analytical models for the first two statistical moments of the output power variable, estimating their limits and the dependence on the wind profile characteristics and the operational parameters of the wind turbine.

Wind speed statistical modelling

The wind speed is continuously changing, making it desirable to be described using statistical models. It is widely agreed that the random behaviour of the wind speeds can be described by two parameters Weibull distribution, characterised by the following probability density and cumulative distribution functions;

$$f_W(v) = \frac{\beta}{\alpha}\left(\frac{v}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{v}{\alpha}\right)^{\beta}\right], \quad (1)$$

$$F_W(v) = 1 - \exp\left[-\left(\frac{v}{\alpha}\right)^{\beta}\right], \quad (2)$$

where α (m/s) is the scale parameter and β (dimensionless) is the shape parameter of the Weibull distribution.

Generally, the scale parameter provides information about the average of the wind speed profile, while the shape parameter provides information about the deviation of the wind speed values around the mean. The shape and the scale parameters are interconnected through analytical expressions of mean and variance of Weibull distributions, given by the following expressions:

$$M(v) = \alpha\Gamma(1+1/\beta), \quad (3)$$

$$Var(v) = \alpha^2\left[\Gamma(1+2/\beta) - (\Gamma(1+1/\beta))^2\right], \quad (4)$$

where $\Gamma(\,)$ is the Euler's gamma function.

Wind turbine's output power modelling

The output power of a wind turbine is a function of the wind speed, the power curve of turbine giving the relation between the wind speed and the electrical power output. These curves come available from the wind turbine manufacturers or there are plotted using recorded wind speed and corresponding output power data.

The power curve of a pitch-regulated wind turbine is characterized by the following three speeds: the cut-in, the rated and the cut-off speed. The wind turbine starts generating power when the speed exceeds the cut-in speed (v_{ci}). The output power increases with the wind speed between the cut-in speed and the rated one (v_r), after that the output power remains constant at the rated power level (P_r). The cut-off wind speed (v_{co}) is the maximum wind speed at which the turbine allows power generation, it being usually limited by safety constraints.

Different wind generators have different output power performances, thus the model used to describe the performances is also different. A literature survey indicates that the most used models for output power are the linear, the quadratic and the cubic models. The output power of a wind turbine generator can be accurately modelled using the quadratic model, as results from many papers [3, 4]:

$$P(v) = \begin{cases} P_r \cdot \left(A_2 v^2 + A_1 v + A_0\right), & for \quad v_{ci} < v < v_r, \\ P_r, & for \quad v_r < v < v_{co}, \\ 0, & otherwise \end{cases} \quad (5)$$

where the coefficients A_2, A_1 and A_0 have the following expressions:

$$A_2 = \frac{1}{\left(v_{ci} - v_r\right)^2}\left[2 - 4\left(\frac{v_{ci} + v_r}{2v_r}\right)^3\right], \quad (6)$$

$$A_1 = \frac{1}{\left(v_{ci} - v_r\right)^2}\left[4\left(v_{ci} + v_r\right)\left(\frac{v_{ci} + v_r}{2v_r}\right)^3 - 3\left(v_{ci} + v_r\right)\right], \quad (7)$$

$$A_0 = \frac{1}{\left(v_{ci} - v_r\right)^2}\left[v_{ci}\left(v_{ci} + v_r\right) - 4v_{ci}v_r\left(\frac{v_{ci} + v_r}{2v_r}\right)^3\right]. \quad (8)$$

Equation (5) expresses the instantaneous value of the electrical power output as a function of the instantaneous wind speed. To consider the effect of the wind speed variation, the next section will consider the probability distribution function of the wind speed.

Proposed models for statistical moments of power

The objective of this section is to combine the output power dependence on the wind speed with the variation of the wind speed at a given site, in order to find the mean and the variance of the output power from a given wind turbine located at the specified site.

The mean and the variance are the first two statistical moments that provide information on the level and dispersion of a set of output power values. The mean value of the electrical power output of a wind turbine can be estimated from its probability density function, representing the generated power at each wind speed value, integrated over all possible power values [5]. In accordance with the statistical approach, the mean of the electrical power output is calculated as follows

$$M(P) = \int_0^\infty P(v) \cdot f_W(v)dv \quad , \quad (9)$$

where $P(v)$ is the power curve function and $f_W(v)$ is the Weibull distribution of the wind speed.

Considering the expression of the power curve from eq. (3), the mean of the output power can be written as

$$M(P) = P_r \cdot \int_{v_{ci}}^{v_r} \left(A_2 v^2 + A_1 v + A_0\right) f_W(v)dv + P_r \cdot \int_{v_r}^{v_{co}} f_W(v)dv \ . \quad (10)$$

The first integral of eq. (10) can be written as a sum of the three similar integrals I_k, with k=1,2,3. The integral I_k, having the following expression, can be easily solved by making the change in variables $y = (v/\alpha)^\beta$, $dy = \beta/\alpha \cdot (v/\alpha)^{\beta-1}$ and respectively $v = \alpha \cdot v^{1/\beta}$, as follows

$$I_k = \int_{v_{ci}}^{v_r} (A_k v^k) f_W(v)dv = \int_{v_{ci}}^{v_r} (A_k v^k) \frac{\beta}{\alpha}\left(\frac{v}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{v}{\alpha}\right)^\beta\right]dv =$$

$$= A_k \int_{y_{ci}}^{y_r} (\alpha v^{1/\beta})^k \exp(-y)dy = \alpha^k A_k \int_{y_{ci}}^{y_r} \left(y^{(k/\beta+1)-1}\right)\exp(-y)dy \cdot \quad (11)$$

The previous integral it is common used in the statistical analysis, being known as lower incomplete gamma function, having the following structure

$$\int_0^t y^{a-1} \exp(-y)dy = \Gamma(a) \cdot P(t,a), \quad (12)$$

where $\Gamma(\ )$ and $P(\ )$ are the gamma and the lower incomplete gamma functions, respectively.

The second integral of the eq. (10) represents the Weibull cumulative distribution function, evaluated between v_r and v_{co} limits. After substituting of integration limits and their reduction to the minimum number of terms, the result became as follows

$$M(P) = P_r\left\{\sum_{k=0}^{2} A_k \alpha^k \Gamma\left(\frac{k}{\beta}+1\right) \cdot \left[P\left(\left(\frac{v_r}{\alpha}\right)^\beta, \frac{k}{\beta}+1\right) - P\left(\left(\frac{v_{in}}{\alpha}\right)^\beta, \frac{k}{\beta}+1\right)\right] + \right.$$
$$\left. + \exp(-(v_r/\alpha)^\beta) - \exp(-(v_{off}/\alpha)^\beta)\right\}. \quad (13)$$

It should be mentioned that the quantity inside the brackets of the eq. (13) is the so-called mean power coefficient (MPC) or, better known, as the capacity factor. The mean power coefficient is defined as the ratio of the average output power over a time period to the rated power of wind turbine generator, eq. (14), being a very important parameter of a wind energy conversion since it determines the total energy production. Likewise, it should be noticed that the MPC is independent of rated power

$$MPC = \sum_{k=0}^{2} A_k \cdot \alpha^k \cdot \Gamma\left(\frac{k}{\beta}+1\right) \cdot \left[P\left(\left(\frac{v_r}{\alpha}\right)^\beta, \frac{k}{\beta}+1\right) - P\left(\left(\frac{v_{ci}}{\alpha}\right)^\beta, \frac{k}{\beta}+1\right)\right] +$$
$$+ \exp(-(v_r/\alpha)^\beta) - \exp(-(v_{co}/\alpha)^\beta). \quad (14)$$

The variance of the output power from a wind turbine is the expected value of the squared difference between the output power values and the mean of electrical power output, integrated over all possible power values

$$Var(P) = \int_0^\infty \left(P(v) - M(P)\right)^2 \cdot f_w(w)dv = M(P^2) - M(P)^2. \quad (15)$$

Considering the same mathematical technique, the variance power coefficient (VPC) is defined as the ratio of the variance output power to the squared of the rated power, as follows

$$VPC = \sum_{k=0}^{4} B_k \cdot \alpha^k \cdot \Gamma\left(\frac{k}{\beta}+1\right) \cdot \left[P\left(\left(\frac{v_r}{\alpha}\right)^\beta, \frac{k}{\beta}+1\right) - P\left(\left(\frac{v_{ci}}{\alpha}\right)^\beta, \frac{k}{\beta}+1\right)\right] + $$
$$+ \exp(-(v_r/\alpha)^\beta) - \exp(-(v_{co}/\alpha)^\beta) - MPC^2, \quad (16)$$

where the new coefficients are evaluated as: $B_4 = A_2^2$, $B_3 = 2A_2A_1$, $B_2 = A_1^2 + 2A_2A_0$, $B_1 = 2A_1A_0$ and $B_0 = A_0^2$.

The (14) and (16) express the relationships between cut-in, rated and cut-off speeds parameters and the mean and variance power coefficients. For a given wind profile, with α and β parameters known, it can be established the values of the wind turbines' operational parameters which lead to a maximal mean power coefficient and/or a minimal variance power coefficient.

Models validation and numerical example

In order to validate the proposed models, their results have been compared with those given by other evaluation technique, namely from the Monte Carlo simulation. Both evaluation techniques were applied on a real wind turbine and a real wind speed database. The considered wind turbine is the 1.5 XLE GE-Energy, having 1.5 MW rated power, 3.5m/s cut-in, 11.5 m/s rated and 20m/s cut-off wind speed parameters. The wind speed database was collected from the north-east of Romania, for the year 2009, the hourly average values being recorded and adjusted to the hub wind turbine height (80m). The parameters of the Weibull distribution used to fitting database have been evaluated with Maximum Likelihood Method and they were founded as the scale parameter α=4.83 m/s and the shape parameter β=1.87.

A Matlab program has been developed in order to evaluate the mean and variance power coefficients. The program has been structured by two main functions. First function is developed based on the eq. (14) and (16). Second function has been developed based on Monte Carlo simulations technique (MCS).

The Monte Carlo technique generates different values of the wind speed, in accordance with their Weibull distribution, these values being used to calculate the output power, based on the power curve of the wind turbine generator. The expected values of the mean and variance power coefficients may be observed from the average and variance of all output power values, over a long number of samples.

The simulation is stopped when a specified degree of confidence has been achieved. The number of the simulations results from the condition that the deviation of the coefficient of variation of MPC and VPC around the expected value to be under a settled value. For a settled value of 0.01%, the convergence process has required about 10,000 necessary samples.

The capacity factor values provided by analytical model and sequential Monte Carlo simulation are given in Table 1 and Table 2.

For a better comparison between models, there were considered two ranges of speed wind distribution parameters into the evaluation of the mean and variance power coefficients.

Table 1. Mean Power Coefficient values from analytical model (AM) and Monte Carlo Simulation (MCS)

MPC (%) for 1.5 xle-GE (v_{ci}=3.5m/s, v_r=11.5m/s, v_{co}=20m/s)								
	∝=3		∝=4		∝=5		∝=6	
β	AM	MCS	AM	MCS	AM	MCS	AM	MCS
0.5	11.311	11.233	12.848	12.771	13.841	13.834	14.504	14.568
1	6.875	6.847	12.325	12.479	17.294	17.260	21.328	21.332
1.5	2.354	2.404	6.760	6.862	12.972	13.004	19.851	19.797
2	1.008	1.016	3.942	3.934	9.226	9.280	16.447	16.430
2.5	0.529	0.519	2.720	2.765	7.188	7.168	14.008	13.943
3	0.310	0.308	2.102	2.081	6.103	6.074	12.510	12.506

Table 2. Variance Power Coefficient values from analytical model (AM) and Monte Carlo Simulation (MCS)

VPC (%) for 1.5 xle-GE (v_{ci}=3.5m/s, v_r=11.5m/s, v_{co}=20m/s)								
	∝=3		∝=4		∝=5		∝=6	
β	AM	MCS	AM	MCS	AM	MCS	AM	MCS
0.5	7.789	7.822	8.787	8.643	9.427	9.398	9.856	9.902
1	3.701	3.686	7.012	7.112	9.838	9.820	11.947	12.015
1.5	0.588	0.601	2.548	2.492	5.787	5.804	9.287	9.259
2	0.103	0.113	0.775	0.781	2.725	2.606	5.949	6.035
2.5	0.028	0.027	0.302	0.309	1.330	1.328	3.652	3.642
3	0.010	0.010	0.153	0.149	0.760	0.771	2.328	2.311

Usually, the scale parameter lies between 3-6 m/s and the shape parameter lies between 1.5-3. It can be seen that the results obtained from both methods are very close. The analytical models provide comparative results with Monte Carlo simulation, these proving the accuracy of the developed analytical models.

Analysis and discussions

The proposed models can be used to assess the effect of optimal selection of wind turbine parameters, based on the maximization of the mean power coefficient, on the variability of the output power generated.

Figure 1 shows the dependence of the mean and variance power coefficients for various normalised turbine rated speed (v_r/α), for a typical turbine with (v_{ci}/v_r=0.275) and (v_{co}/v_r=1.75) [6]. As can be seen, a maximum value of mean power coefficient is achieved for a local minimum value of the variance power coefficient. Therefore, the wind turbine design considering the maximum value of the mean power coefficient involves a minimum variability of the output power.

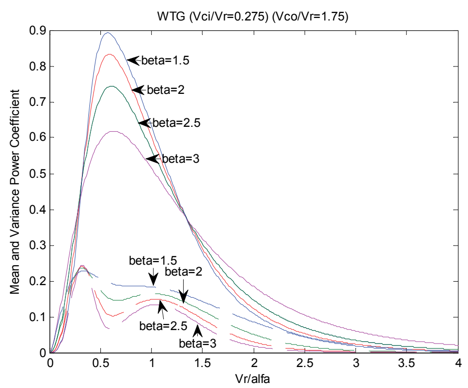

Fig. 1. The mean and variance power coefficients from Monte Carlo simulation

The case study shows that the turbines having a high mean power coefficient generate power in a lower intermittent behaviour. Furthermore, the model allows the calculation of the maximum value of the output power variation. For the considered case, the biggest variation of hourly electrical power output is evaluated to be about 25% (equivalent to a standard deviation by 5%).

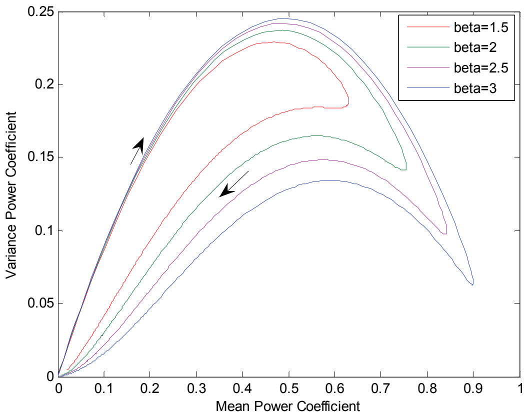

Fig. 2. The variance power coefficients as function of mean power coefficient

Figure 2 illustrates the dependence between the first two moments of the output power. As can be seen, the sites with a lower shape parameter are characterised by the higher mean and variance power coefficients than those sites with a larger shape parameter. This is true only if the average wind speed is the same at each site. However, if two sites are characterised by the same average wind speed, the site with the lower shape parameter will have a larger energy production, but also, a higher intermittent behaviour.

Conclusions

The mean and variance power coefficients are important factor into the analysis of the wind energy potential and the generating behaviour of a wind turbine located in a specific area.

There are presented analytical models to evaluate the mean and variance power coefficients for a wind turbine, located in a specific site. The results were validated using the Monte Carlo simulations providing that the proposed models give more accurate estimations. These analytical models can be used in a preliminary evaluation of the annual energy production and of the biggest variation of generated power, based on the characteristics of the wind profile and the operational parameters of the wind turbines.

The important point is that the annual energy maximization design rule reduces the variance of the electrical power output to a local minimum value, the decrease of the variance being more emphasised for the wind profile having higher value of the shape parameter.

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References

1. **Azubalis M., Azubalis V., Slusnys D.** Estimation of the Feasible Wind Power in a Smapp Power System Energy // Electronics and Electrical Engineering. – Kaunas: Technologija, 2011. – No. 1(107). – P. 79–72.
2. **Kunicina N., Galkina A., Zhiravecka A., Chaiko Y., Ribickis L.** Increasing Efficiency of Power Supply System for Small Manufactures in Rural Regions using Renewable Energy Resources // Electronics and Electrical Engineering. – Kaunas: Technologija, 2009. – No. 8(96). – P. 19–22.
3. **Kaigui Xie, Roy Billinton**. Energy and reliability benefits of wind energy conversion systems // Renewable Energy, 2011. – Vol. 36. – P. 1983–1988.
4. **Albadi M. H., El–Saadany E. F.** Optimum turbine–site matching // Energy, 2010. – Vol. 35. – P. 3593–3602.
5. **Nemes C., Munteanu F.** Development of reliability model for wind farm power generation // Advances in Electrical and Computer Engineering, 2010. – Vol. 10. – P. 24–29.
6. **EL–Shimy M.** Optimal site matching of wind turbine generator: Case study of the Gulf of Suez region in Egypt // Renewable Energy, 2010. – Vol. 35. – P. 1870–1878.

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In this paper, the authors develop the analytical models for the mean and variance of the output power based on the power curve of wind turbine and the Weibull distribution of wind speed at an investigated site. In order to validate the model, the results are compared with those given by Monte Carlo simulation technique. The analytical models have the advantage to put into the evidence the effects of the wind profile and of the wind turbines' operational parameter on the values of the statistical moments. The models can be used in the preliminary planning of the electrical power system with new energy wind sources, to evaluate the amount of energy and the variability of the generated electrical power. Ill. 2, bibl. 6, tabl. 2 (in English; abstracts in English and Lithuanian).

C. Nemes, M. Istrate. Vėjo turbinos išėjimo galios statistinė analizė // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 4(120). – P. 31–34.

Remiantis galios kreive ir vėjo greičio Veibulo skirstiniu tiriamojoje vietovėje, sukurti vėjo turbinos išėjimo galios vidurkio ir dispersijos analitiniai modeliai,. Siekiant patikrinti modelį, rezultatai palyginti su Monte Karlo metodu gautais rezultatais. Analitiniai modeliai yra pranašesni, nes įvertina vėjo profilio efektus ir vėjo turbinų eksploatacinius parametrus. Modeliai gali būti panaudoti preliminariam elektros energijos sistemos su naujais vėjo energijos šaltiniais planavimui, skaičiuojant generuojamos energijos kiekį ir kintamumą. Il. 2, bibl. 6, lent. 2 (anglų kalba; santraukos anglų ir lietuvių k.).