ISSN 1392 – 1215 ELEKTRONIKA IR ELEKTROTECHNIKA. 2003. Nr. 4(46)

T 190 ELEKTROS INŽINERIJA

Thermal Analysis of Electrical Wires by Finite Volume Method

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Introduction

Thermo-electrical investigations of electrical wires, cable harness and electrical fuses are very important topics for present car design. Related questions have gained increasing attention by a number of researches [1,2,3]. Normally, heat transfer questions in electrical conductors are solved by different numeric methods if non-linear boundary conditions, arising from convection and radiation, have to be considered. However, non-linear phenomena require iterative calculation processes, which consume large computer time resources. Therefore, having numerical calculation results approximated by simplified mathematic expressions helps to speed up the analysis of thermo – electrical characteristics. In this paper, a single round wire with convection and radiation at the boundaries is treated to illustrate this procedure

The equation for heat dissipation is applied for round insulated wires, which are placed in the air. It is a second order partial derivative equation (PDE). Since, length of the wire is much bigger than the diameter; the matter can be treated as one-dimensional problem. The energy balance equation is solved numerically. Analytical solution of the PDE is not discussed in the paper.

In order to deal with non-linear boundary condition and other non-linearities numerical methods have to be applied. Several methods to construct finite differences of PDE can be used, like direct approximation by finite differences, Taylor Series or Finite Volume (control volume) approach. The FV method has been chosen, because of its flexibility in case the shape and location of control volumes has to be modified. In addition this method has advantage, that by direct discretization of the integral form of the conservation law, the basic quantities mass, momentum and energy will also remain conserved at the discrete level.

The differential equations are solved by using a semi-implicit scheme. Semi-implicit schemes have the advantage over the explicit schemes, because of not being restricted to the size of the time steps. Since calculation has to be performed over a large number of time steps, the calculation time may become prohibitively large. The semi-implicit schemes help to avoid these shortcomings.

Thermo-electrical characteristics of insulated round wire are approximated by using polynomial fitting algorithm based on the least-square method. This method enables to calculate polynomial coefficients of second order polynomial equation. Second-order polynomial equations approximates reasonably precise the characteristics of isolated electrical wire.

All results obtained from numerical calculation have been validated by physical models. Experimental data obtained from measurements of different types and sizes of wires are used to validate thermal model of the electrical wire.

Limitations of the analytical calculation

General equation of thermal conduction. For the derivation of the thermal conduction equation a solid body with volume distributed heat sources is considered. In our case the heat source is a direct current. The equation of thermal conduction (1) is written in conservative form, which is required by the FV method. The conservative form means, that the derivative of variable coefficient λ does not appear in the equation. The physical model and temperature profile in the metallic wire and its isolation is shown in the figures no. 1 and 2 respectively.

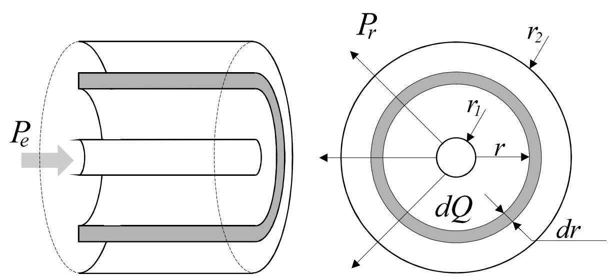

Fig. 1. Axial heat conduction of round isolated wire. Here: P_e – electrical power applied to the wire; P_r – dissipated electrical power in radial direction; r – radius of finite element of insulation r_1 – radius of metallic conductor, r_2 – radius of insulation; dr – dimension of finite element of insulation; dQ – heat energy

The general form of heat transfer equation energy conservation form:

$$div(\lambda \, gradT) + p - \gamma \frac{\partial T}{\partial \tau} = 0 \, ; \qquad (1)$$

here $p = EJ$ [W/m^3] - specific heat flux; λ - heat conductivity of conductor or insulation in W/m·K; γ - specific heat capacity of conductor or insulation in J/m^{3}K.

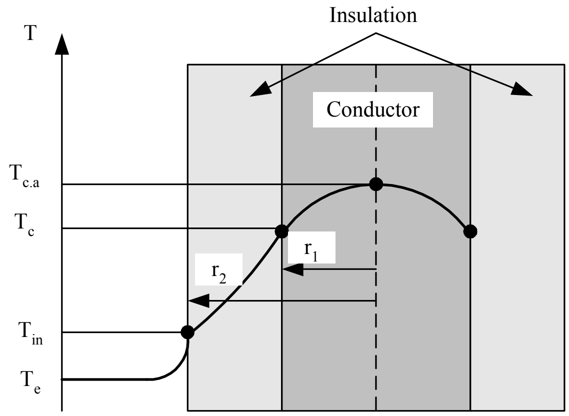

Fig. 2. Temperature profile in the metallic wire and its isolation. The following notations are used: $T_{c.a}$ – temperature in the axis of the wire, T_c – temperature at the interface between wire and insulation (an assumption was made, that the temperature at the surface of the wire is equal to the temperature at the inner side of the insulation), T_{in} – temperature on the outer side if the insulation, T_e – temperature of environment

Taking in to account heat flux only in radial direction, the equation (1) takes the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda \cdot r\frac{\partial \Delta T}{\partial r}\right)+p-\gamma\frac{\partial \Delta T}{\partial \tau}=0 \qquad (2)$$

here $\Delta T(r,t)=T(r,t)-T_{env}; T_e$ – temperature of environment; E - electrical field [V/m]; J – electrical current density [A/m^2].

Transient regime of heat transfer equation. The temperature distribution in transient state in the metallic wire and its insulation is given by the equations (2), where heat conductivity and heat capacity coefficients λ and γ respectively, have to be considered for conductor and insulation differently.

The following initial and boundary conditions apply:

- initial condition

$$\Delta T(r,0)=T_e(r)\,; \qquad (3)$$

- boundary conditions

$$\left(\frac{\partial \Delta T_c}{\partial r}\right)_{r=r_1}=0\,; \qquad (4)$$

(no heat flux in the axis of the conductor)

$$-\lambda_{in}\left(\frac{\partial \Delta T(r,t)}{\partial r}\right)_{r=r_1}=\alpha(d,\Delta T)(T_{in}-T_e)+\varepsilon\sigma\left(T_{in}^4-T_e^4\right); \qquad (5)$$

here α - convection heat transfer coefficient of wire insulation surface in W/m^{2}K. The equation of α can be found in [4]; d – diameter of conductor in m; T_{in} – absolute temperature of insulation in K; ε - emissivity coefficient ; σ - Stefan-Boltzmann constant, $\sigma = 5{,}67\ 10^{-8}$ W/m^{2}K^4.

The two conditions of the continuity equation for heat transfer at the point $r = r_1$ have to be stated:

- continuity equation for the heat flux (heat flux, which goes out of the surface of the metallic conductor enters the inner side of insulation)

$$-\lambda_c(\Delta T)\left(\frac{\partial \Delta T(r,t)}{\partial r}\right)_{r=r_1}=-\lambda_{in}\left(\frac{\partial \Delta T(r,t)}{\partial r}\right)_{r=r_1}; \qquad (6)$$

- continuity equation for the temperature of conductor ΔT_c and insulation ΔT_{in}

$$(\Delta T_c)_{r=r_1}=(\Delta T_{in})_{r=r_2}\,. \qquad (7)$$

Steady state regime of heat transfer equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda \cdot r\frac{\partial \Delta T}{\partial r}\right)+p=0 \qquad (8)$$

Initial and boundary conditions for steady state regime are the same as for transient regime.

Existing problems of solving heat transfer equation. The main problem of solving the equation is the variable coefficients, which lead to the difficulties of solving the heat conduction equation. The main difficulties arise from:

a) *non-linear behaviour of electrical resistance of conductor material.* In the heat equation (3) the electrical field strength coefficient E is influenced by electrical resistance and is therefore a function of temperature:

$$E=E_0[1+\alpha_\rho(T(r,t)-T_0)+\beta_\rho(T(r,t)-T_0)^2] \qquad (9)$$

here E_0 - electrical field strength at reference temperature T_0; α_ρ - linear temperature coefficient of copper specific resistance at T_0. $\alpha_\rho = 3.83\times10^{-3}$ in 1/K at T_0=20°C; β_ρ - square temperature coefficient of copper specific resistance at T_0. $\beta_\rho = 6\times10^{-6}$ in 1/K^2; T_0 - reference temperature in K.

b) *non-linear behaviour of heat conductivity coefficient λ_c of conductor material.* This coefficient is a non-linear function of temperature [4].

c) *non-linear dependence of specific heat capacity coefficient of conductor and insulation materials γ_c and γ_{ins}.* These coefficients are also non-linear functions of temperatures and can be found in the literature [4].

d) *non-linear boundary condition of wire insulation Eq. (10).* The non-linearity is caused by convection and radiation to environment. Heat convection also depends on extremely non-linear heat transfer coefficient of wire insulation surface α, which is a non-linear function of conductor diameter and temperature.

All these non-linear phenomena, mentioned above can not be considered when solving the heat equation in analytical way. Therefore, it is inevitable to apply some numeric technique in order to scope with above declared problems.

Numerical approach of solving heat conduction equation

Since it is not possible to have analytical solution of the heat transfer equation if non-linear boundary conditions as well as non-linear physical constants of the materials have to be considered, a numerical way had to be used to solve the heat transfer problem in insulated round wires.

As already mentioned earlier, the FV method is used to construct finite difference approximations of PDE. This method is based on the conservation of a specific physical quantity as thermal energy. The approach employs numerical balances of a conserved variable over small control volumes.

The first step in the FV method is to state the governing Eq. (2) in its integral form [5]. The region of application for the conservation principle must be also given. After the control volumes are defined, the Eq. (2) has to be applied to this volume (see Fig. 3) and the conservation statement becomes

$$\frac{\partial}{\partial r}\int_{i-1/2}^{i+1/2} \lambda r \frac{\partial \Delta T}{\partial r} dr + \int_{i-1/2}^{i+1/2} r p_i dr - \gamma \frac{\partial}{\partial \tau}\int_{i-1/2}^{i+1/2} r T_i dr = 0 . \quad (10)$$

Various finite difference approximations for $\partial^2 \Delta T / \partial r^2$, $\partial \Delta T / \partial r$ and $\partial \Delta T / \partial \tau$ are used for the surface integrals. Approximating the derivatives by the central differences we get

$$\lambda_{i+1/2}^{n} \frac{r_i^2}{2} \frac{\Delta T_{i+1}^{n} - \Delta T_i^{n}}{\Delta r^2} + \lambda_{i-1/2}^{n} \frac{r_i^2}{2} \frac{\Delta T_i^{n} - \Delta T_{i-1}^{n}}{\Delta r^2} . (11)$$

The initial condition in numerical terms for the Eq. (11) is given by:

$$\Delta T_i^1 = \Delta T(r_i), \quad \forall = 1, n_r - 1$$

$$+ \frac{r_i^2}{2} p_i - \gamma_i^n \frac{r_i^2}{2} \frac{\Delta T_i^n - \Delta T_i^{n-1}}{\tau} = 0 . \quad (12)$$

The boundary condition, which is convective and radiative, has to be also written considering FV method:

$$\lambda_i \int_{i-1/2}^{i+1/2} \frac{\partial \Delta T}{\partial r} dr + \alpha(d, \Delta T)\left(T_i^n - T_e\right) + \varepsilon\sigma\left(\left(T_i^n\right)^4 - T_e^4\right) = 0 . \quad (13)$$

Using central differences to approximate the derivatives, we can write:

$$\lambda_i \frac{\Delta T_{i+1}^n - \Delta T_i^n}{\Delta r} + \alpha(d, \Delta T)\left(T_i^n - T_e\right) + \varepsilon\sigma\left(\left(T_i^n\right)^4 - T_e^4\right) = 0 . \quad (14)$$

The equations (11,14) are solved implicitly, where at one time level the equation for all space nodes have to be solved simultaneously. This method has advantage of being typically unconditionally stable and the schemes are not restrictive to the size of the time step. The price, which has to be paid, is the need to solve either linear or non-linear systems of equations.

When defining the numeric scheme of temperature profile in the wire (Fig. 3) it was made an assumption, that in the metallic conductor there is no temperature distribution because of copper heat conductivity, which is about 10^3 times higher then polyvinylchloride (PVC) heat conductivity. Therefore, overall computation time is reduced.

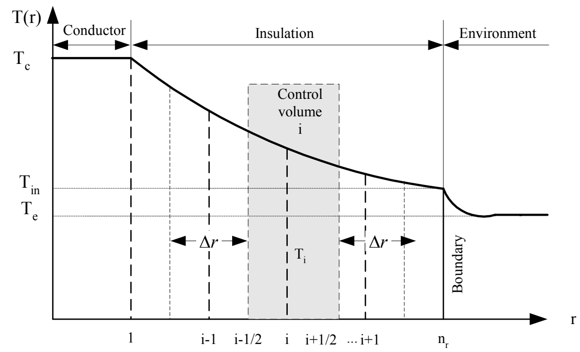

Fig. 3. Numerical scheme of temperature profile in insulated round wire

When equation (11) is solved numerically, for different layers; conductor, insulation and environment, different material coefficients have to be applied.

For conductor the following coefficients apply: λ_c - heat conductivity of conductor; γ_c - specific heat capacity of conductor; p - specific heat flux.

For insulation the following coefficients apply: λ_{ins} - heat conductivity of conductor; γ_{ins} - specific heat capacity of conductor.

In the following three pictures (Fig.4, 5, 6) are shown the results obtained from numerical calculation. For the investigation round insulated wire of 16 mm^2 cross-section has been chosen. The environment temperature is 65°C, maximal nominal temperature of the wire insulation is 90°C.

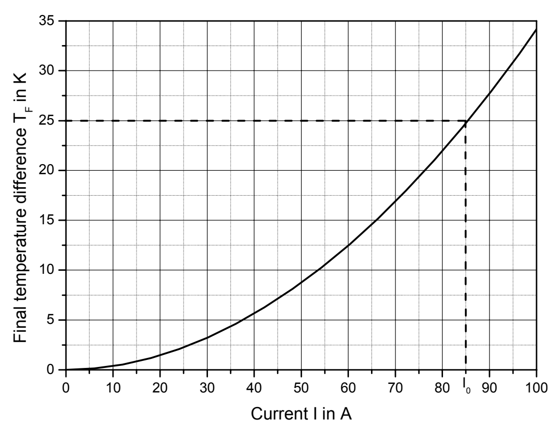

Fig. 4. Thermo-electrical characteristic of round insulated wire

Fig. 4 shows temperature increase in the conductor of electrical wire when the power source is direct current. The picture represents temperature difference between conductor and environment temperature. Fig. 5 depicts electrical field strength in dependence on electrical current. It can be seen from the chart, that first part of the curve does not have linear behaviour, which is caused by specific resistance of copper, which is not linear in the respect to the temperature. The second part of curve is linear, because the numerical algorithm stopped temperature increase in the conductor at 90°C. This part of the curve depicts the electrical field strength for the maximum wire temperature. Fig. 6 gives the so called heating-up time as function of the current, which is the time until the maximal permissible temperature of the wire is reached. It is a logarithmic function, where time approaches zero if current goes to infinite.

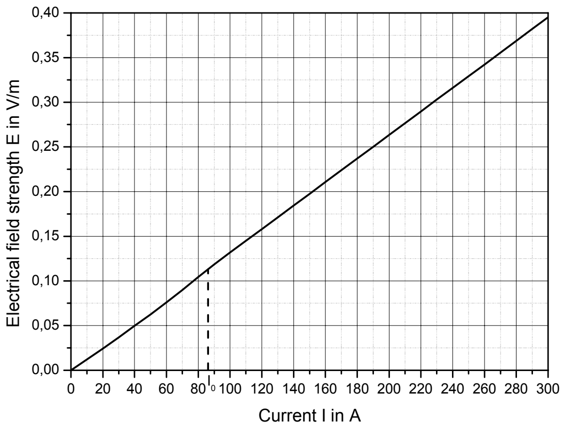

Fig. 5. The characteristic of electrical field strength versus current of round insulated wire

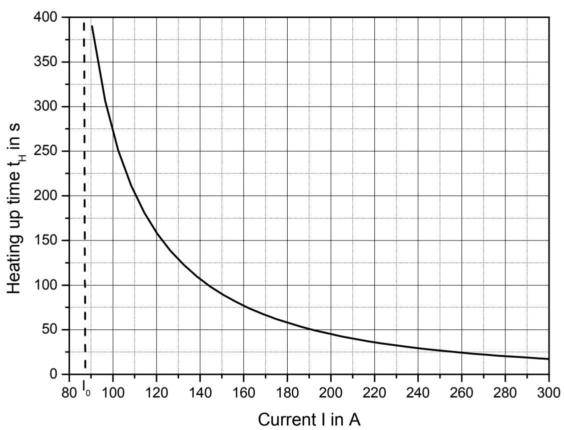

Fig. 6. Heating up time characteristic of round insulated wire

This characteristic is very important when designing electrical fuses, because fuse has the same characteristic and both curves have to mach in order to protect the cable against short circuit currents.

Experimental verification of heat transfer model

The numerical model of the electrical wire was validated by experimental results. Different sizes of electrical wires were loaded with direct current and the voltage drop across the wire measured. Simultaneously the temperature on the surface of the metallic conductor was read by thermocouples. On the basis of this experimental result it was possible to validate the heat conductivity value λ_{ins} of the insulation material and the temperature coefficient α_{ro} of metallic conductor.

Fitting algorithm of thermo-electrical wire characteristics

The next is to simplify the thermal model of the isolated wire by applying so-called “simplified equations” with only two coefficients for each equation, which describes wire characteristics (Fig. 4, 5 ,6) with a reasonable precision within the relevant range. The fitting algorithm is employed in order calculate these characteristic coefficients. The simplified equations are given bellow:

- thermo-electrical behaviour of the wire (here $I \ge 0$):

$$\Delta T\left(I \le I_0\right) = a\,I + b\,I^2\,; \qquad (15)$$

- electrical field strength of the wire (non-linear and linear of the characteristic, here $I \ge 0$)

$$E\left(I \le I_0\right) = c\,I + d\,I^2 \;\text{ or }\; E\left(I \ge I_0\right) = \left(c + d\,I_0\right)I\;; \qquad (16)$$

- heating-up time of the wire:

$$t\left(I \ge I_0\right) = \tau \ln \frac{I^2}{I^2 - I_0^2}\,. \qquad (17)$$

All 6 coefficients of equations (18, 19, 20) I_0, a, b, c, d, τ, are valid only for one wire type. The Least-Square Method is used in order to obtain these coefficients. The following equations are applied:

- Final Temperature per Current coefficient a:

$$a = \frac{\sum_{n=1}^{n} I_n^4 \cdot \sum_{n=1}^{n} I_n \cdot \Delta T_n - \sum_{n=1}^{n} I_n^3 \cdot \sum_{n=1}^{n} I_n^2 \cdot \Delta T_n}{\sum_{n=1}^{n} I_n^2 \cdot \sum_{n=1}^{n} I_n^4 - \left(\sum_{n=1}^{n} I_n^3\right)^2}\,; \qquad (18)$$

- Final Temperature per Current square coefficient b:

$$b = \frac{\sum_{n=1}^{n} I_n^2 \cdot \sum_{n=1}^{n} I_n^2 \cdot \Delta T_n - \sum_{n=1}^{n} I_n^3 \cdot \sum_{n=1}^{n} I_n \cdot \Delta T_n}{\sum_{n=1}^{n} I_n^2 \cdot \sum_{n=1}^{n} I_n^4 - \left(\sum_{n=1}^{n} I_n^3\right)^2}\,; \qquad (19)$$

- Field Strength per Current coefficient c:

$$c = \frac{\sum_{n=1}^{n} I_n^4 \cdot \sum_{n=1}^{n} I_n \cdot E_n - \sum_{n=1}^{n} I_n^3 \cdot \sum_{n=1}^{n} I_n^2 \cdot E_n}{\sum_{n=1}^{n} I_n^2 \cdot \sum_{n=1}^{n} I_n^4 - \left(\sum_{n=1}^{n} I_n^3\right)^2}\,; \qquad (20)$$

- Field Strength per Current square coefficient d

$$d = \frac{\sum_{n=1}^{n} I_n^2 \cdot \sum_{n=1}^{n} I_n^2 \cdot E_n - \sum_{n=1}^{n} I_n^3 \cdot \sum_{n=1}^{n} I_n \cdot E_n}{\sum_{n=1}^{n} I_n^2 \cdot \sum_{n=1}^{n} I_n^4 - \left(\sum_{n=1}^{n} I_n^3 \right)^2}; \qquad (21)$$

- Time Constant τ

$$\tau = \frac{\sum_{n=1}^{n} t_{n_n} \ln \frac{I_n^2}{I_n^2 - I_0^2}}{\sum_{n=1}^{n} \left(\ln \frac{I_n^2}{I_n^2 - I_0^2} \right)^2}; \qquad (22)$$

here $\forall = 1, n$; where n is the number of calculating points in the wire characteristics;

$$I_0 = \frac{\Delta T}{a} \left(1 + \frac{b \Delta T}{a^2} \right). \qquad (23)$$

Applications and Extensions

The presented algorithm to calculate thermo-electrical characteristic coefficients of insulated wires is applied to the single electrical wire, which is placed in the air. The numerical algorithm has been presented here in a summarised form for illustration. However it shows that non-linear heat conductivity of the conductor and all other phenomena as non-linear convection, radiation and electrical resistances were considered.

Extensions of the proposed method to the two-dimensional model analysis of conductors with limited length like fuses would allow calculating axial and radial temperature distributions.

In essence the polynomial fitting method, which is presented here, is also applicable to any thermal analysis problem of electrical conductors e.g. flat cable structures, flat and cable harnesses.

Conclusions

The proposed methodology to treat thermal analysis problems in electrical conductors has proven to provide fast and efficient solutions, which completely satisfy experimental results.

In this paper the algorithm is presented to calculate thermo-electrical characteristics of round insulated wires. The method is also valid for other wire geometries and multidimensional temperature distribution.

Acknowledgement

This work was performed at University of Federal Armed Forces Munich and had been supported by the cable harness supplier Intedis GmbH, the car manufacturer DaimlerChrysler and the Bavarian Research Foundation.

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Pateikta spaudai 2003 03 05

A. Ilgevičius, H. D. Liess. Elektros laidų terminė analizė baigtinių tūrių metodu // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2003. – Nr. 4(46). – P. 87-92.

Aprašytas apvalių izoliuotų elektros laidininkų termoelektrinių charakteristikų apskaičiavimo algoritmas, įvertinant šilumos srautų pasiskirstymą radialine laidininko kryptimi. Kadangi šilumos pernešimo lygties koeficientai yra netiesiniai, uždaviniui spręsti turi būti taikomi iteraciniai metodai. Kai reikia apskaičiuoti didelio skaičiaus laidų charakteristikas, kompiuterio skaičiavimo trukmė neleistinai padidėja. Todėl yra pritaikytas termoelektrinių charakteristikų aproksimacijos antrojo laipsnio polinomais algoritmas. Gautos aproksimacijos išraiškos yra antrojo laipsnio polinomo lygtys, kurios labai paspartina skaičiavimą. Nagrinėjamas atskiras izoliuotas elektros laidininkas laikomas ore, įvertinant šiluminę kraštų konvekciją bei spinduliavimą. Matematinis modelis yra sudarytas remiantis energijos konservavimo dėsniu, kuris yra pritaikytas elektros laidininkui. Energijos balansų lygtis, kuri aprašo šilumos perdavimą laidininke yra homogeninė dalinių išvestinių lygtis su nepastoviais netiesiniais koeficientais. Netiesiškumai yra įvertinti apskaičiuojant laidininko elektrinę varžą, specifinę šiluminę talpą bei kraštinę lygties sąlygą: konvekciją ir išspinduliavimą. Šilumos perdavimo lygčiai spręsti skaitiniu būdu buvo taikomas baigtinių tūrių metodas. Remiantis šiuo metodu, buvo sudarytos lygties dalinių išvestinių baigtinių skirtumų aproksimacijos. Mažiausių kvadratų metodu apskaičiuoti polinominių lygčių koeficientai I_0, *a, b, c, d, τ.* Il. 6, bibl. 5 (lietuvių kalba; santraukos lietuvių, anglų ir rusų k.).

A. Ilgevičius, H. D. Liess. Thermal Analysis of Electrical Wires by Finite Volume Method // Electronics and Electrical Engineering. – Kaunas: Technology, 2003. – No. 4(46). – P. 87-92.

In this paper is proposed the algorithm to calculate thermo-electrical characteristics of round insulated wire, considering temperature distribution in radial direction. However, non-linear phenomena require iterative calculation processes, which consume large computer time resources. Therefore, having numerical calculation results approximated by simplified mathematic expressions helps to speed up the analysis of thermo – electrical characteristics. The procedure of obtaining polynomial coefficients from thermo-electrical characteristics is the major emphasis of this paper. In order to illustrate the procedure for implementation, only a single round wire with convection and radiation at the boundaries is treated. The mathematical model is obtained from energy conservation law and is applied to the round wire. The energy balance equation, which describes heat transfer in a round wire, is a homogenous partial derivatives equation (PDE), where non-linear electrical resistance and specific heat capacity are considered. Finite Volume Method (FVM) is applied for the development of finite-difference equations, thus to solve iterative problem, which is caused by non-linear phenomena in electrical resistance, convection and radiation. The PDE is solved by semi-implicit method. Finally, least-square algorithm is used to obtain polynomial coefficients I_0, a, b, c, d, τ. Ill. 6, bibl. 5 (in Lithuanian; summaries in Lithuanian, English, Russian).

А. Илгявичюс, Д. Лесс. Термоанализ электрических проводов методом конечных объемов // Электроника и электротехника. – Каунас: Технология, 2003. - № 4(46). – С. 87-92.

Предлагается алгоритм для вычисления термоэлектрических свойств круглой изолированнщй проволоки, учитывая радиальное направление распределения температуры. Однако нелинейное явление требует итеративного вычитывания, которое приводит к использованию огромных ресурсов ПК и времени. Следовательно имея прочитаные данные, аппроксимируемые упрощенными математическими выражениями, можно ускорить анализ термоэлектрических характеристик. Одним из самых главных элементов данного доклада является процесс получения полиномиальных коэффициентов из термоэлектрических свойств. Для того, чтобы проиллюстрировать осуществление данного процесса, исследуется только одна круглая проволока с конвекцией и радиацией по концам. Математическая модель достигается на основе закона сохранения энергии и применяется к круглой проволоке. Уравнение энергетического баланса, которое характеризует теплопередачу в круглой проволоке является гомогенным уравнением частной производной (partial derivatives equation PDE), где нелинейное электрическое сопротивление и определенное теплоемкость должно быть взято во внимание. Метод конечных объемов (FVM) используется для вывода конечно-разностного уравнения, чтобы решить итеративную проблему, вызванную нелинейными явлениями электрического сопротивления, конвекции и радиации. PDE решается полу-неразрешенным методом. В заключение используется алгоритм с наименьшими квадратами для вычисления полиномиальных коэффициентов I0, a, b, c, d, t. Ил. 6, библ. 5 (на литовском языке; рефераты на литовском, английском и русском яз.).

DOI: 10.5755/j02.eie.11254