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Design of Forward Adaptive Piecewise Uniform Scalar Quantizer with Optimized Reproduction Level Distribution per Segments

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Introduction

The primary goal of the quantizer design is to determine the reproduction levels and the partition regions or cells such as to provide the minimum possible distortion for a fixed number of quantization levels N, or equivalently a fixed resolution $R=log_2N$ [1, 3]. A quantizer support region can be divided into a variety of ways. If a quantizer support region consists of several segments, each of which contains several quantization cells and reproduction levels corresponding to a uniform quantizer, the quantizer is a piecewise uniform one [1, 2]. The piecewise uniform scalar quantizers (PUSQ) are widely used in practice due to their simple encoding procedure, which, in contrast to that of the nonuniform quantizer models, does not require the full search of the quantizer code book [1, 2]. The prevailing international standard for digital telephony, known as G.711 standard, defines a symmetric PUSQ by 8 bits of resolution and L=8 positive segments increased in length by a factor of 2 for each successive segments having 16 cells [1]. Particularly, the G.711 quantizers based on a piecewise uniform approximation to the A-law and μ-law compressor characteristics [1] divide the support region into a $2L$=16 unequal segments, each of which has equal number of cells. Accordingly, the G.711 quantizer can be considered as a set of $2L$ uniform quantizers having an equal number of quantization levels, but the unequal support regions. Namely, each uniform quantizer, from the set of $2L$ quantizers composing the PUSQ, is designated to quantize the signals belonging to the corresponding segments so that, when the sample of the input signal to be quantized is within the i-th segment, the corresponding i-th uniform quantizer is then used. Along with the support region partition, according to the mentioned piecewise linear compressor characteristics, there are some novel propositions of the support region partition, i.e. of the piecewise uniform quantizer design. For instance, in the reference [4], the robustness conditions of the PUSQ based on a piecewise uniform approximation to the optimal compressor law are analysed. Additionally, in the reference [5], there is a proposition of the piecewise uniform vector quantizer that considers an unequal number of cells within the segments. However, in the reference [5], in the conclusion of the paper, the authors have highlighted that the main drawback of their method is that they do not have a manner for deciding how to determine the segments into which to divide the support region of a piecewise uniform quantizer for an arbitrary signal distribution. This comment has motivated us to propose an intuitively obtained solution, i.e. to research the performance of the PUSQ that defines the equidistant support region partion and the optimized distribution of the cells (or reproduction levels) within such defined segments. What is actually proposed in this paper is a PUSQ composed of $2L$ uniform quantizers having equal support regions, but possibly different number of quantization cells. Moreover, in order to provide an optimal manner of reproduction level distribution per segments, the granular distortion optimization is proposed, while such a constrained optimization problem is solved using the method of Lagrange multipliers [1].

Generally, the optimization of a quantizer design is based on the exact statistical knowledge of the signal to be quantized [1, 2]. However, if the signal statistics is not completely known, or it varies with time, it is of practical interest to study the performance degradation of the quantizer designed for certain input statistics, but used for a different set of statistics. Accordingly, for the considered PUSQ, which is optimally designed for the unit variance, the performance determined in the wide variance range of the input speech signals are analyzed in the third section of the paper. Moreover, since the main trade off in scalar quantizer design is making the quantization step size large enough to accommodate the maximum peak-to-peak range of the input signal while keeping this step size small enough to minimize the quantizer distortion, this paper provides an additional performance analysis of the forward adaptive version of the proposed PUSQ model. The third section of the paper is also devoted to the conclusions which summarise the contribution achieved in the paper.

The novel piecewise uniform scalar quantizer with an optimized reproduction level distribution per segments

In this section, the fundamental concepts of a PUSQ are introduced, followed by an explanation of the novel PUSQ model. After that, the brief theory of the forward adaptive technique is presented, then it is applied during the proposed PUSQ design. The PUSQ quantizer partitions the quantizer support region into $2L$ subsets named quantization segments. Within each segment, the quantizer appears to be uniform with the particular cell length that may differ from segment to segment. The novel PUSQ proposes the equidistant support region partion

$$t_i^{\mathrm{seg}} = i\frac{x_{\max}}{L},\ i = 0,1,...,L, \tag{1}$$

where t_i^{seg}, i=0,1,…,L denote the segment thresholds and $x_{\max}$ denotes the support region threshold. In other words, we propose equal lengths of the segments, but possibly different number of reproduction levels within them. Mind the fact that due to the symmetry of the PUSQ, only the positive segment thresholds are defined. Under the assumption of the equidistant reproduction level distribution per segments, each cell length of the considered PUSQ can be determined by

$$\Delta_i = \Delta_{i,j} = \frac{t_i^{\mathrm{seg}} - t_{i-1}^{\mathrm{seg}}}{N_i} = \frac{x_{\max}}{LN_i},\ i = 1,...,L,\ j = 1,...,N_i, \tag{2}$$

where N_i is the reproduction level number within the corresponding i-th segment. Observe that indexes i and j indicate the j-th cell within the i-th segment $(t_{i-1}^{\mathrm{seg}}, t_i^{\mathrm{seg}}]$. If a current amplitude value of the input signal falls in the j-th cell within the i-th segment $(t_{i,j-1}, t_{i,j}]$

$$t_{i,j} = t_{i-1}^{\mathrm{seg}} + j\Delta_i,\ i = 1,...,L,\ j = 1,...,N_i, \tag{3}$$

the quantization rule provides its coping onto the near allowed value $y_{i,j}$ defined by the code book of the proposed quantizer

$$y_{i,j} = t_{i-1}^{\mathrm{seg}} + \frac{(2j-1)}{2}\Delta_i,\ i = 1,...,L,\ j = 1,...,N_i. \tag{4}$$

Let us now define the granular distortion of the considered PUSQ [1, 2]

$$D_{\mathrm{g}} = 2\sum_{i=1}^{L}\frac{\Delta_i^2}{12}P_i, \tag{5}$$

where P_i denotes the probability of belonging the input sample x of variance σ^2 to the i-th segment. It is commonly accepted that the Laplacian probability density function (pdf) is a good approximation to the actual distribution of speech samples. For the assumed Laplacian pdf

$$p(x) = \frac{\sqrt{2}}{2\sigma}e^{-\frac{|x|\sqrt{2}}{\sigma}}, \tag{6}$$

we can derive the following closed form expressions for the probablities P_i, i=1,...,L and the overload distortion, respectively:

$$P_i = \int_{t_{i-1}^{\mathrm{seg}}}^{t_i^{\mathrm{seg}}} p(x)dx = \frac{1}{2}\left(\exp\left(-\frac{\sqrt{2}t_{i-1}^{\mathrm{seg}}}{\sigma}\right) - \exp\left(-\frac{\sqrt{2}t_i^{\mathrm{seg}}}{\sigma}\right)\right), \tag{7}$$

$$D_{\mathrm{o}} = 2\int_{x_{\max}}^{\infty}\left(x - y_{L,N_i}\right)^2 p(x)dx, \tag{8}$$

$$D_{\mathrm{o}} = \exp\left(-\frac{\sqrt{2}x_{\max}}{\sigma}\right)\left(\left(\frac{\Delta_L}{2} + \frac{\sigma}{\sqrt{2}}\right)^2 + \left(\frac{\sigma}{\sqrt{2}}\right)^2\right). \tag{9}$$

Now we can highlight the fact that for a given support region $[-x_{\max}, x_{\max}]$ and the number of segments $2L$, under the given constraint that the total number of reproduction levels is N, the granular distortion of the considered quantizer for the input signal of variance σ^2 is the function of the reproduction level distribution per segments. The common approach to solving constrained optimization problems is based on the method of Lagrange multipliers [1, 2]. This method converts the constrained optimization problem

$$\min_{N_i}\{D_{\mathrm{g}}\} \text{ subject to constraint } 2\sum_{j=1}^{L}N_j = N \tag{10}$$

into an unconstrained one:

$$J = D_{\mathrm{g}} + \lambda\sum_{j=1}^{L}N_j,\ \frac{\partial J}{\partial N_i} = 0,\ i = 1,...,L, \tag{11}$$

$$\frac{\partial J}{\partial N_i} = \frac{\partial\left(\frac{x_{\max}^2}{6L^2}\sum_{j=1}^{L}\frac{P_j}{N_j^2}\right)}{\partial N_i} + \frac{\partial\left(\lambda\sum_{j=1}^{L}N_j\right)}{\partial N_i} = 0,\ i = 1,...,L, \tag{12}$$

where λ is a Lagrange multiplier. Since the probablities P_i, i=1,...,L are not a function of N_i, we can simply obtain the following expression

$$N_i = \sqrt[3]{\frac{P_i x_{\max}^2}{3\lambda L^2}},\ i = 1,...,L, \tag{13}$$

which further, in combination with the constraint given by (10), lead to the closed form formula for the optimal number of reproduction levels per segments

$$N_i = \frac{NP_i^{1/3}}{2\sum_{i=1}^{L}P_i^{1/3}},\ i = 1,...,L. \tag{14}$$

In such a way, the considered PUSQ is completely specified and hence, both the granular and the overload distortion can be determined along with the signal to quantization noise ratio [1, 2]

$$\mathrm{SQNR} = 10\log\left(\frac{\sigma^2}{D}\right) = 10\log\left(\frac{\sigma^2}{D_{\mathrm{g}} + D_{\mathrm{o}}}\right). \tag{15}$$

Although the probability density function of speech signal is known and can be used in a quantizer design process, the variations in speech statistics can reduce the performance of any fixed quantizer [6, 7]. As it was briefly mentioned earlier, this can be overcome by using the forward adaptive technique, i.e. by estimating the variance of the current speech segment prior to quantization and hence, adjusting the quantizer levels accordingly [1, 2]. The forward adaptation can be performed by normalizing the input sequence, by further quantizing with fixed quantizer and finally by performing the denormalization procedure with the same quantized value of the calculated gain that was used for the normalization [1, 2]. The same effect can be achieved by using an adaptive quantizer with a code book obtained by multiplying the code book of the aforementioned fixed quantizer with the quantized value of the estimated gain. Hence, adapting the codebook of the PUSQ, the same formulas (5), (9), (15) can be used to estimate the performance of the forward adaptive PUSQ.

In this paper, we consider forward adaptive scheme (Fig. 1) composed of a buffer, an adaptive N-level PUSQ, the gain estimator and the N_g-level scalar quantizer for gain quantizing (SQ_{gq}). Particularly in this paper, the log-uniform scalar quantizer for gain quantizing rather than the uniform scalar quantizer is implemented, since we have recently demonstrated that it could provide higher SQNR [8]. The design procedure of the proposed forward adaptive PUSQ is composed of the following steps:

- *Step 1. Design of non adaptive (fixed) PUSQ for the reference variance* ($\sigma_{\mathrm{ref}}^2=1$) is based on finding the thresholds $t_{i,j}$, $i=1,...,L$, $j=1,...,N_i$ (3) and the reproduction levels $y_{i,j}$ $i=1,...,L$, $j=1,...,N_i$ (4).
- *Step 2. Buffering of the input signal and the gain estimation* **-** Buffering frame after frame enables an estimation of the gain, defined as $g=\sigma/\sigma_{\mathrm{ref}}$, i.e. as a ratio of the squared root of the current frame variance and squared root of the reference variance.
- *Step 3. Quantization of the estimated gain* by using the N_g-level log-uniform scalar quantizer SQ_{gq}

$$20\log(\hat{g}=\hat{g}_k)=20\log(\sigma_{\min})+(2k-1)\frac{\Delta^{\mathrm{lu}}}{2},\ k=1,...,N_g, \quad (16)$$

where the variance range of the input signal in decibels $B=20\log(\sigma_{\max}/\sigma_{\min})$ is divided into N_g cells having equal lengths $\Delta^{\mathrm{lu}}=B/N_g$.

- *Step 4. Design of adaptive PUSQ*- The finding of decision thresholds and the reproduction levels of the adaptive PUSQ is performed by multiplying the appropriate thresholds and the reproduction levels of the non adaptive PUSQ (obtained in the step 1) with the quantized gain $\hat{g}$

$$t_{i,j}^a=\hat{g}t_{i,j},\ y_{i,j}^a=\hat{g}y_{i,j},\ i=1,...,L,\ j=1,...,N_i. \quad (17)$$

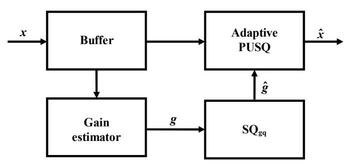

Fig. 1. Forward adaptive quantization scheme

Results

What is presented and discussed in this section are the performance that we have ascertained by applying the considered PUSQ and its forward adaptive version in quantization of signals having Laplacian pdf and a wide variance range (B=40 dB). Since the support region width has a great influence on the quantizer design and, consequently, on its performance [9], we have started our research with the support region optimization of the fixed PUSQ having L=8 positive segments and N=128 quantization levels. By designing the PUSQ for the unit variance and different support regions, we have numerically determined the value of the optimal support region threshold $x_{\max}^{\mathrm{PUSQ}}$= 8.9 that minimizes PUSQ distortion. In the same manner, we have optimized the support region threshold of the asymptotically optimal nonlinear compandor $x_{\max}^{\mathrm{c}}$= 9.5 having the same number of quantization levels [1]. Assuming such obtained support region thresholds, we have determined the appropriate SQNR characteristics, shown in Figs. 2. and 3. Namely, one way to determine how well the asymptotically optimal nonlinear compandor and the PUSQ match is to compare SQNR characteristics for the same input statistics and the number of quantization levels, but for a different number of segments (Fig. 2). Regarding such obtained SQNR characteristics one can conclude that starting from L=1 to L=16 SQNR characteristic of the PUSQ approaches to the one of the asymptotically optimal nonlinear compandor. Pay attention to the fact that in case of L=1, the considered PUSQ is a uniform quantizer. This notice justifies the obvious large performance degradation observed in such a case.

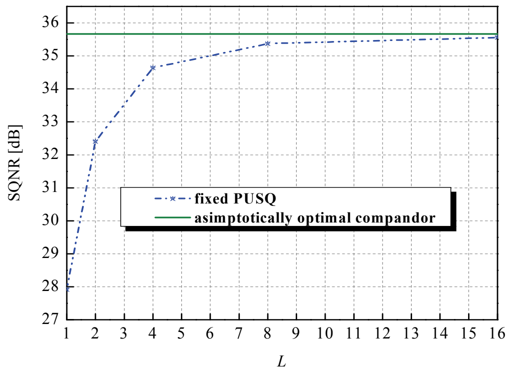

Fig. 2. Signal to quantization noise ratio as a function of segment number L for the bit rate of R=7 bit/sample

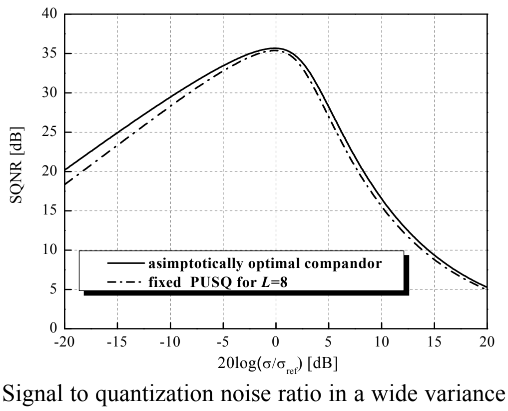

Fig. 3. Signal to quantization noise ratio in a wide variance range for the bit rate of R=7 bit/sample

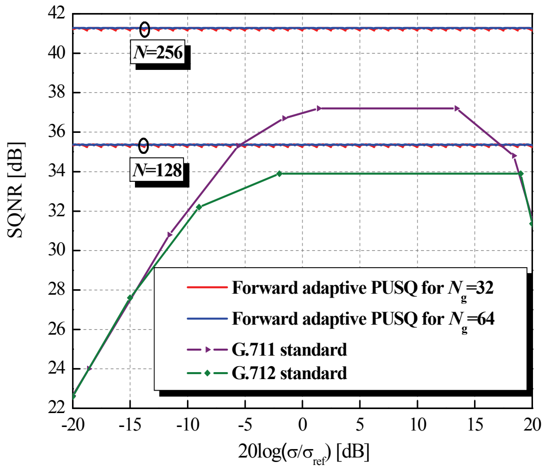

Fig. 4. Illustration of the signal to quantization noise ratio characteristics in the wide variance range

As it is obvious that the number of quantization levels N must be a large relative to L so that all of the segments are finely quantized, we have decided, for our further analysis, to assume L=8, as in the case of the G.711 quantizer. The Fig. 3 ilustrates the SQNR characteristics that we have obtained in the wide variance range under the mentioned asumption for the fixed PUSQ and the asymptotically optimal nonlinear compandor. It can be concluded that the fixed PUSQ achieves the performance near the one of the asymptotically optimal compandor, where the better match of the SQNR characteristics is achieved for the variances $\sigma^2>\sigma_{\mathrm{ref}}^2$, then in the case when $\sigma^2<\sigma_{\mathrm{ref}}^2$. We have also determined the SQNR charactersistics of the forward adaptive PUSQ (Fig. 4.) for the cases when the adaptive PUSQ and the log-uniform SQ_{gq} have N=128, N=256, N_g=32 and N_g=64 quantization levels, respectively. It is obvious that in the case of adaptive PUSQ having N=128 quantization levels, the proposed forward adaptive PUSQ can not completely overreach G.711 Recommendation. However, for the proposed model, this is not an issue in the case of adaptivne PUSQ having N=256 quantization levels. In such a case, from the Fig. 4., one can notice that the proposed forward adaptive quantizer provides a more constant and higher level of SQNR, which can be considered via the gain in the average SQNR of about 9 dB.

Conclusions

Since we have ascertained that the proposed quantizer satisfies the G.712 Recommendation [10] in the considered variance range, one can believe that it will find practical implementation in the high quality quantization of signals, which, as well as speech signals, have statistics modeled by the Laplacian pdf.

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The problem we address in this paper is the design of nearly optimal scalar quantizer in a wide variance range of the Laplacian input signals, using the piecewise uniform quantizers while restricting the class of quantizers to be forward adaptive. Particularly, the design procedure of the piecewise uniform quantizer with an equidistant support region partion and the optimized reproduction level distribution per segments is presented along with the design procedure of its forward adaptive version. Reproduction level optimization is performed by optimizing the granular distortion of the proposed quantizer using the method of the Lagrange multipliers. For the proposed model we study the influence of the segment number on the SQNR, as well as the SQNR robustness in a wide variance range. Since the results obtained for the assumed Laplacian distribution indicate the SQNR improvement over the G.711 standard, one can expect that the proposed quantizer will be effective in the quantization of signals having the same distribution and the time varying characteristics. Ill. 4, bibl. 10 (in English; abstracts in English and Lithuanian).

J. Nikolic, Z. Peric, A. Jovanovic, D. Antic. Adaptyvaus tolygiojo skaliarinio kvantoriaus, kurio reprodukcijos lygio pasiskirstymas segmentuose, yra optimizuotas, projektavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 3(119). – P. 19–22.

Nagrinėjamos problemos, su kuriomis susiduriama projektuojant artimą optimaliam skaliarinį kvantorių plačiame Laplaso įėjimo signalų ruože naudojant apibrėžtus tolygiuosius kvantorius. Pateikiamos tolygiųjų kvantorių su vienodai nutolusia palaikymo srities dalimi ir optimizuoto reprodukcijos lygio pasiskirstymo segmentuose projektavimo procedūros. Reprodukcijos lygio optimizavimas atliktas Lagranžo daugiklių metodu optimizuojant pasiūlyto kvantoriaus iškraipymus. Ištirta pasiūlytojo modelio SQNR segmentų skaičiaus įtaka, taip pat SQNR atsparumas plačiame kitimo ruože. Kadangi tariamam Laplaso skirstiniui gauti rezultatai rodo SQNR pranašumą, palyginti su G.711 standartu, galima tikėtis, kad pasiūlytasis kvantorius leis efektyviau kvantuoti signalus, turinčius tą pačią sklaidą ir laikui bėgant kintančias charakteristikas. Il. 4, bibl. 10 (anglų kalba; santraukos anglų ir lietuvių k.).