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Solar Irradiance Model for Solar Electric Panels and Solar Thermal Collectors in Lithuania

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Introduction

As the technology evolves and the price of fossil fuel resources grows rapidly, the increasing focus on renewable energy resources is observed. Much attention is paid to wind power plants [1, 2], systems composed of wind turbines, solar cells and batteries [3].

For the detailed analysis of electronic systems, using solar cells and solar thermal collectors, mathematical models of their constituent components must be developed. The primary input signal for such model is solar energy flux. Since the statistics about solar energy are collected in a limited amount of geographical areas, one must first develop a mathematical model of insolation reaching Lithuanian territory.

The paper describes the developed solar insolation model, based on the clear sky standard and used for electronic systems, which are managing solar cells and solar thermal collectors. Research data on the solar energy flux distribution in Lithuanian territory is presented.

Solar radiation

Solar radiation is a result of a fusion reaction where the hydrogen turns into helium. The amount of solar energy, falling on a unit area depends on latitude, altitude, year and time of a day.

Solar spectrum consists of wavelengths, which fall in the range of visible light, infrared and ultraviolet radiation. Solar radiation is similar to the ideal black body heated to 5800° K, radiation [4]. Solar spectrum above the Earth's atmosphere is calculated on the basis of the Planck's law as an absolute black body spectral radiance of electromagnetic radiation at all wavelengths $I(\lambda,T)$

$$I(\lambda,T)=\frac{\mathrm{d}W}{\mathrm{d}S\,\mathrm{d}t\,\mathrm{d}\lambda}=\frac{8\pi\mathrm{hc}}{\lambda^5}\cdot\frac{1}{\mathrm{e}^{\frac{\mathrm{hc}}{\lambda\mathrm{k}T}}-1}, \quad (1)$$

here dW – energy emitted from area dS during the time dt, where the wavelengths are in range from λ to λ+dλ; h - Planck's constant, c – speed of light, k – Boltzmann constant, T – absolute temperature of the black body.

Solar insolation *I(T)* and its spectral radiance $I(\lambda,T)$ related as follows

$$I(T)=\int_0^\infty I(\lambda,T)\,\mathrm{d}\lambda=\frac{8\pi^5\mathrm{k}^4}{15\mathrm{c}^3\mathrm{h}^3}T^4=\frac{4\sigma}{\mathrm{c}}T^4. \quad (2)$$

This relationship is obtained following a Stefan and Boltzmann Law. In expression (2) σ denotes the Stefan Boltzmann constant. The coefficient $4\sigma/\mathrm{c}$ is called radiation constant.

Extraterrestrial solar insolation

Earth rotates around the Sun in elliptic orbit. Therefore, when calculating extraterrestrial solar energy flux, elliptic orbit is assumed using eccentricity factor ε. Insolation $I_\mathrm{S}(n)$, in the chosen day of the year n is calculated by the following expression

$$I_\mathrm{S}(n)=I_\mathrm{S0}\varepsilon(n), \quad (3)$$

here $I_\mathrm{S0}=1367$ W/m^2 is a solar constant [4].

Sufficiently accurate results are obtained using the eccentricity factor $\varepsilon(n)$ calculated using following expression [4]:

$$\varepsilon(n)=1,00011+0,034221\cos\Gamma+0,001280\sin\Gamma+ \\ +0,000719\cos 2\Gamma+0,000077\sin 2\Gamma, \quad (4)$$

here Γ denotes the day angle in radians

$$\Gamma=\frac{2\pi(n-1)}{365}, \quad (5)$$

here $n=1, 2, 3, \ldots 365$ is number of the day in a year (January 1 $n=1$, December 31 is $n=365$).

According to expressions (3) – (5) it is possible to calculate that approximately on July 5 ($n=187$) the Earth is at the furthest point from the Sun – aphelion, and the first days of January and the end of December – closest to the sun – perihelion. For this reason, in the northern hemisphere summers are cooler and winters are warmer.

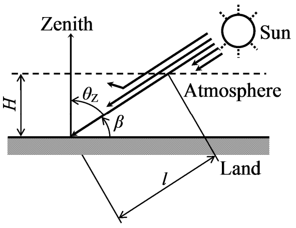


Fig. 1. Solar radiation falling to the ground plane

Model of the solar energy flux falling on Earths horizontal surface

On a way to Earth's surface solar rays have to pass Earth's atmosphere (Fig. 1). Travelling through the atmosphere part of solar rays is absorbed, reflected and scattered. Solar energy flux that is falling directly to the ground plane is estimated using the following expression

$$I_E(n) = I_S(n)\alpha\cos\theta_Z, \qquad (6)$$

here α – attenuation of solar energy flux in atmosphere, θ_Z – Zenith angle. This angle describes the height of the sun above the horizon. Zenith angle equals the angle formed by the sun's rays falling to land surface and the direction of zenith.

The absorption, reflection and scattering of solar rays depend on the weather and other conditions. Generated model simplification is relied on the clear sky standard definition. As a clear sky the gas medium is assumed, obtained mixing an optically black, gray and transparent gas in equal proportions. Solar flux attenuation at a clear sky atmosphere is calculated as follows [4]

$$\alpha = a_0 + a_1 e^{-\frac{k}{\cos\theta_Z}}, \qquad (7)$$

here a_0, a_1, and k – are the factors characterizing the attenuation of solar radiation in the atmosphere. These factors vary from mist (visibility) in the atmosphere and the observed surface height above sea level. The coefficients a_0, a_1, and k for a plane up to 2.5 km above sea level and at 23 km visibility, are calculated according to the following empirical expressions:

$$a_0 = 0{,}4237 - 0{,}00821(6-h)^2, \qquad (8)$$

$$a_1 = 0{,}5055 - 0{,}005958(6{,}5-h)^2, \qquad (9)$$

$$k = 0{,}2711 - 0{,}01858(2{,}5-h)^2, \qquad (10)$$

here h denotes the height above sea level in kilometres.

For calculations of solar energy flux at a horizontal plane, it is convenient to change the Zenith angle θ_Z to solar altitude (elevation) angle β (Fig. 1). These angles are linked by the relationship

$$\cos\theta_Z = \cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta. \qquad (11)$$

Zenith and solar altitude angles depend, as shown in Fig. 2, from observer location A geographic coordinates, distance from the sun to equator (declination) and time of day. Zenith and solar altitude angles used in the model are calculated using following expression [5]

$$\cos\theta_Z = \sin\beta =$$

$$= \cos\left(\frac{\pi}{180}\phi_{LA}\right)\cos\Delta\phi_M\cos\delta + \sin\phi_{LA}\sin\delta, \qquad (12)$$

here ϕ_{LA} – latitude angle of location A, in deg., $\Delta\phi_M = \phi_{MN} - \phi_{MA}$ – longitude difference angle between local noon ϕ_{MN} and location A ϕ_{MA}, rad, δ – distance from the sun to equator (declination) angle, rad.

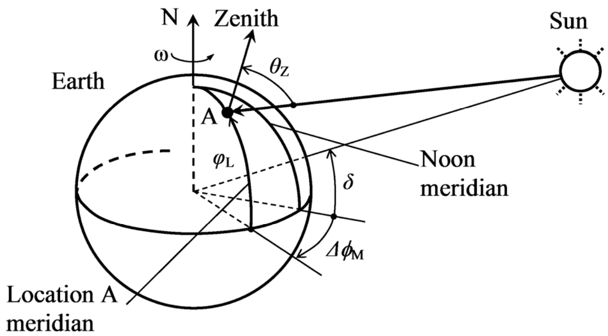


Fig. 2. Factors influencing zenith angle θ_Z at a location A

Declination angle δ – is the angle between the Earth equatorial plane and a line joining the Sun and the Earth centres. It defines the axis of the Earth heel, which is a key factor in determining the change of seasons. During the year it is changing, as shown in Fig. 3, from -23.45 ° (December 21 - Winter Solstice) and 23.45 ° (June 21 - Summer Solstice). The values of declination angle in radians are calculated using the following simplified expression [5]

$$\delta = 0{,}4093\sin\left[\frac{2\pi(n-81)}{365}\right]. \qquad (13)$$

As the Earth rotates around its axis, an angle $\Delta\phi_M$ used in the (12) changes by 2π radians during the day. For the practical applications it is useful to relate the model with solar (local) time. This is done by replacing the longitude difference angle between local noon meridian and the location $\Delta\phi_M$ with the following expression

$$\Delta\phi_M(t_{ST}) = \omega(12 - t_{ST}), \qquad (14)$$

here $\omega = \pi/12$ – an angular velocity of the Earth's rotation around its axis, rad/h, 12 – local noon time, h, t_{ST} – solar (local) time, $t_{ST} \in [0, 24]$, h. Until noon, $0 \le t_{ST}$

<12, $\Delta\phi_M(t_{ST})$ has positive values, but after the noon, $12 < t_{ST} \le 24$, $\Delta\phi_M(t_{ST})$ has negative values.

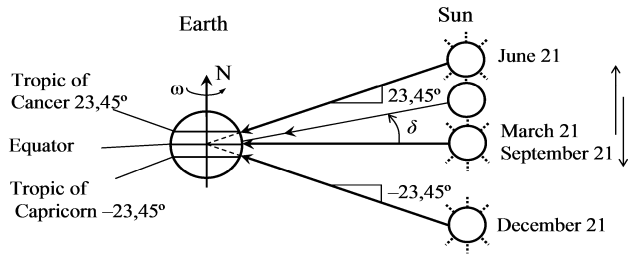


Fig. 3. Solar declination angle δ change during the year

Solar altitude angle cannot be negative. When $\beta < 0$ the sun is behind the horizon and its rays do not fall to the plane located at A. From these conditions it follows that zenith angle must always be positive and less than $\pi/2$, $0 < \theta_Z \le \pi/2$. For this reason, the condition is introduced in the model that separates day and night time periods:

$$I_E(n) = \begin{cases} I_S(n)\alpha\cos\theta_Z, & 0 < \theta_Z \le \pi/2, \\ 0, & \text{other } \theta_Z \text{ values.} \end{cases} \quad (15)$$

The results of analysis of solar insolation in Lithuania territory

Lithuania is located between 20°57' and 26°51' east longitude meridians and between 53°54' and 56°27' north latitude parallels. Using the model, developed in MATLAB environment according to (3) - (15), solar energy flux rates in 313369 locations of Lithuania were calculated. The geographic coordinates ϕ_{MA}, ϕ_{LA} and altitudes of analyzed locations there combined in three-dimensional matrix of 623×503. Distances between areas in meridian direction were taken every 0.01°, and in parallel direction – every 0.006°. Solar energy flow in the territory of Lithuania on 21 June ($n = 173$) calculation results are presented in Fig. 4, a.

For calculations it is accepted that $\Delta\phi_M(t_{ST}) = 0$ in expression (12). Therefore values of solar insolation at each location of Lithuania at noon are presented. The analysis of the Fig. let to show that at noon of June 21 average solar insolation in Lithuania territory is 407 W/m^2. Minimal values of insolation of 375 W/m^2 are observed in low-lying areas, while maximal values of 439 W/m^2, are at highlands (for example on Juozapinės hill). Thus the deviation of insolation values from the mean value is ± 8% in territory of Lithuania. Comparing obtained results with data presented in [6], it can be stated, that model gives appropriate results for modelling electronic systems. More detailed conclusions on the accuracy of the model will be made upon receipt of data from the solar thermal collector control system [7].

Fig. 4., b shows the solar insolation variation during the day in Vilnius, when the Earth is at critical points in its orbit around the sun. The abscissa axis represents the time zone (clock) time, which is adjusted for 1 h during summer.

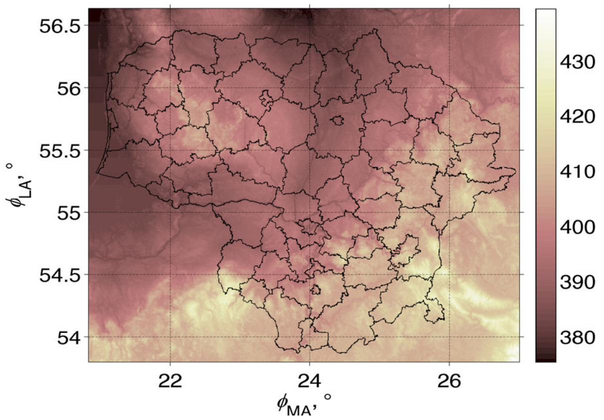
Following expression links Solar and time zone times

$$t_{ST} = t_{CT} - \Delta t_{CTi} + v_{ST}(\phi_{MTZ} - \phi_{MA}) + \Delta t_\varepsilon(n), \quad (16)$$

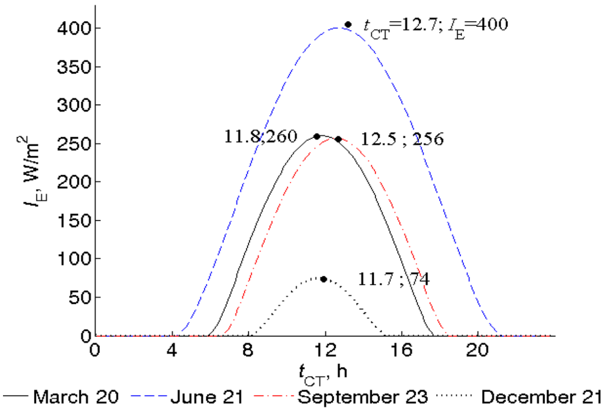
here Δt_{CTi} – daylight savings time correction, h. During the summer, $i = 1$, clocks are turned forward by 1 hour. During the winter, $i = 0$, – $\Delta t_{CT0} = 0$. $v_{ST} = 1/15$ – speed of solar time change in time zone deg/h, ϕ_{MTZ} – longitude of meridian for the local time zone, deg west, $\Delta t_\varepsilon(n)$ – solar noon time correction due to Earth's elliptical orbit around the sun on n–th day, h. The correction is calculated as follows

$$\Delta t_\varepsilon(n) = 0{,}165\sin\left[2\frac{2\pi(n-81)}{365}\right] - 0{,}126\cos\left[\frac{2\pi(n-81)}{365}\right] - \\ -0{,}025\sin\left[\frac{2\pi(n-81)}{365}\right]. \quad (17)$$

Fig. 4 b shows that the solar insolation significantly changes during the year and the day. Those changes mostly depend on the change of zenith angle θ_Z and altitude angle β. For maximal use of solar energy, planes of solar panels and solar thermal collectors should be perpendicular to solar beams. $\cos\theta_Z$ in expression (12) can be easily linked with the solar azimuth and tilt angles.



a)



b)

Fig. 4. Solar insolation values in territory of Lithuania on June 21 at noon (a) and their changes during the day in Vilnius, while Earth is at critical points of its orbit around the Sun (b)

Thus, the model is useful for calculation of optimal angles of solar panels and solar thermal collectors in any location of Lithuania [8].

Conclusions

1. The developed model based on the clear sky standard allows calculating solar insolation in each location of Lithuania and is suitable for modelling electronic systems for control of cells and solar thermal collectors.

2. Average insolation in the territory of Lithuania on June 21 is equal to 407 W/m^2 at noon. Lowest insolation of 375 W/m^2, is in low-lying areas, while in high areas highest insolation of 439 W/m^2 is calculated. Thus the deviation of insolation values from the mean value in territory of Lithuania is ± 8%.

3. The changes of solar insolation mostly depend on the change of zenith angle θ_Z and altitude angle β. For maximal use of solar energy, planes of solar panels and solar thermal collectors should be perpendicular to solar beams. Model is suitable for calculation of optimal angles of solar panels and solar thermal collectors in any location of Lithuania.

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Presented the model based on the clear sky standard, which allows calculation of solar insolation values at each location in Lithuania. Model is associated with geographical coordinates of Lithuania and local time of every day of the year. It is made for modeling electronic systems for management of solar panels and solar thermal collectors. Model was tested by calculating solar insolation at 313 369 locations of Lithuania. It is shown that the lowest insolation in territory of Lithuania on noon of June 21 is 375 W/m^2 and the highest is – 439 W/m^2. Deviation of solar insolation values from average is ± 8%. In addition, it is shown that the changes in solar insolation are caused by zenith or altitude angle variations. Model is suitable for calculation of optimal angles of solar panels and solar thermal collectors in any location of Lithuania. Ill. 4, bibl. 8 (in English; abstracts in English and Lithuanian).

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Pateikiamas giedro dangaus standartu pagrįstas modelis, leidžiantis apskaičiuoti kiekvieną Lietuvos vietovę pasiekiančio saulės energijos srauto vertes. Modelis susietas su Lietuvos geografinėmis koordinatėmis bei kiekvienos metų dienos juostiniu laiku ir skirtas elektroninėms sistemoms, valdančioms saulės baterijas ir kolektorius, modeliuoti. Sudarytas modelis išbandytas apskaičiuojant saulės energijos srautą 313 369 Lietuvos vietovėse. Parodyta, kad saulės energijos srauto, pasiekiačio Lietuvos teritoriją birželio 21 d. vidurdienį, minimali vertė yra 375 W/m^2, o maksimali – 439 W/m^2. Saulės energijos srauto verčių nuokrypa nuo vidutinės siekia ±8 %. Be to, parodyta, kad saulės energijos srautas keičiasi dėl zenito arba saulės altitudės kampų pokyčių. Modelis tinka saulės baterijų ar kolektorių optimaliems padėties kampams nustatyti kiekvienoje Lietuvos vietovėje. Il. 4, bibl. 8 (anglų kalba; santraukos anglų ir lietuvių k.).