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Diversity System for Demodulation BFSK Signals in the Presence of Rayleigh Fading

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Introduction

During broadcasting of BFSK signals through communication systems, under the influence of different inconvenience, is coming to decreasing of quality of received signal.

In wireless communications systems the biggest effect on the quality of received signal has fading which originate due to broadcasting of signals on several different paths, reference to reflection and scattering of signals on different object which produce that received signals have different phasing and another attributes. That could cause significant reduce of power signals on the reception. Modeling of fading can be performed on different ways and in case of short-lived variations of signal and when there is no optical visibility among emitter and receiver, is used Rayleigh model fading with the probability density function:

$$p(A) = \frac{A}{\sigma_1^2} \cdot \exp\left(-A^2 / 2 \cdot \sigma_1^2\right). \tag{1}$$

For elimination of effects of fading on quality of received signal, mostly is used so-called distance diversity which uses a lot of antennas at the receiver and at the emitter, too. On that way, it is broadcasting several duplicate signals, some of them less or more are weakening and the error probability on the reception is going to be less.

By using mentioned technique, on the reception will be several duplicate signals so is needed to be made their combining and determining. For that are used different techniques which in several cases are giving optimal results. The least complicated for realization is selection combining (SC) which chooses the branch with the highest signal-to-noise ratio and the processing is performed only on one of the diversity branches and no channel information is required. The probability density function of the signals on the output is equal to the sum of product of the probability density function at the one output and the cumulative distributions signals at other outputs.

On this article is analyzed dual SC diversity system for noncoherent demodulation BFSK signals in presence of Rayleigh fading. For this case, the expressions for the probability density functions of the output signals and error probability are given in the closed form.

The model of system

At the model for noncoherent demodulation BFSK signals, there are a band pass filter and a detector of envelope in the each branches of the receiver like is shown in Fig.1.

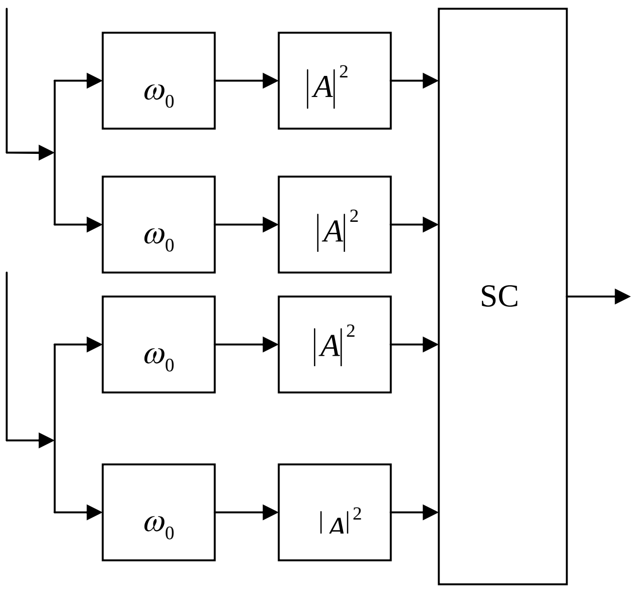

Fig. 1. Model of system

On the input of receiver beside useful signal, appears Gaussian noise with constant power spectral density, too. On the output band pass filter there is band pass Gaussian noise. The output signal of envelope detector corresponds to the envelope of signal on its input, actually, in the first branch of receiver, the signal corresponds to the amplitude of sum of useful signal and band pass of Gaussian noise and in the second branch to amplitude of band pass of Gaussian noise.

Input receiver signal can be presented as:

$$s(t) = A\cos\omega t\ , \qquad (2)$$

where A is the amplitude signal, ω is the radian frequency of signal. The radian frequency is different in the different digit intervals. Under influence of fading signal of amplitude vary and has Rayleigh distribution. Such signal is added to band pass Gaussian noise. The envelops of signals on the output of detector of envelope in some branches of receiver are z_{01} , z_{02}, z_{11} and z_{12}, where the first index shows the receiver branch and the second index diversity branch.

The probability density function of output signal

For mentioned model of diversity system for non-coherent modulation of BFSK signal probability density function of signal on the output at the first branch of the first receiver has Rice distribution:

$$p_{z_{01}/A}(z_{01}/A) = \frac{z_{01}}{\sigma^2}\cdot\exp\left(-\frac{z_{01}^2+A^2}{2\sigma^2}\right)\cdot I_0\left(\frac{z_{01}A}{\sigma^2}\right)\ . \qquad (3)$$

Since the signal amplitude under influence of fading has Rayleigh distribution, the probability density function of output envelope detector signal at the first branch of the first receiver is:

$$\begin{cases} p_{z_{01}}(z_{01}) = \int\limits_0^\infty p_{z_{01}/A}(z_{01}/A)\cdot p(A)\cdot dA, \\ p_{z_{01}}(z_{01}) = \int\limits_0^\infty \frac{z_{01}}{\sigma^2}\cdot\exp\left(-\frac{z_{01}^2+A^2}{2\sigma^2}\right)\cdot I_0\left(\frac{z_{01}A}{\sigma^2}\right)\times \\ \times\frac{A}{\sigma_1^2}\cdot\exp\left(-A^2/2\cdot\sigma_1^2\right)\cdot dA, \\ p_{z_{01}}(z_{01}) = \frac{z_{01}}{\sigma^2\cdot\sigma_1^2}\cdot\exp\left(-\frac{z_{01}^2}{2\sigma^2}\right)\times \\ \times\int\limits_0^\infty A\cdot\exp\left(-\frac{\sigma^2+\sigma_1^2}{2\sigma^2\cdot\sigma_1^2}\cdot A^2\right)\cdot I_0\left(\frac{z_{01}A}{\sigma^2}\right)\cdot dA. \end{cases} \qquad (4)$$

The modified Bessel function in the last expression can be also written as in the following form:

$$I_0\left(\frac{z_{01}\cdot A}{\sigma^2}\right) = \sum_{k=0}^\infty \frac{z_{01}^{2k}\cdot A^{2k}}{\sigma^{4k}\cdot 2^{2k}\cdot(k!)^2}\ . \qquad (5)$$

By substituting of expression for the modified Bessel function (5) in relation for the probability density function of output envelope detector signal, we get:

$$\begin{cases} p_{z_{01}}(z_{01}) = \frac{z_{01}}{\sigma^2\cdot\sigma_1^2}\cdot\exp\left(-\frac{z_{01}^2}{2\sigma^2}\right)\times \\ \times\int\limits_0^\infty\sum_{k=0}^\infty \frac{z_{01}^{2k}\cdot A^{2k}}{\sigma^{4k}\cdot 2^{2k}\cdot(k!)^2}\cdot A\cdot\exp\left(-\frac{\sigma^2+\sigma_1^2}{2\sigma^2\cdot\sigma_1^2}\cdot A^2\right)\cdot dA, \\ p_{z_{01}}(z_{01}) = \exp\left(-\frac{z_{01}^2}{2\sigma^2}\right)\cdot\sum_{k=0}^\infty \frac{z_{01}^{2k+1}}{\sigma^{4k+2}\cdot\sigma_1^2\cdot 2^{2k}\cdot(k!)^2}\times \\ \times\int\limits_0^\infty A^{2k+1}\cdot\exp\left(-\frac{\sigma^2+\sigma_1^2}{2\sigma^2\cdot\sigma_1^2}\cdot A^2\right)\cdot dA. \end{cases} \qquad (6)$$

Using expression for Gamma function:

$$\Gamma(z) = \int\limits_0^\infty t^{z-1}\cdot e^{-t}\cdot dt\ , \qquad (7)$$

the probability density function on the output at the first branch of the first receiver is:

$$p_{z_{01}}(z_{01}) = \exp\left(-\frac{z_{01}^2}{2\sigma^2}\right)\cdot\sum_{k=0}^\infty \frac{z_{01}^{2k+1}}{\sigma^{4k+2}\cdot\sigma_1^2\cdot 2^{2k}\cdot(k!)^2}\times$$

$$\times\left(\frac{2\sigma^2\cdot\sigma_1^2}{\sigma^2+\sigma_1^2}\right)^{k+1}\cdot\Gamma(k+1)\ . \qquad (8)$$

Change of probability density function for different values of parameters σ and σ_1 is shown in Fig. 2.

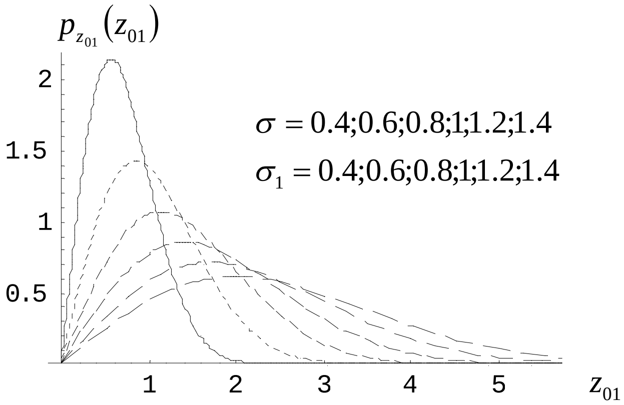

Fig. 2. Probability density function of the signal z_{01}

The cumulative distribution of signal on output envelope detector on the first branch output of the first receiver is:

$$F_{z_{01}}(z_{01}) = \int\limits_0^{z_{01}} p_{z_{01}}(x)\cdot dx\ ; \qquad (9)$$

$$F_{z_{01}}(z_{01}) = \sum_{k=0}^\infty \frac{\Gamma(k+1)}{\sigma^{4k+2}\cdot\sigma_1^2\cdot 2^{2k}\cdot(k!)^2}\cdot\left(\frac{2\sigma^2\cdot\sigma_1^2}{\sigma^2+\sigma_1^2}\right)^{k+1}\times$$

$$\times\int\limits_0^{z_{01}} x^{2k+1}\cdot\exp\left(-\frac{x^2}{2\sigma^2}\right)\cdot dx\ . \qquad (10)$$

Using:

$$\gamma(n,x)=\int_0^x t^{n-1}\cdot e^{-t}\cdot dt\,, \tag{11}$$

cumulative distribution is:

$$F_{z_{01}}(z_{01})=\sum_{k=0}^{\infty}\frac{\Gamma(k+1)}{\sigma^{4k+2}\cdot\sigma_1^2\cdot 2^{2k}\cdot(k!)^2}\cdot\left(\frac{2\sigma^2\cdot\sigma_1^2}{\sigma^2+\sigma_1^2}\right)^{k+1}\times$$

$$\times\gamma\left(k+1,\frac{z_{01}^2}{2\cdot\sigma^2}\right). \tag{12}$$

Change of this cumulative distribution signal for different values of parameters is shown in Fig. 3.

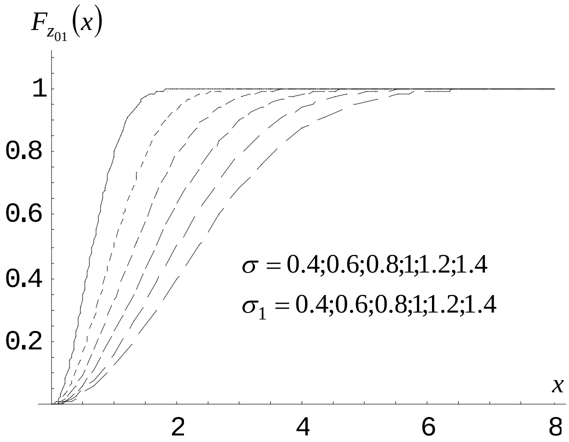

Fig. 3. Cumulative distribution of the signal z_{01}

On the similar way we get the probability density function on the output signal at the first branch of the second receiver is:

$$p_{z_{11}}(z_{11})=\exp\left(-\frac{z_{11}^2}{2\sigma^2}\right)\cdot\sum_{k=0}^{\infty}\frac{z_{11}^{2k+1}}{\sigma^{4k+2}\cdot\sigma_1^2\cdot 2^{2k}\cdot(k!)^2}\times$$

$$\times\left(\frac{2\sigma^2\cdot\sigma_1^2}{\sigma^2+\sigma_1^2}\right)^{k+1}\cdot\Gamma(k+1). \tag{13}$$

The cumulative function of the output signal at the first branch of the second receiver is:

$$F_{z_{11}}(z_{11})=\sum_{k=0}^{\infty}\frac{\Gamma(k+1)}{\sigma^{4k+2}\cdot\sigma_1^2\cdot 2^{2k}\cdot(k!)^2}\cdot\left(\frac{2\sigma^2\cdot\sigma_1^2}{\sigma^2+\sigma_1^2}\right)^{k+1}\times$$

$$\times\gamma\left(k+1,\frac{z_{11}^2}{2\cdot\sigma^2}\right). \tag{14}$$

The probability density function and the cumulative distribution of the output signal of envelope at the second branch of the first receiver are:

$$p_{z_{02}}(z_{02})=\frac{z_{02}}{\sigma^2}\cdot\exp\left(-\frac{z_{02}^2}{2\sigma^2}\right), \tag{15}$$

$$F_{z_{02}}(z_{02})=1-\exp\left(-\frac{z_{02}^2}{2\sigma^2}\right) \tag{16}$$

and on the output signal of the second branch of the second receiver:

$$p_{z_{12}}(z_{12})=\frac{z_{12}}{\sigma^2}\cdot\exp\left(-\frac{z_{12}^2}{2\sigma^2}\right), \tag{17}$$

$$F_{z_{12}}(z_{12})=1-\exp\left(-\frac{z_{12}^2}{2\sigma^2}\right). \tag{18}$$

According to results of the probability density function of the output SC combiner signal is the sum of product of the probability density function at one and the cumulative distributions at other outputs:

$$p_z(z)=p_{z_{01}}(z)\cdot F_{z_{11}}(z)\cdot F_{z_{02}}(z)\cdot F_{z_{12}}(z)+$$

$$+p_{z_{11}}(z)\cdot F_{z_{01}}(z)\cdot F_{z_{02}}(z)\cdot F_{z_{12}}(z)+$$

$$+p_{z_{02}}(z)\cdot F_{z_{01}}(z)\cdot F_{z_{11}}(z)\cdot F_{z_{12}}(z)+$$

$$+p_{z_{12}}(z)\cdot F_{z_{01}}(z)\cdot F_{z_{11}}(z)\cdot F_{z_{02}}(z). \tag{19}$$

Change of probability density function of the output SC combiner signal for different values of parameters is shown in Fig. 4.

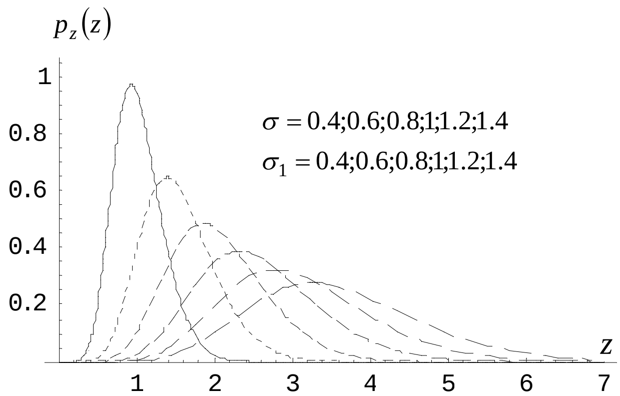

Fig. 4. Probability density function of the signal z

The error probability of system

In case that the output signal at the first branch of the first receiver z_{11} is bigger than z_{01}, z_{02}, and z_{12}, or output signal at the second branch of the second receiver z_{12} is bigger than z_{11}, z_{01} and z_{02}, the probability of signal error at output of SC diversity system is:

$$P_e=P\{z_{11}>z_{01},z_{11}>z_{02},z_{11}>z_{12}\}+$$

$$+P\{z_{12}>z_{11},z_{12}>z_{01},z_{12}>z_{02}\}; \tag{20}$$

$$
\begin{cases}
P_e = 2\cdot\int_0^\infty p_{z_{11}}(x)\cdot F_{z_{01}}(x)\cdot F_{z_{12}}^2(x)\cdot dx\,; \\
P_e = 1-2\cdot P\{z_{01} > z_{11}, z_{01} > z_{02}, z_{01} > z_{12}\}\,; \\
P_e = 1-2\cdot\int_0^\infty p_{z_{01}}(x)\cdot F_{z_{11}}(x)\cdot F_{z_{02}}^2(x)\cdot dx\,; \\
P_e = 1-2\cdot\int_0^\infty \exp\left(-\frac{x^2}{2\sigma^2}\right)\cdot\sum_{k_1=0}^{\infty}\frac{x^{2k_1+1}}{\sigma^{4k_1+2}\cdot\sigma_1^2\cdot 2^{2k_1}\cdot(k_1!)^2}\times \\
\times\left(\frac{2\sigma^2\cdot\sigma_1^2}{\sigma^2+\sigma_1^2}\right)^{k_1+1}\cdot\Gamma(k_1+1)\cdot\sum_{k_2=0}^{\infty}\frac{\Gamma(k_2+1)}{\sigma^{4k_2+2}\cdot\sigma_1^2\cdot 2^{2k_2}\cdot(k_2!)^2}\times \\
\times\left(\frac{2\sigma^2\cdot\sigma_1^2}{\sigma^2+\sigma_1^2}\right)^{k_2+1}\cdot\gamma\left(k_2+1,\frac{x^2}{2\cdot\sigma^2}\right)\times \\
\times\left(1-\exp\left(-\frac{x^2}{2\sigma^2}\right)\right)^2\cdot dx\,. \qquad (21)
\end{cases}
$$

Finally, from preceding expression the error probability output receiver signal is:

$$
P_e = 1-\frac{2}{\sigma^4\cdot\sigma_1^4}\cdot\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{\Gamma(k_1+1)\cdot\Gamma(k_2+1)}{\sigma^{4k_1+4k_2}\cdot 2^{2k_1+2k_2}\cdot(k_1!)^2(k_2!)^2}\times
$$

$$
\times\left(\frac{2\cdot\sigma^2\cdot\sigma_1^2}{\sigma^2+\sigma_1^2}\right)^{k_1+k_2+2}\cdot\int_0^\infty\exp\left(-\frac{x^2}{2\cdot\sigma^2}\right)\cdot x^{2k_1+1}\times
$$

$$
\times\left(1-\exp\left(-\frac{x^2}{2\cdot\sigma^2}\right)\right)^2\cdot\gamma\left(k_2+1,\frac{x^2}{2\cdot\sigma^2}\right)\cdot dx. \qquad (22)
$$

Conclusion

In this paper was analyzed dual SC diversity system for non-coherent demodulation of BFSK signals in presence of Rayleigh fading, and for this case was determined the expression for the probability density function of signals on the output of system. On the figures are shown changes of this probability density functions for different values of parameters. The expressions for the error probability of system were given in the closed form, too.

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G. D. Tomović, S. N. Suljović. Diversity System for Demodulation BFSK Signals in the Presence of Rayleigh Fading // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 8(88). – P. 59–62.

Dual diversity system for noncoherent demodulation BFSK signals in the presence of Rayleigh fading is considered. On the reception selective combining is used which in that case giving optimal results. For this model the closed form expression for the probability density function and error probability of output signals are given and change of probability density function of the output SC combiner signal for different values of parameters are shown. Ill. 4, bibl. 6 (in English; summaries in English, Russian and Lithuanian).

Г. Д. Томович, С. Н. Сулевич. Система диверсификации для демодуляции сигналов BFSK в присутствии гашения Рэлея // Электроника и электротехника – Каунас: Технология, 2008. – № 8(88). – С. 59–62.

Анализируется система двойной диверсификации для некогерентной демодуляции сигналов BFSK в присутствии гашения типа Рэлея. Для приема сигналов используется отборное объединение, которое в этом случае дает оптимальные результаты. Для этой модели дается закрытая форма выражения функции плотности вероятности и вероятности ошибки выходных сигналов, а также показано изменение функции плотности вероятности на выходе объединителя сигналов при разных величинах параметров. Ил. 4, библ. 6 (на английском языке; рефераты на литовском, английском и русском яз.).

G. D. Tomović, S. N. Suljović. Diversifikuota BFSK signalų demoduliacijos, esant Reilėjaus slopinimui, sistema // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 8(88). – P. 59–62.

Nagrinėjama dvigubos diversifikacijos nekoherentinė BFSK signalų demoduliacija esant Reilėjaus slopinimui. Imtuve panaudotas selektyvus signalų sujungimas, kuris šiuo atveju leido gauti optimalius rezultatus. Pateikta šį modelį apibūdinančios tikimybės tankio funkcijos bei išėjimo signalo klaidos tikimybės išraiška. Analizuota, kaip keičiasi selektyvaus sumatoriaus išėjimo signalo tikimybės tankio funkcija priklausomai nuo skirtingų įėjimo parametrų verčių. Il. 4, bibl. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).