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Impact of Control System Structure and Performance of Inventory Goods Flow System with Long-Variable Delay

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***Abstract*—In this paper, we use a mathematical model of a inventory system with time-varying delivery delay and three control systems in order to compare their properties for the goods flow optimization problem in the inventory systems.**

Structures of the chosen control systems are based on mathematical discrete equations: a periodic inventory system with adaptive maximal inventory level and perpetual inventory system with adaptive order quantity level as well as the methods proposed by the authors in previous works based on the Smith predictor. The selection of the control systems parameters is done by solving optimization tasks for a specific scenario of time-varying market demand using a genetic algorithm in Matlab/Simulink.

In this article, we mainly want to compare which of the control structures is able to achieve a high service level with maintaining of a given inventory maximal level in response to assumed consumer demand scenario.

***Index Terms*—Inventory control; Delay systems; Discrete-time systems; PD control.**

I. Introduction

Supply-chain risks can become full-fledged supply-chain problem, causing unanticipated changes in flow due to disruptions or delays. The proper control system is essential. Decisions concerning inventory control are associated with order batching and when to order. Acting as a buffer to smooth production in response to demand fluctuations is determined as the main role of an inventory. In fact, there are plenty of reasons for keeping inventories. The most significant is to satisfy the demand during the replenishment period in order to prevent loss of orders. More important reasons for obtaining and holding inventory can be find in [1]: predictability, fluctuations in demand, unreliability of supply, price protection, quantity discounts, lower ordering costs. It is utmost important to maintain the right balance between demand and orders with a view to minimizing costs. What is more, inventory is a protection against fluctuations in demand – it facilitates satisfying customer demands. Inventory costs generally fall into ordering costs and holding costs [1]. In the worst-case scenario, customer service goes down, sales are lost, lead times lengthen, costs go up, maximal level of inventory is increasing.

Over the years models of inventory systems have been being created in a variety of areas. In this paper, the model of inventory proposed by the authors is considered. We want to extend our research [2]–[4] over the control systems taken from literature and compare it with originally created system which has a better or similar performance.

There are more and more methods of improving the flow of goods in the inventory systems, which in turn use more and more advanced control techniques [3]–[10]. In [7] linear stationary discrete system with a fixed delay to the effective control of storage systems with perishable goods, using methods based on sliding-mode control is considered. In [8] it is used a linear-quadratic control in order to reduce the risk of bullwhip effect.

The paper presents a comparative analysis of the impact of the control system structure and the performance of the inventory goods flow system. Control systems structures based on mathematical discrete equations are given for optimization: a periodic inventory system with adaptive maximal inventory level and perpetual inventory system with adaptive order quantity level as well as the methods proposed by the authors in previous works based on the Smith predictor. Parameters were selected for each of the control system structures through solving optimization tasks for a specific scenario of variable market demand using a genetic algorithm.

II. The Mathematical Model

A hybrid discrete automata model is used to describe the system. Number of products that could potentially be sold from the store is modelled as a certain, unknown in advance limited function of time: $0 \le d(k) \le d_{\max}$, where $d_{\max}$ is the maximum number of products sold per unit of time. Time-varying values of variable $d(k)$ depend on the market demand. If the stock level at moment k is sufficiently high, then demand will be fulfilled, it means that $d(k) = h(k)$. Shipped products from the inventory $h(k)$ depends on the demand, as well as the available stocks $y(k)$ and following inequalities are held:

$$0 \le h(k) \le d(k) \le d_{\max},\ 0 \le y(k) \le y_{\max}. \quad (1)$$

Regardless of constant changes in customer demand, it is

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important to maintain accurate stocks in the inventory. A crucial matter is to avoid a situation in which stocks levels are critically low or the quantity of the stored products is excessive, or even exceeds the storage capacity y_{max}. In order to take into account variable time delays in the model associated with product accumulation in anticipation of transport, we introduce shipping rate at moment k defined as follows

$$q(k) = \begin{cases} 0 \text{ - shipment,} \\ 1 \text{ - waiting for transport.} \end{cases} \quad (2)$$

The product quantity awaiting shipment at time k is defined by the following relationship

$$x(k) = q(k-1)x(k-1) + u(k-\tau_p). \quad (3)$$

The product quantity stored in the inventory at moment k, called the stock, is therefore given as follows

$$y(k) = y(k-1) + (1-q(k))x(k-\tau_s) - h(k), \quad (4)$$

where $x(k) \geq 0$, $u(k) \geq 0$, τ_p – production delay – related to the time required to produce or complete the orders, τ_s – forwarding delay – the time interval indispensable to transport the ordered products to the inventory without waiting time for transport.

For the needs of this work, it is assumed that the inventory model has one supplier. The results can be easily extended to the case with more suppliers with the similar delivery time, i.e. total production, shipping and forwarding delays from each supplier. Moreover, on the basis of the balance of products in the inventory, it is clear that products only have to cumulate or be sold to the customers.

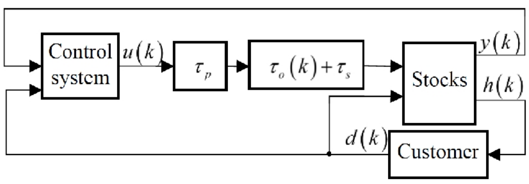

Fig. 1. Block diagram of inventory system with control.

Assuming that known are the delays τ_p, τ_s, function $q(k)$ for $k \in \mathbb{N}$ the system is discrete-time, non-stationary, linear with signals saturations, for which methods can be used, given in inter alia [11]–[13]. The block diagram of the analysed system is shown in Fig. 1. The system consists of three main blocks: production, transport and storage. Time-dependent delay $\tau_o(k)$ is associated with waiting for a mean of transport and accumulation of orders.

III. THE CONTROL SYSTEMS DEFINITIONS

The two basic questions any inventory control system must answer are when and how much to order. There are several replenishment techniques attempt to answer this fundamental questions.

Control system determines a certain amount of products to be ordered on the basis of market demand. As a result, a class of inventory models has been designed to cope with situations where demand level fluctuates.

The two classic systems for managing independent demand inventory are periodic review and perpetual review systems [14].

It is essential to create a mathematical description of investigated control systems:

A. Periodic Inventory System with Adaptive Maximal Inventory Level

It is a modified version of the classical periodic inventory system which gives an opportunity to the adaptation of the maximum level of inventory in stock. The block diagram is depicted in Fig. 2. It has been put additional factor k_1 in order to enable adaptation depending on the market demand. Factor k_1 together with k_2 make an affine function of reference stock level depending on market demand, given in the following form

$$u(k) = \begin{cases} k_1 d(k) + k_2 - y(k), & \text{for} \quad q(k-\tau_p) = 0, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

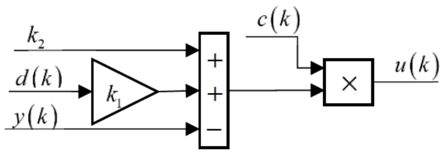

Fig. 2. A block diagram of the control system for the periodic inventory system with adaptive reference stock level.

B. Perpetual Inventory System with Adaptive Order Quantity Level.

A perpetual system is responsible for continuous record-keeping of the inventory level, and it replenishes when the stock drops to certain level k_3. The reorder point is fixed, but review period, order quantity and maximal stock level are variable (max inventory level depends on demand). The structure of the system is shown in Fig. 3. Factor k_1 together with k_2 make an affine function of maximal stock level depending on market demand, given in the following form

$$u(k) = \begin{cases} 0, & \text{for} \quad y(k) > k_3, \\ k_1 d(k) + k_2, & \text{for} \quad y(k) \leq k_3, \end{cases} \quad (6)$$

where k_3 – reorder point.

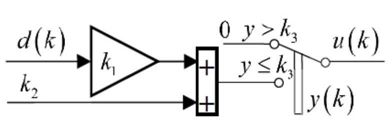

Fig. 3. A block diagram of the control system for the perpetual inventory system with adaptive order quantity level.

C. Proportional-Derivative Inventory Control System with Smith Predictor and Adaptive Reference Stock Level

Proposed control system consists of a classical structure of inventory model with Smith predictor. It is a type of predictive controller for control systems with long and inevitable delays. Its structure is based on implementations of the model without delay and with delay.

Based on the control concepts for systems with delays using a Smith predictor it is assumed that an estimated model of the system without delay is given in the form

$$\hat{y}_p(k) = \hat{y}_p(k-1) + u(k-1) - h(k). \tag{7}$$

Error of approximate model without delay and error of approximate model with delay is given by:

$$\varepsilon(k) = y_{ref}(k) - \hat{y}_p(k), \tag{8}$$

and the PD controller is evaluated in the following form:

$$u(k) = k_2\varepsilon(k) + k_3(\varepsilon(k) - \varepsilon(k-1)), \tag{9}$$

It is assumed that the reference value of stocks $y_{ref}(k)$ is a linear function of the demand [3] given in the form of

$$y_r(k) = k_1 d(k). \tag{10}$$

Figure of the control system is shown in Fig. 4. The variables $k_1 - k_3$ are parameters of the control system.

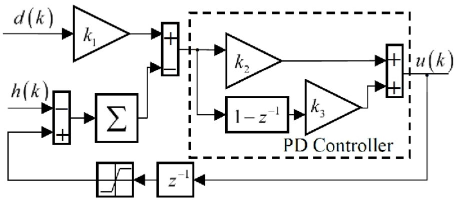

Fig. 4. A block diagram of the control system for the proportional-derivative inventory control system with Smith predictor and adaptive reference stock level.

Similarities between the considered class of systems and engineering processes allow to apply control-theoretic methods in the design and analysis of strategies managing the flow of goods in inventories.

IV. OPTIMIZATION CRITERION

It is extremely important to use the simplest optimization model for a certain problem in numerical optimization issues – as simple as possible. Simplification of optimization model not only contribute to reduce the calculation time but also enables you to find solutions closer to the global minimum repeatedly in the case of multimodal issues.

The issue of optimal control requires a mathematical formulation of the process performance index to be optimal. Consider the problem of finding the optimal values of the parameters k_i $(i = 1,2...4)$ of a dynamical system with fixed its structure from Fig. 2–Fig. 4. In the case of the inventory system indicators can be described by the following relations:

$$j_1 = \frac{1}{N}\sum_{k=n_0}^{N}(d(k) - h(k)), \tag{11}$$

$$j_2 = \frac{1}{N}\sum_{k=n_0}^{N} y(k), \tag{12}$$

where $n_0 = \tau_p + \tau_s + \tau_o(\tau_p)$ is a cumulative delay for $u(k=0)$, N is the length of the time horizon.

The equation (13) represents a lost opportunity to make sales. In turn, the expression (14) concerns use of space in the inventory. These indicators are associated with financial costs. Its form is based on the physical interpretation of the problem.

In the present case, scalarization of the objective function is used to form of the weighted sum

$$j = w_1 j_1 + w_2 j_2, \tag{13}$$

where w_1 and w_2 are weighting factors and j can be evaluated in arbitrary currency.

For the model described by relationships (1)–(4) and the control systems described by equations (5)–(12) and a quality indicator in the form of (13)–(15) the optimization problem can be defined in the following form

$$\min_{\mathbf{k}} j, \tag{14}$$

where optimization variables and constraints are based on the controller structure:

$$\mathbf{k} = [k_1, k_2],\ k_1 \geq 0,\ k_2 \geq 0 \ \text{ for eq. (5)},$$
$$\mathbf{k} = [k_1, k_2, k_3],\ k_1 \geq 0,\ k_2 \geq 0,\ k_3 \geq 0 \ \text{ for eq. (6)},$$
$$\mathbf{k} = [k_1, k_2, k_3],\ k_1 \geq 0,\ k_2 \geq 0,\ k_3 \geq 0 \ \text{ for eq. (7)–(10)}.$$

The parameters k_1 – k_3 of controller are non-negative because it is assumed that returns are not taken into consideration. According to this and to the requirements of negative feedback loop, including stability issues, the output signal of the controller cannot be negative. The proposed system is aimed at determining the optimal size of deliveries. It minimizes the cost ratio consisting of the average weighted total inventory costs including the cost of stocking and maintenance costs and lost benefits, reducing the risk of stoppages.

V. SIMULATION RESEARCH AND ANALYSIS

In this section we conduct computer simulations. The structures of control systems in Fig. 2–Fig. 4 are applied. The main purpose of this section is to compare properties of three different control structures: periodic inventory system with adaptive maximal inventory level and perpetual inventory system with adaptive order quantity level, present in economic environment and proportional-derivative inventory control system with Smith predictor and adaptive reference stock level proposed by authors.

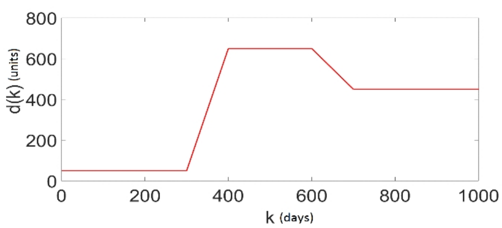

Fig. 5. Presumed function of market demand for products – trapezoidal signal.

With a view to simulation research of the control systems for a discrete, non-stationary hybrid model described by equations (1)–(4), the control systems described by equations (5)–(12) and the quality indicator in the form of

(13)–(15), the following values of the system parameters are held: $\tau_p = \tau_s = 14$, $w_2 = 1$, $n_0 = 42$, $N = 1000$. For the simulations purposes, it is assumed that w_1 is either 60 or 530.

The sampling period is 1 day. Specific time-varying market demand functions shown in Fig. 5 and Fig. 6 are taken into consideration. Demand signal is shown in Fig. 5 is the assumed scenario proposed by the authors. The presumed function of market demand consists of step change at k=0. Next, it is constant until k=300. After that, it linearly increases with slope 6 products per day for 100 days, then it is constant for 200 days. From k=600 the presumed demand decrease with slope –1.5 products per day until k=700 and then it is constant until the end of the scenario (k=1000). This demand signal has these different values in order to show the control system performance at different conditions. The signal in Fig. 5 is used for verifying optimal controller parameters.

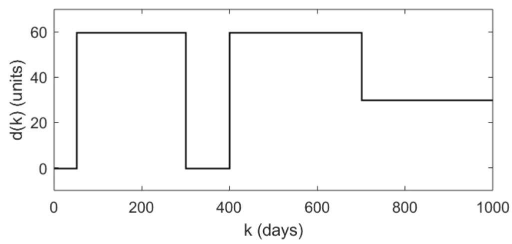

Fig. 6. Presumed function of market demand for products – rectangular signal.

As a function that controls the periodic time-variability of the model $q(k)$ it is assumed following periodic function

$$q(k) = \begin{cases} 0 & \text{for} \quad \operatorname{rem}(k,14) = 0, \\ 1 & \text{for} \quad \operatorname{rem}(k,14) \neq 0, \end{cases} \tag{15}$$

where the function rem is the remainder of the division.

Tuning of the control system is based on the criterion (1) and trapezoidal demand signal plotted in Fig. 5. On the basis of the results we try to evaluate: how does the controller structure impact on the properties of the inventory control system. In order to verify the solution for different scenario – rectangular demand test signal plotted in Fig. 6 are employed.

Before developing the theoretical argumentation, it is important to first establish definitions and foundational concepts underlying this research.

To solve the optimization problem (17) in order to determine parameters of controllers, a genetic algorithm with parameters: population size 50, elite count: 2, crossover fraction: 0.8 is used. The genetic algorithm has been receiving great attention and it has successfully been applied to other problems in the supply chain environment [15]–[18]. All variables used for simulation are summarized in the Table I.

Pareto front with shortages cost j_1 and holding cost j_2 calculated for 11 logarithmic spaced weights in the range $< 40,1000 >$ and trapezoidal signal is depicted in Fig. 7.

Results for PIS-AOQL control system are marked by blue triangles and blue smoothing line. Points of Pareto front for PIS-AMIL are marked by green 'x' and green smoothing line whereas for PDIS-SP-ARSL control system are marked by yellow squares and yellow smoothing line.

TABLE I. VARIABLES USED FOR SIMULATION.

Variable	Value
τ_p production delay	14 days
τ_s forwarding delay	14 days
N simulation horizon	1000 days
w_1 weighting factor of shortage costs	60 or 530
w_2 weighting factor of holding costs	1
n_0 cumulative delay	42 days
population size	50
elite count	2
crossover fraction	0.8

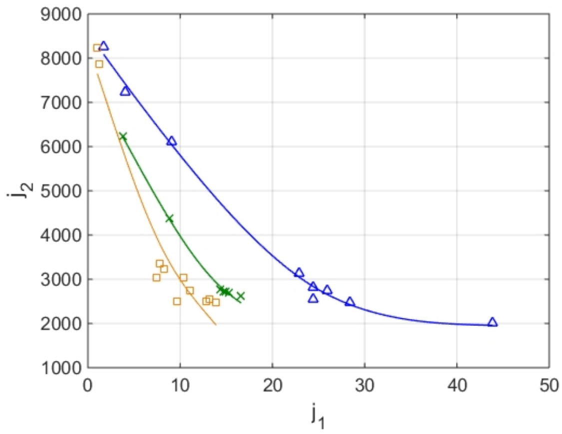

Fig. 7. Pareto front for inventory control system for trapezoidal signal.

It can be noticed from Fig. 7 that solutions for PDIS-SP-ARSL control system are non-dominated although results for 'x' and PDIS-SP-ARSL are close to each other. The solutions for PIS-AOQL are dominated by PDIS-SP-ARSL and PIS-AMIL however they are also relatively near the non-dominated solution of PDIS-SP-ARSL.

In order to compare the different control systems, consequently marked by square, 'x' and triangle some additional evaluations are made. One of them are plot in the objective space for rectangular demand shown in Fig. 8 with controllers optimized for trapezoidal demand depicted in Fig. 6. The results do not differ significantly from each other.

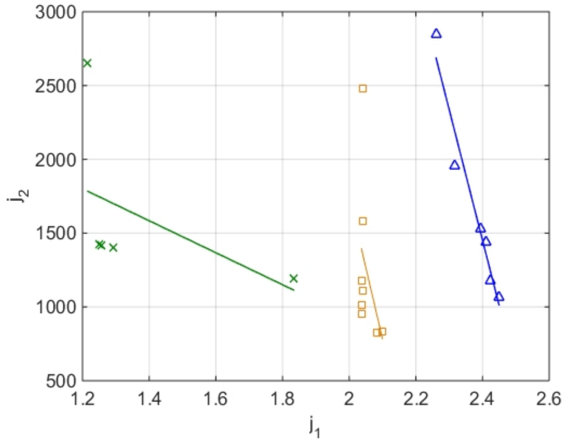

Fig. 8. Pareto front for inventory control system for rectangular signal.

Solutions of PDIS-SP-ARSL control system are non-dominated over the solutions for other two control systems structures. It means that the phenomenon of shortages and inventory redundancy occurs less than in other two systems.

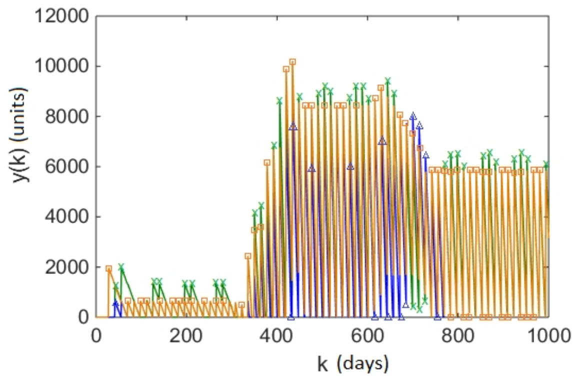

Fig. 9. The level of stocks of three control systems vs. the discrete time for the presumed function of market demand for trapezoidal signal, where w_1=60.

System responses $y(k)$ of the control system are shown in Fig. 9–Fig. 12. Responses for w_1=60 are close each other. Although for the weight w_1=530 indicates the weaknesses of the PIS-AOQL structure. The main reason for is that the system response in Fig. 11 is characterized by inappropriately large increase of stock in the initial period of the simulation. The peak is almost 4 times larger than in the structures PIS-AMIL and PDIS-SP-ARSL. A similar situation can be seen in Fig. 12.

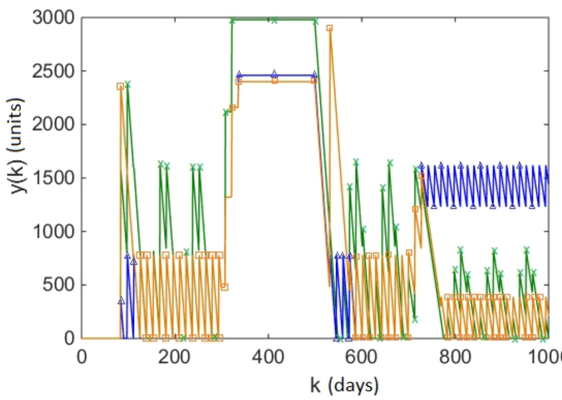

Fig. 10. The level of stocks of three control systems vs. the discrete time for the presumed function of market demand for rectangular signal, where w_1=60.

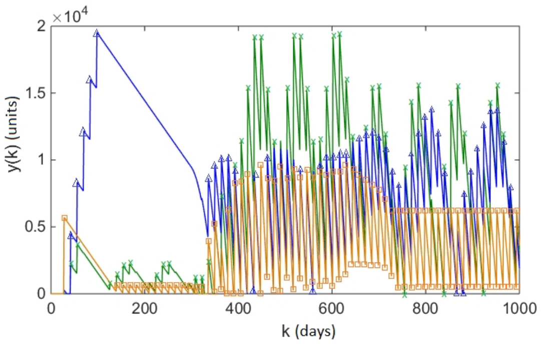

Fig. 11. The level of stocks of three control systems vs. the discrete time for the presumed function of market demand for trapezoidal signal, where w_1=530.

In order to finalize the comparison we analyse the stock level response for different control systems, weights w_1 and customer demands. Let's now move on to the system response which represents inventory levels. On the basis of the inventory stock, we can decide how control systems react for the demand and have an opportunity to differ them.

One the other hand, the same formulation in case of triangular signal shows that the non-dominated set contains subsets of solutions for PDIS-SP-ARSL and PIS-AMIL. It can therefore be concluded that the PDIS-SP-ARSL and PIS-AMIL control system have better robustness for the changes in the assumed scenario of customer demands.

Time responses for systems PIS-AMIL and PDIS-SP-ARSL remain adequate level of orders and stocks, also in a wide range of changes of w_1. Inventory control system has to provide weighted balance between this two indicators: j_1 and j_2.

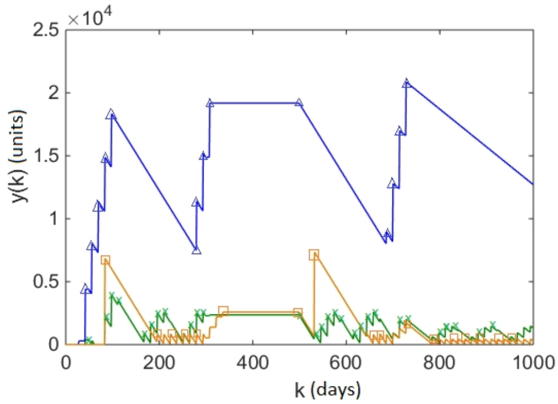

Fig. 12. The level of stocks of three control systems vs, the discrete time for the presumed function of market demand for rectangular signal, where w_1=530.

The analysed results for the structures PIS-AMIL and PDIS-SP-ARSL are similar. However, by comparing the obtained value of the cost indicator j, presented in Fig. 7 and Fig. 8 it can be noted that for small values of w_1 is the little advantage of the system PDIS-SP-ARSL for both trapezoidal and rectangular signal.

Our analysis shows that adaptive reference stock level has a smoothing effect on the order variability.

CONCLUSIONS

The overall objective of inventory management is to achieve satisfactory levels of customer service while keeping inventory costs within reasonable bounds. The balancing act between liquidity and profitability is key to good inventory management.

Regarding the objective, a decision maker has to make two fundamental decisions: the timing and size of orders (i.e., when to order and how much to order).

The risk of shortage can be reduced by holding safety stock, which is a stock in excess of average demand to compensate for variability in demand and lead time. It is important, that safety stock cannot be high.

The main factor of the control selection is to minimize costs and losses relating to holding costs. Shortage is extremely disadvantageous situation – the demand is greater than the amount of products, because orders have not keep pace with the time-variable needs of customers.

On the basis of empirical analysis, it was found that the structure of the control system has a great impact on the performance of the inventory goods flow system with long-variable delay. The goal was to minimize the total cost of the system and maximize service level simultaneously. The structure of the PIS-AOQL provides the worst reaction for high varying demands, the reference inventory level is higher almost 3-4 times than in other systems. The system with adaptive reference stock level PIS-AMIL has much better performance. Pareto front for inventory control system for trapezoidal signal in Figure 6 shows that PDIS-SP-ARSL has the best values of indicators which represents lost opportunities to make sales and use of space in the inventory.

For future study, we will focus on more complex systems e.g. hybrid systems associated with supply chain and taking into account the specific effects occurring in such objects.

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