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Dual-Hop System Analysis in Nakagami-m Environment

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Introduction

Dual-hop transmission represents a technique used for dividing transmitter-receiver channel into two shorter channels connected by a relay. This principle found broad application now days in cooperative wireless communication systems. Main idea behind this method is solving the problem of signal transmission from transmitter to receiver, where major attenuation is present, by transmitting the signal over two or, in general case multiple, separate links [1]. Fading is a serious problem in mobile communications and it represents variation of signal strength at receiving point. Its value can be up to 30 dB [2] and it is crucial to consider it when doing any system design. Fading varies depending on path length, frequency, climate conditions as well as terrain profile. Nakagami-m distribution is a statistical distribution which can accurately model different types of environments and is used for modelling of fading that occurs in mobile communications. Although empirical and therefore susceptible to different approximations and settings, Nakagami-m distribution proved to be very useful due to easy manipulation and wide range of applicability [3–5].

System analysis

In Fig. 1 dual-hop wireless system is presented. The communication between transmitting terminal S and receiving terminal D is realized by relay terminal R. If envelopes of desired signal and interference at the input of terminal R are denoted with s_1 and i_1 at the input of terminal D with s_2 and i_2 respectively, then signal-to-interference ratio (SIR) at the input of terminals R and D can be defined as following:

$$\lambda_1 = \frac{s_1^2}{i_1^2} = \frac{z_1}{y_1} \;, \quad (1)$$

$$\lambda_2 = \frac{s_2^2}{i_2^2} = \frac{z_2}{y_2} \;. \quad (2)$$

Envelopes of desired signals s_1 and s_2 as well as envelopes of interference i_1 and i_2 have Nakagami-m distribution. Therefore their squares z_1 and z_2 as well as y_1 and y_1, will have Gamma distribution.

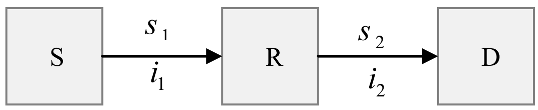

Fig. 1: Graphic illustration of wireless system where terminal R represents intermediate terminal between terminal S and terminal D

Distributions of the square envelope of desired signal and interference at the input of the terminal R have following form:

$$\begin{cases} p_{z_1}(z_1) = \dfrac{1}{\Gamma(m_{11})}\left(\dfrac{m_{11}}{\Omega_{11}}\right)^{m_{11}} z_1^{m_{11}-1} e^{-\frac{m_{11}}{\Omega_{11}} z_1}, \; z_1 \geq 0, \\ p_{y_1}(y_1) = \dfrac{1}{\Gamma(m_{12})}\left(\dfrac{m_{12}}{\Omega_{12}}\right)^{m_{12}} y_1^{m_{12}-1} e^{-\frac{m_{12}}{\Omega_{12}} y_1}, \; y_1 \geq 0, \end{cases} \quad (3)$$

while their form at the input of terminal D is as follows:

$$\begin{cases} p_{z_2}(z_2) = \dfrac{1}{\Gamma(m_{21})}\left(\dfrac{m_{21}}{\Omega_{21}}\right)^{m_{21}} z_2^{m_{21}-1} e^{-\frac{m_{21}}{\Omega_{21}} z_2}, z_2 \geq 0, \\ p_{y_2}(y_2) = \dfrac{1}{\Gamma(m_{22})}\left(\dfrac{m_{22}}{\Omega_{22}}\right)^{m_{22}} y_2^{m_{22}-1} e^{-\frac{m_{22}}{\Omega_{22}} y_2}, y_2 \geq 0, \end{cases} \quad (4)$$

where Ω_{11} and Ω_{21} represent average powers of desired signal, Ω_{12} and Ω_{22} represent average powers of interference and m_{11}, m_{12}, m_{21} and m_{22} represent Nakagami-m parameters ($m \geq 0.5$) which determine fading severity. As parameter *m* increases, the fading severity decreases.

Probability density function (PDF) expression for SIR for S-R segment can be calculated using the equation

$$p_{\lambda_1}(\lambda_1) = \int_0^\infty |J_1| p_{z_1}(\lambda_1 y_1) p_{y_1}(y_1) dy_1, \quad (5)$$

where Jacobian function can be calculated by following equation

$$|J_1| = \left|\frac{dz_1}{d\lambda_1}\right| = y_1. \quad (6)$$

Cumulative distribution function (CDF) for the same segment has following form

$$F_{\lambda_1}(\lambda_1) = \int_0^{\lambda_1} p_{\lambda_1}(x) dx. \quad (7)$$

In similar way the equation for PDF for R-D segment can be obtained

$$p_{\lambda_2}(\lambda_2) = \int_0^\infty |J_2| p_{z_1}(\lambda_2 y_2) p_{y_2}(y_2) dy_2, \quad (8)$$

where again Jacobian function can be calculated by

$$|J_2| = \left|\frac{dz_2}{d\lambda_2}\right| = y_2. \quad (9)$$

Equation for CDF for second segment has following form

$$F_{\lambda_2}(\lambda_2) = \int_0^{\lambda_2} p_{\lambda_2}(x) dx. \quad (10)$$

After series of numerical transformations, PDF for λ_1 and λ_2 have following forms:

$$p_{\lambda_1}(\lambda_1) = \frac{\Gamma(m_{11}+m_{12})}{\Gamma(m_{11})\Gamma(m_{12})} m_{11}^{m_{11}} m_{12}^{m_{12}} \times \times \frac{\lambda_1^{m_{11}-1}}{\left(\frac{1}{\gamma_1} m_{11}\lambda_1 + m_{12}\right)^{m_{11}+m_{12}}} \left(\frac{1}{\gamma_1}\right)^{m_{11}}, \quad (11)$$

$$p_{\lambda_2}(\lambda_2) = \frac{\Gamma(m_{21}+m_{22})}{\Gamma(m_{21})\Gamma(m_{22})} m_{21}^{m_{21}} m_{22}^{m_{22}} \times \times \frac{\lambda_2^{m_{21}-1}}{\left(\frac{1}{\gamma_2} m_{21}\lambda_2 + m_{22}\right)^{m_{21}+m_{22}}} \left(\frac{1}{\gamma_2}\right)^{m_{21}}. \quad (12)$$

In the equations γ_1 and γ_2 represent ratios of average powers of desired and interference signal:

$$\gamma_1 = \frac{\Omega_{11}}{\Omega_{12}}, \quad (13)$$

$$\gamma_2 = \frac{\Omega_{21}}{\Omega_{22}}. \quad (14)$$

CDF for λ_1 and λ_2 will than be:

$$F_{\lambda_1}(\lambda_1) = \frac{\Gamma(m_{11}+m_{12})}{\Gamma(m_{11})\Gamma(m_{12})} \left(\frac{m_{11}}{m_{12}}\right)^{m_{11}} \frac{\lambda_1^{m_{11}}}{m_{11}} \left(\frac{1}{\gamma_1}\right)^{m_{11}} \times \times {}_2F_1\left(\left(m_{11}+m_{12}\right), m_{11}; 1+m_{11}; -\frac{\lambda_1}{\gamma_1}\frac{m_{11}}{m_{12}}\right), \quad (15)$$

$$F_{\lambda_2}(\lambda_2) = \frac{\Gamma(m_{21}+m_{22})}{\Gamma(m_{21})\Gamma(m_{22})} \left(\frac{m_{21}}{m_{22}}\right)^{m_{21}} \frac{\lambda_2^{m_{21}}}{m_{21}} \left(\frac{1}{\gamma_2}\right)^{m_{21}} \times \times {}_2F_1\left(\left(m_{21}+m_{22}\right), m_{21}; 1+m_{21}; -\frac{\lambda_2}{\gamma_2}\frac{m_{21}}{m_{22}}\right). \quad (16)$$

System failure can occur in two cases:

a) in case of a disconnection in S-R channel which happens in case the value of λ_1 is below the threshold required for Quality of Service (QoS) to be satisfied.

b) in case of a disconnection in R-D channel which happens in case the value of λ_2 is below above mentioned threshold.

In order to analyze the system, it is necessary to find distribution of minimum of these two values $\lambda = \min(\lambda_1, \lambda_2)$.

PDF of λ can be calculated by substituting (13), (14), (17) and (18) into the equation

$$p_\lambda(\lambda) = p_{\lambda_1}(\lambda)\left(1 - F_{\lambda_2}(\lambda)\right) + p_{\lambda_2}(\lambda)\left(1 - F_{\lambda_1}(\lambda)\right). \quad (17)$$

Using one of mathematical packages Mathematica or MatLab it can be confirmed that $p_\lambda(\lambda)$ is non-negative function and that $\int_0^\infty p_\lambda(\lambda) = 1$ is true therefore confirming regularity of above mentioned equation.

Highly important and widely accepted system performance indicator is outage probability which is defined as probability of having SIR value lower than pre-determined threshold which defines required QoS.
Outage probability can be obtained using following formula

$$P_{out} = \int_0^{\lambda_0} p_\lambda(\lambda) d\lambda, \quad (18)$$

where λ_0 represents established threshold.

Using the equation different fading environment can be simulated and analyze system behaviour.

Numerical results

With help of program packages Mathematica 6.0 and Origin 8, based on above derived equations, the figures are obtained which are used for analysis of above described system.

In Fig. 2 PDF of λ for different values of system parameters is shown. Based on the results presented in the figure, it is obvious that PDF of λ is higher as γ becomes greater and as fading severity lowers for desired signal. That implies greater values for m_{11} and m_{21}.

In order to analyze system performance, the figure obtained using (18) will be observed. First case to be considered is when fading severity is constant and average values of SIR are equal in both segments and changing.

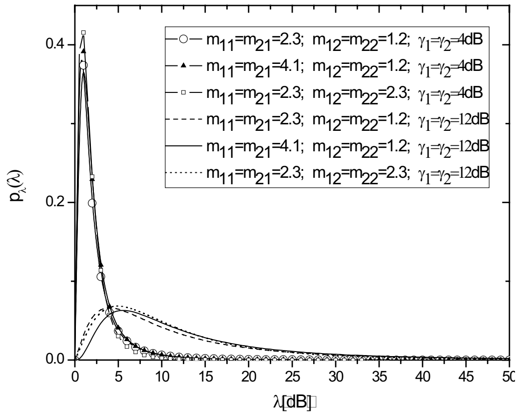

Fig. 2. PDF of minimum SIR or different values of parameters m_{11}, m_{12}, m_{21}, m_{22}, γ_1 and γ_2

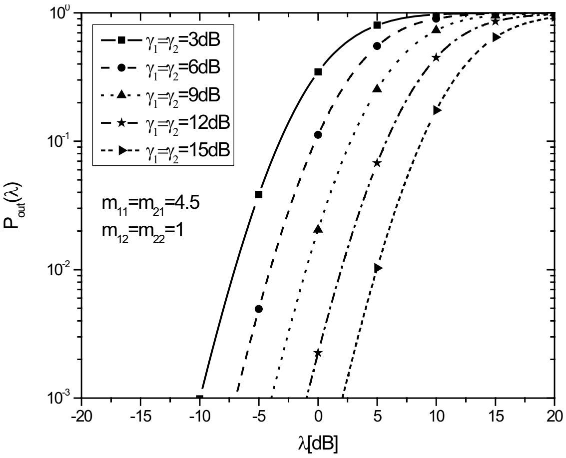

Fig. 3. Outage probability depending on threshold for different average SIR values

By analyzing Fig. 3, it can be concluded that outage probability is higher for higher values of γ therefore performance of the system becomes worse as SIR value decreases.

Next case for analysis considers different average values of SIR at first and second transmission segment while all the parameters are constant, except γ_2.

Fig. 4 shows that outage probability of the system becomes higher as γ_2 decreases. If the value of the same parameter increases and becomes greater than certain threshold, the outage probability therefore the system conditions don't change significantly.

The following case discusses the impact of interference fading severity change on the system for equal and constant value signal fading severity and average SIR value on both segments.

Fig. 5 shows that outage probability is higher for lower values of fading interference parameters but it is shown as well that by increasing the value of λ, the outage probability increases for higher values of fading parameters.

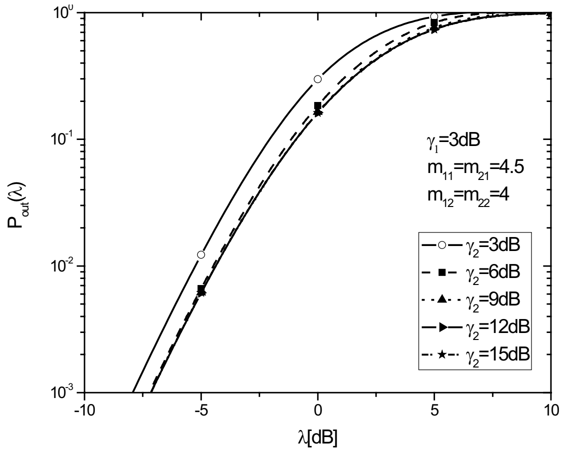

Fig. 4. Outage probability depending on threshold for different average SIR values at second transmission segment

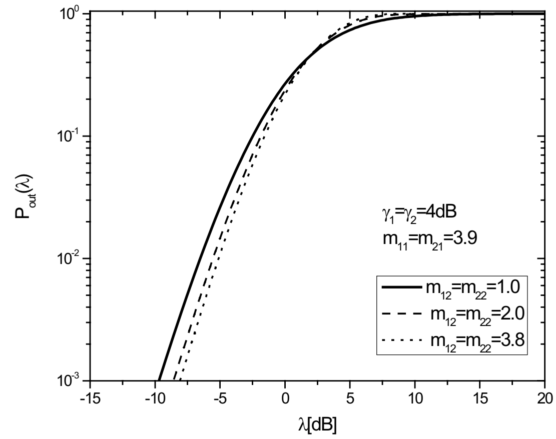

Fig. 5. Outage probability depending on fading severity interference

Conclusions

This paper presents the analysis of dual-hop system in Nakagami-m environment for different values of system parameters. Realistic cases of the system were considered therefore it refers to different values of parameters in first and second transmitting segment of the system. The paper includes derivations of equations for minimum PDF of instantaneous SIR values as well as outage probability used for system analysis. Presented figures show that the outage probability is higher for lower average SIR values as well as for higher fading severity to which the desired signal is exposed. Interference fading severity influence on the system performance depends on the threshold.

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In this paper, performance analysis of dual-hop transmission system is presented. The source-relay (S-R) and relay-destination (R-D) channels experience Nakagami-m fading. The generalized case in which desired signal and cochannel interference (CCI) experience mutually different amount of fading is considered. Furthermore, analysis includes the case in which fading severity in S-R and R-D links can be unequal which is real scenario in practice. Outage probability, as an important performance measure, is considered. As an illustration of mathematical formalism, numerical results are presented to show the effects of system and channel parameters on overall performance. Ill. 5, bibl. 5 (in English; abstracts in English and Lithuanian).

A. D. Cvetkovic, M. C. Stefanovic, N. M. Sekulovic, E. S. Mekic, D. Manic. Dvipakopės sistemos analizė Nakagami-m aplinkoje // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2011. – Nr. 2(108). – P. 47–50.

Analizuojamas dvipakopės perdavimo sistemos našumas Nakagami-m aplinkoje. Paprastai norimo signalo ir gretimo kanalo interferencija yra nykstamai skirtingos. Toks reiškinys dažnai pasitaiko praktikoje. Našumas įvertinamas naudojant išėjimo tikimybę. Pateiktos matematinės išraiškos parodo kanalo parametrų įtaką sistemos našumui. Il. 5, bibl. 5 (anglų kalba; santraukos anglų ir lietuvių k.).