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The Method of Markov Processes with Macro States for Analysis of Unreliable Telecommunication Systems

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I. Introduction

The goal of this paper is to propose the analysis method of processes in an unreliable telecommunication system by the means of Markov changes. Moreover, our proposed analysis method based on macro states of Markov chains is much simpler than the detailed Markov processes applied previously by authors for analysis of such unreliable telecommunication systems [1–8]. A lot of effort has gone into the study of unreliable telecommunication systems. A rich literature exists in the area of data packets transmission process analysis in the telecommunication system by the means of Markov chains [9]. The majority of them analyze the processes in the system using detailed Markov chains [6, 7]. These papers present some examples of elementary Markov model for unreliable systems. Complex systems give rise to large and complicated Markov models. Several techniques are used for the automatic generation for large Markov chains models and balance equations [8, 10]. The literature on the research topic has hundreds of papers with exact analysis of such systems. One possible way to solve this problem is to use macro state Markov models. An implementation of macro state Markov model can simplify the evaluation of performance measures of complex systems, such as telecommunication networks. The present paper, on the other hand, obtains exact analytical results for the evaluation of telecommunication system performance measures using the investigation method based on macro state Markov process. We extend the application of Markov model for unreliable system investigation.

The paper is organized as follows. The section I introduces to the necessity of the macro state Markov model for telecommunication system investigation. The unreliable data packet loss system with two data packet transmission channels is presented in the section II. The analysis, employing macro states Markov chains, of system with two unreliable channels loss system is described in the section III. The comparison of system performance measures is presented and discussed in the section IV. In the section V we give our conclusions.

Analysis of an unreliable telecommunication system with data packet loss

Consider a system that consists of two unreliable transmission channels with losses and Poisson data packet arrival rate λ as shown in the Fig. 1. Data packet transmission durations over each channel are distributed exponentially with intensities μ_1 and μ_2. Each channel is characterized by the failure rates γ_1, γ_2 and the failure's repair rates r_1, r_2.

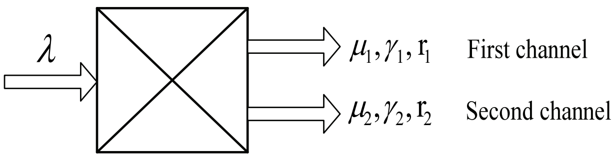


Fig. 1. The structure of telecommunication system with two unreliable channels

The detailed diagram of the system state transition is depicted in the Fig. 2.

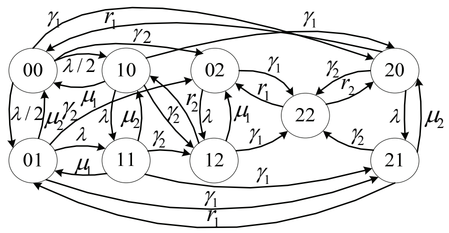


Fig. 2. Detailed Markov chains of an unreliable system with two channels, data packet losses and Poisson data packet arrival

The processes in such system can by presented by the Markov chains, which show the changes between the following system states:

- 00 – both channels are free;
- 10 – first channel is occupied, second channel is free;
- 01 – first channel is free, second channel is occupied;
- 11 – both channels are occupied;
- 20 – first channel is failed, second channel is free;
- 02 – first channel is free, second channel is failed;
- 12 – first channel is occupied, second channel is failed;
- 21 – first channel is failed, second channel is occupied;
- 22 – both channels are failed.

Using the global balance concept, we can easily write down the following equations of the Markov states (Fig. 2) probabilities P_{XY} for the evaluation of the unreliable system:

$$\begin{cases}(\lambda+\gamma_1+\gamma_2)P_{00}-\mu_1 P_{10}-\mu_2 P_{01}-r_1 P_{20}-r_2 P_{02}=0;\\ (\lambda+\mu_1+\gamma_1+\gamma_2)P_{10}-\frac{\lambda}{2}P_{00}-\mu_2 P_{11}-r_2 P_{12}=0;\\ (\lambda+\mu_2+\gamma_1+\gamma_2)P_{01}-\frac{\lambda}{2}P_{00}-\mu_1 P_{11}-r_1 P_{21}=0;\\ (\mu_1+\mu_2+\gamma_1+\gamma_2)P_{11}-\lambda P_{01}-\lambda P_{10}=0;\\ (\lambda+\gamma_1+r_2)P_{02}-\mu_1 P_{12}-\gamma_2 P_{00}-\gamma_2 P_{01}-r_1 P_{22}=0;\\ (\mu_1+\gamma_1+r_2)P_{12}-\lambda P_{02}-\gamma_2 P_{10}-\gamma_2 P_{11}=0;\\ (\lambda+\gamma_2+r_1)P_{20}-\mu_2 P_{21}-\gamma_1 P_{10}-\gamma_2 P_{00}-r_2 P_{22}=0;\\ (\mu_2+\gamma_2+r_1)P_{21}-\lambda P_{20}-\gamma_1 P_{01}-\gamma_1 P_{11}=0;\\ (r_1+r_2)P_{22}-\gamma_1 P_{02}-\gamma_1 P_{12}-\gamma_2 P_{20}-\gamma_2 P_{21}=0;\\ P_{00}+P_{10}+P_{01}+P_{11}+P_{02}+P_{12}+P_{20}+P_{21}+P_{22}=1.\end{cases} \quad (1)$$

To solve these equations, we obtain the unreliable system state probabilities P_{XY} and then proceed to find the following system performance measures:

1. Data packet loss probability

$$P_{loss}=P_{11}+P_{12}+P_{21}. \quad (2)$$

2. Served traffic intensity in the first and second channels (data packet transmission channel utilization):

$$\begin{cases}Y_1=P_{10}+P_{11}+P_{12};\\ Y_2=P_{01}+P_{11}+P_{21}.\end{cases} \quad (3)$$

3. Probabilities of failure of the first and second channel:

$$\begin{cases}P_{1F}=P_{20}+P_{22}+P_{21};\\ P_{2F}=P_{02}+P_{22}+P_{12}.\end{cases} \quad (4)$$

4. System faulty probability:

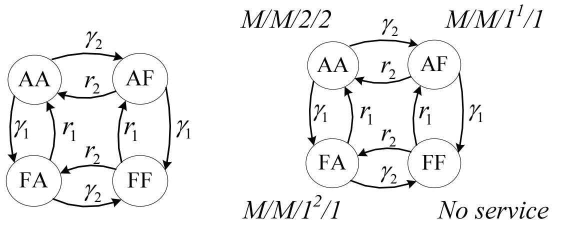
$$P_{SF}=P_{22}. \quad (5)$$

Analysis of the system with two unreliable channels using the Markov chains with macro states

In this chapter we present the analysis of loss system with two unreliable data packet transmission channels by the means of Markov chains between the system macro states (Fig. 3) for the telecommunication system shown in Fig. 1.

In the macro state AA system works as a classical *M/M/2/2* system with loss.

In the macro state AF system transmits data packets as a classical loss system $M/M/1^1/1$ via the first channel.



a) b)

Fig. 3. Markov chains for the macro states of an unreliable system

In the macro state FA system transmits data packets as a classical loss system $M/M/1^2/1$ via the second channel.

In the macro state FF system is faulty.

Let's denote the macro state probabilities of the unreliable system:

- P_{AA} – probability, that both channels are available;
- P_{AF} – probability, that second channel is failed;
- P_{FA} – probability, that first channel is failed;
- P_{FF} – probability, that both channels are failed.

Using the global balance concept, we can easily write down the following equations for the Markov states probabilities P_{XY} for the Markov process shown in the Fig. 3:

$$\begin{cases}(\gamma_1+\gamma_2)P_{AA}-r_2 P_{AF}-r_1 P_{FA}=0;\\ (\gamma_1+r_2)P_{AF}-\gamma_2 P_{AA}-r_1 P_{FF}=0;\\ (\gamma_2+r_1)P_{FA}-\gamma_1 P_{AA}-r_2 P_{FF}=0;\\ (r_1+r_2)_{FF}-\gamma_1 P_{AF}-\gamma_2 P_{FA}=0;\\ P_{AA}+P_{FA}+P_{AF}+P_{FF}=1.\end{cases} \quad (6)$$

To solve these equations, we obtain the unreliable system state probabilities and then proceed to find the system performance measures.

Let's take case when $\mu_1=\mu_2=\mu$ and the number of channels $v=2$, then the performance measures of such system in each macro state are calculated as follow.

In the macro state AA system performance measures are defined as:

1. Data packet loss is given by Erlang's B formula

$$P_{Loss_{AA}}=\frac{\frac{(\lambda/\mu)^v}{v!}}{\sum_{k=0}^{v}\frac{(\lambda/\mu)^k}{k!}}. \quad (7)$$

In our case, when $v=2$, we have

$$P_{Loss_{AA}}=\frac{\frac{(\lambda/\mu)^2}{2}}{1+\frac{\lambda}{\mu}+\frac{(\lambda/\mu)^2}{2}}. \quad (8)$$

2. Served traffic intensity by each channel (data packet transmission channel utilization)

$$Y_{1_{AA}} = Y_{2_{AA}} = \frac{(\lambda/\mu)(1-P_{LossAA})}{2}. \qquad (9)$$

3. System faulty probability

$$P_{SF} = P_{FF}. \qquad (10)$$

Data packet transmission quality measures in the system macro state FA and macro state AF:

1. Data packet loss probabilities due to Erlang's B formula, when v=1, is given by

$$P_{Loss_{FA}} = P_{Loss_{AF}} = \frac{\lambda/\mu}{1+\lambda/\mu}. \qquad (11)$$

2. Served data packet traffic intensities by each channel are given by

$$Y_{1_{FA}} = Y_{2_{FA}} = (\lambda/\mu)(1-P_{Loss\,FA,AF}). \qquad (12)$$

Then performance measures of the unreliable system can be written as:

1. Data packet loss probability

$$\overline{P}_{Loss} = P_{AA}\cdot P_{Loss_{AA}} + P_{AF}\cdot P_{Loss_{AF}} + P_{FA}\cdot P_{Loss_{FA}}. \qquad (13)$$

2. Mean value of each data packet transmission channel utilization:

$$\begin{cases} Y_1 = Y_{1_{AA}}\cdot P_{AA} + Y_{1_{AF}}\cdot P_{AF}; \\ Y_2 = Y_{2_{AA}}\cdot P_{AA} + Y_{2_{FA}}\cdot P_{FA}. \end{cases} \qquad (14)$$

3. Probability of failure of the first and second channel:

$$\begin{cases} P_{1F} = P_{FA} + P_{FF}; \\ P_{2F} = P_{AF} + P_{FF}. \end{cases} \qquad (15)$$

Comparison of system performance measures results

In this chapter we compare the results, which were provided by the proposed macro state Markov model and the detailed state Markov model. From the results of the tests one may observe that our proposed method exactly reproduces the performance measures of the investigated unreliable system.

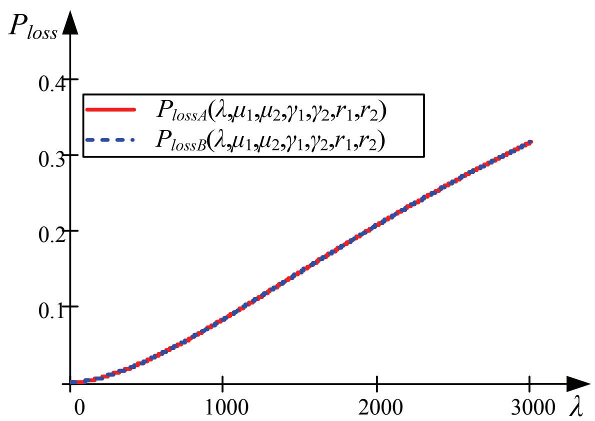


Fig. 4. Results of data packet transmission loss obtained by the means of detailed states Markov model – P_{lossA} and the macro states Markov model – P_{lossB} as a function of data packet arrival rate λ, when $\mu_1=\mu_2=\mu=$ 2000 pack/s, γ_1=1/5000 [hours^{-1}], γ_2=1/1000 [hours^{-1}], $r_1=r_2$=1/24[hours^{-1}]

Comparison of data packet loss in the system is taken when the initial parameters of the investigated system are λ=1000 pack/s, $\mu_1=\mu_2=\mu=$ 2000 pack/s, γ_1=1/5000 [hours^{-1}], γ_2=1/1000 [hours^{-1}], $r_1=r_2$=1/24[hours^{-1}]:

A. For the detailed Markov model P_{loss}=0.0840915.

B. For the macro state Markov model P_{loss}=0.0840916.

More detailed comparison of the results, obtained using both Markov models, as a function of data packet arrival intensity λ are shown in the Fig. 4 and Fig. 5.

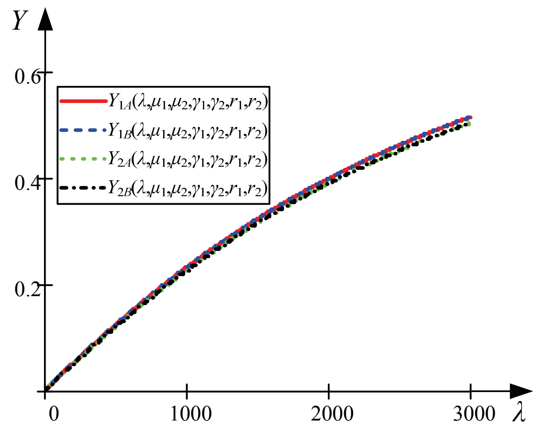


Fig. 5. Results of data packet transmission channels utilization obtained by the means of detailed states Markov model – Y_{1A},Y_{2A}, and the macro states Markov model – Y_{1B},Y_{2B} as a function of data packet arrival rate λ, when $\mu_1=\mu_2=\mu=$ 2000 pack/s, γ_1=1/5000 [hours^{-1}], γ_2=1/1000 [hours^{-1}], $r_1=r_2$=1/24[hours^{-1}]

The comparison of the results, which are presented in the Fig. 6 and Fig. 7, shows very negligible differences.

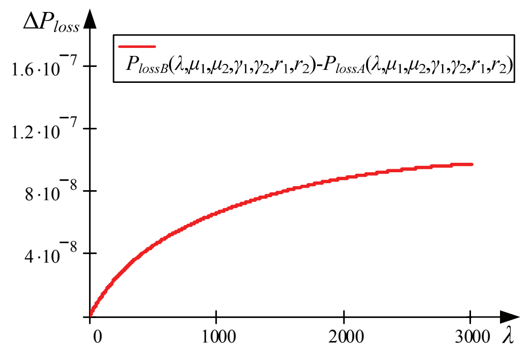


Fig. 6. Difference between the P_{loss} results from the Fig. 4

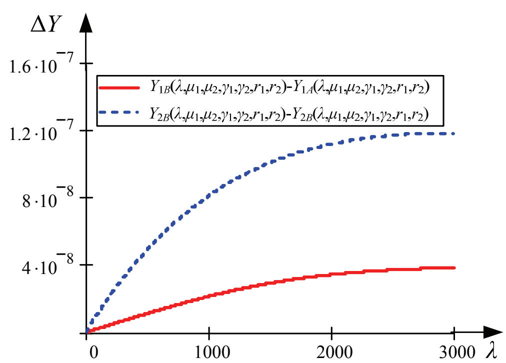


Fig. 7. Difference between the data packet transmission channels utilization results from the Fig. 5

Conclusion and future work

In this paper we have made an attempt at applying our proposed macro state Markov chains model in investigating an unreliable telecommunication system. It is well known that for large telecommunication system, the exact calculation of network reliability is complicated. We argued that our proposed method is applicable for analysis of complex telecommunication technologies. Its realization is less complex than the detailed Markov state models, but the comparison of results show that differences between the results of macro and detailed states Markov models are very negligible.

Our future work will focus on investigating the proposed macro state Markov model for complex telecommunication system analysis.

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Our proposed analysis method of macro state Markov model has the advantages over detailed state Markov model. The data packet loss in telecommunication system with the unreliable data packet transmission links is modeled as a continuous time and the discrete states Markov process. The advantages of the macro state Markov model method were demonstrated by modeling two channels unreliable telecommunication system with losses and providing some numerical illustrations. The application of the proposed method for analysis of an unreliable data packet transmission system and queueing system is discussed. All investigated systems are presented according Kendall's notation. On the basis of the reached outcomes it is possible to say that there are negligible differences between the results of macro and detailed states Markov models for the wide value parameters of investigated unreliable loss system M/M/2/2. Ill. 7, bibl. 10 (in English; abstracts in English and Lithuanian).

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Pasiūlytas Markovo procesų su makrobūsenomis analizės metodas turi pranašumų, palyginti su detalių būsenų Markovo modeliu. Telekomunikacijų sistema su duomenų paketų praradimais, esant nepatikimoms duomenų perdavimo grandims, modeliuojama tolydaus laiko ir diskrečių būsenų Markovo grandinėmis. Makrobūsenų Markovo modelio analizės pranašumai pagrįsti analizuojant dviejų nepatikimų duomenų perdavimo kanalų su duomenų paketų praradimais telekomunikacijų sistemą, o palyginimui pateikiamos skaitinės iliustracijos. Aptariamos galimybės pasiūlytąjį metodą pritaikyti nepatikimoms sistemoms su duomenų paketų praradimais ir jų eiliavimu nagrinėti. Nagrinėjamos sistemos pateikiamos remiantis Kendalo žymėjimais. M/M/2/2 dviejų nepatikimų kanalų sistema su duomenų paketų praradimais buvo modeliuojama naudojant makro- ir detalių būsenų Markovo grandines, o gautų rezultatų skirtumai yra labai nedideli. Il. 7, bibl. 10 (anglų kalba; santraukos anglų ir lietuvių k.).