ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2008. No. 8(88)
ELEKTRONIKA IR ELEKTROTECHNIKA

T121 SIGNAL TECHNOLOGY
SIGNALŲ TECHNOLOGIJA

Blind Source Separation of a Mixture of Pseudoperiodic Chaotic Signals

K. Pukėnas
Department of Sports Biomechanics, Information Science and Engineering, Lithuanian Academy of Physical Education, Sporto str. 6, LT-44221, Kaunas, Lithuania, phone: +370 37 302668; e-mail: k.pukenas@lkka.lt

Introduction

Blind source separation (BSS) and independent component analysis (ICA) are generally based on a wide class of algorithms [1]-[3] and they found potential applications in many areas from engineering to neuroscience. BSS methods consider the separation of observed (often called sensor) signals into their underlying independent source signals knowing neither the source signals nor the mixing process. Most of linear BSS models in the simplest forms can be expressed algebraically as some specific problems of matrix factorization [3]: Given observation matrix $\mathbf{X} = [\boldsymbol{x}(1), \cdots, \boldsymbol{x}(N)] \in \mathbb{C}^{M \times N}$ perform the matrix factorization

$$\mathbf{X} = \mathbf{AS}, \qquad (1)$$

where N is the number of available samples, M is the number of observations, K is the number of sources, $\mathbf{A} \in \mathbb{C}^{M \times K}$ represents the unknown mixing matrix, $\mathbf{S} = [\boldsymbol{s}(1), \cdots, \boldsymbol{s}(N)] \in \mathbb{C}^{K \times N}$ contains the corresponding latent (hidden) components which represent unknown source signals.

Generally BSS is obtained by finding an $K \times M$ linear transformation (separating) matrix $\mathbf{W} = \hat{\mathbf{A}}^{+}$, where $\mathbf{A}^{+}$ means some well-defined pseudo-inverse of $\mathbf{A}$ such that the output signal matrix $\hat{\mathbf{S}} = [\hat{\boldsymbol{s}}(1), \cdots, \hat{\boldsymbol{s}}(N)] \in \mathbb{C}^{K \times N}$, by $\hat{\mathbf{S}} = \mathbf{WX}$, contains components that are as independent as possible, as measured by an information-theoretic cost function or other criteria [3]. The most popular approach exploits as the cost function some measure of signals statistical independence and non-Gaussianity and involve the higher-order statistics (HOS) to solving the BSS problem. If sources have temporal structures, then each source has non-vanishing temporal correlation, and second-order statistics (SOS) are often sufficient to estimate the mixing matrix and sources [1]. Recently BSS has attracted much attention in chaotic signal field [4], [5], as many complex signals obtained from high-dimensional nonlinear dynamical systems are not purely simple chaotic signals – they may be contaminated by different components of individual chaotic signals or internal noise signals. Due to the some singularity of the chaotic signals the techniques mentioned above may not be applicable to chaotic signals implicitly [4] and must be verified.

In this paper the BSS algorithm based on nonlinear phase-space reconstruction and nonorthogonal joint approximate diagonalization (JAD) [6]-[8] of several time-delayed covariance matrices is investigated. The time-delayed covariance matrices were estimated with one data matrix of first embedding dimension and second data matrix of every another embedding dimension. The case $K = M$ is considered. It is demonstrated, that algorithm gives a good performance in the separation of mixed pseudoperiodic chaotic or similar to pseudoperiodic signals. This class of chaotic signals – pseudoperiodic – has aroused great interest due to their close relation to some important natural and physiological systems [5], [9]. The performance of the algorithm is compared with the widely known BSS algorithms such as AMUSE [3], dAMUSE [10], [11] and ICA algorithm FastIca [12], also algorithm, described in [4] for an artificial mixture of synthetic signals.

Description of the algorithm

Given a group of M sensor signals with N samples a reconstructed phase space matrix $\mathbf{X}^{(k)}$, $k = 1, \cdots, M$ with d rows and $L = N - (d-1)\tau$ columns (called a trajectory matrix) for the mixture received by k^{th} sensor is defined by [13]

$$\mathbf{X}^{(k)} = \begin{bmatrix} x_1^{(k)} & x_2^{(k)} & \cdots & x_{N-(d-1)\tau}^{(k)} \\ x_{1+\tau}^{(k)} & x_{2+\tau}^{(k)} & \cdots & x_{N-(d-2)\tau}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1+(d-1)\tau}^{(k)} & x_{2+(d-1)\tau}^{(k)} & \cdots & x_N^{(k)} \end{bmatrix}, \qquad (2)$$

where d – the embedding dimension and τ – time delay are chosen according to certain optimization criterion [13].

For M sensors, we obtain M embedding matrices generally with the same values for τ and d. Using the i^{th}, $i = 1, \cdots, d$ rows of the embedding matrices $\mathbf{X}^{(k)}$ we can form a data matrix for all sensors for every embedding dimension, i. e.

$$\mathbf{X}_i = \begin{bmatrix} x^{(1)}_{1+(i-1)\tau} & x^{(1)}_{2+(i-1)\tau} & \cdots & x^{(1)}_{N-(d-i)\tau} \\ x^{(2)}_{1+(i-1)\tau} & x^{(2)}_{2+(i-1)\tau} & \cdots & x^{(2)}_{N-(d-i)\tau} \\ \vdots & \vdots & \ddots & \vdots \\ x^{(K)}_{1+(i-1)\tau} & x^{(K)}_{2+(i-1)\tau} & \cdots & x^{(K)}_{N-(d-i)\tau} \end{bmatrix}, \tag{3}$$

where $i = 1, \cdots, d$.

The time-delayed correlation matrixes $\mathbf{R}_{\mathrm{j}} \in C^{M \times M}$ has the form

$$\mathbf{R}_j = \frac{1}{L-1} \mathbf{X}_1 \mathbf{X}^T_{j+1}, \tag{4}$$

where $j = 1, \cdots, d-1$.

Given a matrix set $\mathcal{R} = \{\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_j\}$, where $\mathbf{R}_j \in C^{M \times M}$, the approximate joint diagonalization problem seeks a nonsingular diagonalizing matrix $\mathbf{U} \in C^{M \times M}$ and associated diagonal matrix $\mathbf{D}_1, \mathbf{D}_2, \cdots, \mathbf{D}_j \in C^{M \times M}$ (which are usually not of interest in the context of BBS) such that the following common structures are best fitted [6]-[8]:

$$\mathbf{R}_j = \mathbf{U}\mathbf{D}_j\mathbf{U}^T, \quad (j = 1, \cdots, d-1). \tag{5}$$

The “goodness of fit” is evaluated by some criterion (cost or objective function). Due to the limitations of orthogonal joint diagonalization, the nonorthogonal joint diagonalization, which do not require the problematic prewhitening, has received increasing attention in recent years [6]-[8].

The algorithm is defined by the following implementation:

1. Perform JAD: $\mathbf{R}_j = \mathbf{U}\mathbf{D}_j\mathbf{U}^T$, $(j = 1, \cdots, d-1)$, i.e., estimate the orthogonal matrix $\mathbf{U}$ using one of the available numerical algorithm. In this work numerical algorithm and appropriate software, described in [7], are used.
2. Estimate unmixing (separating) matrix as $\mathbf{W} = \mathbf{U}$ and the source signals matrix as

$$\hat{\mathbf{S}} = \mathbf{U}\mathbf{X}, \tag{6}$$

in which each row represent a separate signal.

Numerical results

The proposed algorithm was applied to artificially mixed synthetic signals. In the first experiment two mixed x components of the Rossler system, defined by

$$\begin{cases} \dfrac{dx}{dt} = -(y+z), \\ \dfrac{dy}{dt} = x + a \cdot y, \\ \dfrac{dz}{dt} = b + z(x-c), \end{cases} \tag{7}$$

with parameters a = 0,398; b = 2; c = 4 and a = 0,2; b = 0,2; c = 4,6 respectively were considered. The embedding dimension of the reconstructed phase space $d = 4$ and time delay $\tau = 16$ for mixed signals were defined and 2000 samples were used in this experiment. In the second experiment two mixed signals of the Mackey-Glass differential-delay equation, defined by

$$\frac{dx}{dt} = \frac{ax(t-\tau d)}{1 + x(t-\tau d)^c} - bx(t) \tag{8}$$

were used. The two sequences are generated with the same parameters (a = 0,2; b = 0,1; c = 10 and $\tau d = 30$) but with different integrating conditions. The equation is solved numerically by using the algorithm described in [14]. The embedding dimension of the reconstructed phase space $d = 4$ and time delay $\tau = 12$ for mixed signals were defined. The lenght of sequences – 1600 points. As mentioned above it is assumed that the number of sensors M is equal to the number of signals K. That is, two sensors are used and the coupling (mixing) matrix in both cases is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0{,}5 \\ 1{,}1 & 1 \end{bmatrix}. \tag{9}$$

In Fig. 1 and Fig. 2, the original signals with their reconstructed versions are superimposed. As can be seen, one is almost not able to tell them apart. For clarity the signals were standardized.

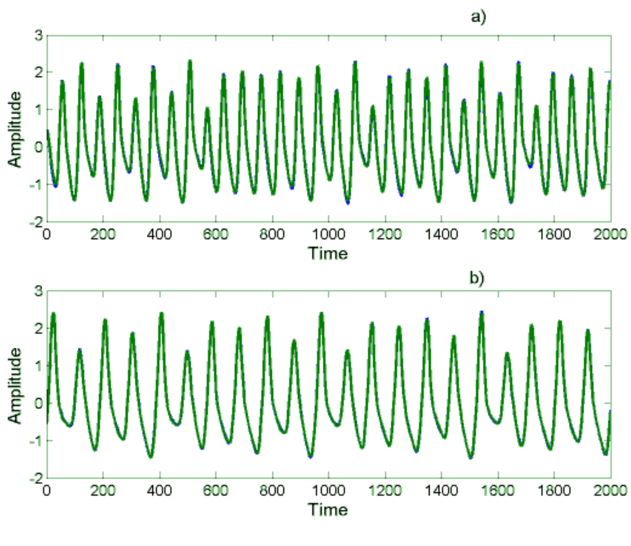

Fig. 1. Comparison of the original Rossler signals (blue) and the reconstructed signals (green): a) first source signal, b) second source signal (the blue signal is covered by the green one)

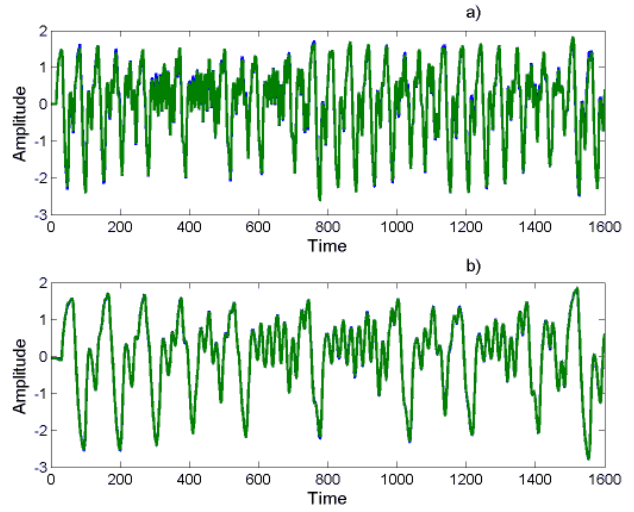

Fig. 2. Comparison of the original Mackey-Glass signals (blue) and the reconstructed signals (green): a) first source signal, b) second source signal (the blue signal is covered by the green one)

In a simulation environment (the true matrix $\mathbf{A}$ is known) the performance of blind separation can be characterized by one single performance index defined by [3], [7]

$$J(\mathbf{P}) = \frac{1}{M}\sum_{i=1}^{M}\left(\frac{\sum_{j=1}^{M} p_{ij}^2}{\max_{j} p_{ij}^2} - 1\right) + \frac{1}{M}\sum_{j=1}^{M}\left(\frac{\sum_{i=1}^{M} p_{ij}^2}{\max_{i} p_{ij}^2} - 1\right), \qquad (10)$$

where the permutation matrix $\mathbf{P} = \mathbf{WA} = \lfloor p_{ij} \rfloor$, $\mathbf{P} \in C^{M \times M}$. The performance index is a number that measures the degree of diagonality of the product of the separation matrix ($\mathbf{W}$) by the mixing matrix ($\mathbf{A}$).

A comparison of the performance of proposed algorithm with some others is shown in Table 1.

Table 1. A comparison of the performance of proposed algorithm

Algorithm	Mixed signals	
	Rossler	Mackey-Glass
Proposed	0,00256	0,00589
AMUSE	0,00036	0,02728
dAMUSE	0,65158	0,44253
FastIca	0,23646	0,00534
Described in [4]	0,00439	0,00511

Conclusions

In this paper the BSS algorithm based on nonlinear phase-space reconstruction and nonorthogonal joint approximate diagonalization of several time-delayed covariance matrices corresponding to the data matrix of first embedding dimension and data matrix of the every another embedding dimension are investigated by applying them to mixed pseudoperiodic chaotic Rossler signals and Mackey-Glass signals.

Simulation results show that this algorithm is able to separate mixed pseudoperiodic chaotic or similar to pseudoperiodic signals which have temporal structures and each source has non-vanishing temporal correlation, i. e. when analysis based on the second-order statistics is applicable. These simulation results also show that algorithm leads to better performances than the AMUSE, dAMUSE, the very efficient fastICA-algorithm and algorithm described in [4]. It will be observed, that the worst performance is shown by dAMUSE algorithm, whereby all the spatial dimensions are stacked into column vectors. However, the stacking operation does not capture the structure inherent a sampling the data [15]. The performance of the investigated algorithm is nearest to the performance of algorithm based on generalized eigenvalue decomposition [4], but is better in this instance for separating the mixed Rossler signals.

References

1. **Belouchrani A., Meraim K. A., Cardoso J. F., Moulines E.** A Blind Source Separation Technique Using Second-Order Statistics // IEEE Trans. Signal Process. – 1997. – Vol. 45, Iss. 2. – P. 434–444.
2. **Parra L., Sajda P.** Blind Source Separation via Generalized Eigenvalue Decomposition // Journal of Machine Learning Research. – 2003. – No. 4. – P. 1261–1269.
3. **Choi S., Cichocki A., Park H.-M., Lee S-Y.** Blind Source Separation and Independent Component Analysis: A Review // Neural Information Processing. – Letters and Reviews. – 2005. – Vol. 6, No. 1. – P. 1–57.
4. **Lo T., Leung H., Litva J.** Separation of a mixture of chaotic signals // IEEE International Conference on Acoustics, Speech, and Signal Processing ICASSP-96. Conference Proceedings. – 1996. – Vol. 3. – P. 1798–1801.
5. **Kuraya M., Uchida A., Yoshimori S., Umeno K.** Blind source separation of chaotic laser signals by independent component analysis // Optics Express. – 2008. – Vol. 16, No. 2. – P. 725–730.
6. **Li X.-L., Zhang X.-D.** Nonorthogonal Joint Diagonalization Free of Degenerate Solution // IEEE Trans. Signal Process. – 2007. – Vol. 55, Iss. 5. – P. 1803–1814.
7. **Ziehe A., Laskov P., Nolte G., Muller K.R.** A Fast Algorithm for Joint Diagonalization with Non-orthogonal Transformations and its Application to Blind Source Separation // Journal of Machine Learning Research. – 2004. – Vol. 5. – P. 777–800.
8. **Degerine S., Kane E.** A Comparative Study of Approximate Joint Diagonalization Algorithms for Blind Source Separation in Presence of Additive Noise // IEEE Trans. Signal Process. – 2007. – Vol. 55, Iss. 6. – P. 3022–3031.
9. **Pukenas K.** Detecting deterministic structure from a high noisy pseudoperiodic time series. // Electronics and Electrical Engineering. – 2008. – No. 4 (84). – P. 80–85.
10. **Tome A. M., Texeira A. R., Lang E. W., Stadlthanner K., Rocha A. P., Almeida R.** Blind Source Separation Using Time-Delayed Signals // Proceedings of IEEE International Joint Conference on Neural Networks. – 2004. – Vol. 3. – P. 2187–2191.
11. **Gruber P., Stadlthanner K., Böhm M., Theis F. J., Lang E. W., Tomé A. M., Teixeira A. R., Puntonet C. G., Gorriz Saéz J. M.** Denoising using local projective subspace me-

thods // Neurocomputing. – 2006. – Vol. 69, Iss. 13-15. – P. 1485–1501.

12. **Hyvärinen A., Oja E.** A Fast Fixed-Point Algorithm for Independent Component Analysis // Neural Computation – 1997. – Vol. 9, No. 7. – P. 1483–1492.
13. **Kantz H., Schreiber T.** Nonlinear Time Series Analysis // Cambridge: Cambridge University Press, 2003. – P. 30–39.
14. The Matlab code for generating Mackey-Glass time series. Publicly available at http://www.cse.ogi.edu/~ericwan/.
15. **Haardt M., Roemer F., Del Galdo G.** Higher-Order SVD-Based Subspace Estimation to Improve the Parameter Estimation Accuracy in Multidimensional Harmonic Retrieval Problems // IEEE Trans. Signal Process. – 2008. – Vol. 56, Iss. 7. – P. 3198–3213.

Received 2008 09 10

K. Pukenas. Blind Source Separation of a Mixture of Pseudoperiodic Chaotic Signals // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 8(88). – P. 77–80.

The straightforward blind source separation (BSS) algorithm based on nonlinear phase-space reconstruction and nonorthogonal joint approximate diagonalization (JAD) of several time-delayed covariance matrices, estimated with one data matrix of first embedding dimension and second data matrix of every another embedding dimension, is investigated by applying it to mixed pseudoperiodic chaotic Rossler signals and Mackey-Glass signals. Simulation results show that this algorithm gives a good performance in the separation of mixed pseudoperiodic chaotic or similar to pseudoperiodic signals when each source has non-zero autocorrelation function for a non-zero time lag, i. e. when analysis based on the second-order statistics (SOS) is applicable. Algorithm leads to better performance than many widely known BSS algorithms including the very efficient iterative FastICA algorithm. Ill 2, bibl. 15 (in English; summaries in English, Russian and Lithuanian).

К. Пукенас. Разделение смеси псевдопериодических хаотических сигналов методом слепого разделения источников // Электроника и электротехника. – Каунас: Технология, 2008. – № 8(88). – С. 77–80.

Исследуется алгоритм слепого разделения источников (*Blind Source Separation* – BSS), основанный на реконструкции фазового пространства и совместной приблизительной неортогональной диагонализации нескольких ковариационных матриц сигналов реконструированного фазового пространства, определенных с использованием матрицы данных первой меры и матрицы данных каждой другой меры фазового пространства. Путем анализа смеси хаотических сигналов Росслера и сигналов Маккей-Гласс показывается, что алгоритм обеспечивает хорошее разделение псевдопериодических хаотических или им подобных сигналов, когда каждый источник обладает ненулевой автокорреляционной функцией при ненулевом сдвиге, т. е. когда возможно применение статистики второго порядка. Алгоритм обеспечивает более высокую эффективность чем ряд известных алгоритмов, включая высокоэффективный алгоритм FastICA. Ил. 2, библ. 15 (на английском языке; рефераты на английском русском и литовском яз).

K. Pukėnas. Pseudoperiodinių chaotinių signalų atskyrimas taikant aklo šaltinių atskyrimo metodą // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 8(88). – P. 77–80.

Tiriamas aklo šaltinių atskyrimo algoritmas fazinės erdvės rekonstrukcijos ir bendros apytikslės kelių rekonstruotos fazinės erdvės signalų kovariacijos matricų neortogonalios diagonalizacijos pagrindu. Kovariacijos matricos sudaromos pirmo rekonstruotos fazinės erdvės mato duomenų matricos ir visų kitų rekonstruotos fazinės erdvės matų duomenų matricų pagrindu. Atlikti tyrimai su sumaišytais chaotiniais Rosslerio signalais ir Mackey-Glass signalais rodo, kad algoritmas įgalina gerai atskirti pseudoperiodinius chaotinius signalus bei į juos panašius, kada kiekvienas atskiriamas šaltinis turi nenulinę autokoreliacinę funkciją esant nenuliniam postūmiui, t. y. kada galima taikyti antros eilės statistikas. Algoritmo efektyvumas viršija daugelio žinomų algoritmų efektyvumą, įskaitant ir FastICA algoritmą. Il. 2, bibl. 15 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

DOI: 10.5755/j02.eie.11350