ISSN 1392 – 1215 ELECTRONICS AND ELECTRICAL ENGINEERING 2008. No. 6(86)
ELEKTRONIKA IR ELEKTROTECHNIKA

T180 TELECOMMUNICATIONS ENGINEERING
TELEKOMUNIKACIJŲ INŽINERIJA

Analyse of Kalman Algorithm for Different Movement Modes of Land Mobile Object

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Introduction

Inertial Navigation System (INS) and Global Positioning System (GPS) technologies have been widely used in a variety of positioning and navigation applications. The integration of GPS with INS can be implemented using a Kalman filter. Efficiency of conventional Kalman algorithm is low, when vehicle dynamics change quickly [1, 2].

Therefore adaptive Kalman algorithms are used to improve overall system performance. The test results demonstrate that the presented adaptive algorithm is much robust to the sudden changes of vehicle motion during maneuvering.

System dynamic model

The 1-D acceleration land mobile object dynamic model equations in state-space formulation are described as [2]:

$$\mathbf{X}_{k+1} = \begin{vmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{vmatrix} \mathbf{X}_k + \begin{vmatrix} w_d \\ w_v \\ w_a \end{vmatrix}, \tag{1}$$

where $\mathbf{X}_k$ –system state vector (distance, velocity, acceleration), w_d , w_v , w_a – vector of system noise with zero mean and variances q_d , q_v , q_a respectively , T – sampling interval.

The measurement model is:

$$\mathbf{Z}_{k+1} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \mathbf{X}_{k+1} + \begin{vmatrix} u_d \\ u_v \\ u_a \end{vmatrix}, \tag{2}$$

where $\mathbf{Z}_{k+1}$ – measurement vector, u_d , u_v , u_a – vector of measurement noise with zero mean and variances r_d , r_v , r_a respectively.

Kalman algorithm description

Basically, the Kalman filtering estimation algorithm comprises two steps, namely prediction and update with external measurements. Kalman filtering can be used to estimate optimally the system states (the unknown elements of the state vector) at the current time basing on a combination of predicted states and actual measurements.

The main Kalman filtering equations are given below [3]:

$$\begin{aligned}
&\text{Updating}: \\
&\mathbf{K}_k = \mathbf{P}_{k(predicted)} \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k(predicted)} \mathbf{H}_k^T + \mathbf{R}_k)^{-1}, \\
&\hat{\mathbf{X}}_k = \hat{\mathbf{X}}_{k(predicted)} + \mathbf{K}_k (\mathbf{Z}_k - \mathbf{H}_k \hat{\mathbf{X}}_{k(predicted)}), \\
&\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k(predicted)}, \\
&\text{Predicting}: \\
&\hat{\mathbf{X}}_{k+1(predicted)} = \mathbf{\Phi}_k \hat{\mathbf{X}}_k, \\
&\mathbf{P}_{k+1(predicted)} = \mathbf{\Phi}_k \mathbf{P}_k \mathbf{\Phi}_k^T + \mathbf{Q}_k,
\end{aligned} \tag{3}$$

where $\mathbf{K}_k$ – Kalman gain matrix, $\mathbf{P}_k$ –state uncertainty covariance matrix, $\mathbf{H}_k$ –measurement sensitivity matrix, $\mathbf{R}_k$ –measurement noise covariance matrix, $\mathbf{\Phi}_k$ –state transition matrix, $\mathbf{Q}_k$ –system noise covariance matrix .

Velocity and distance estimation during sudden vehicle dynamic change

The simulation made for the cases when initial vehicle velocity was 10m/s and acceleration (5m/s^2 or 10m/s^2) took place from t_1=30s till t_2=50s with different levels of system noise. The random error variance due to the distance measurement noise was (15m)^2, and the random error variance due to the velocity measurement noise was (5 m/s)^2.

Modeling results using standard Kalman filtering algorithm (3) are presented in Fig.1-6.

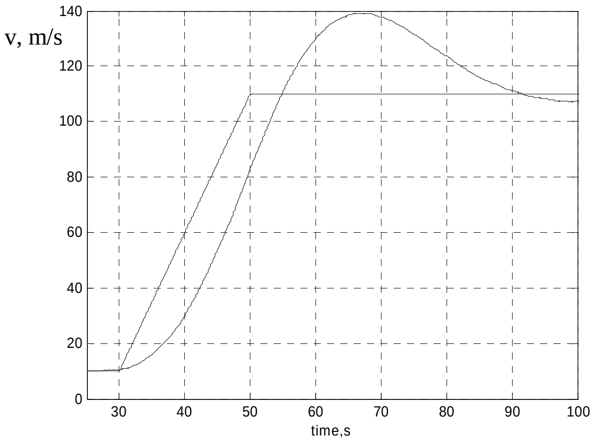

Fig. 1. Mobile object real and estimated velocity when acceleration a_x=5m/s^2 take place from t=30s till t=50s and with small system noise ($q_d = q_v = q_a$=10^{-6})

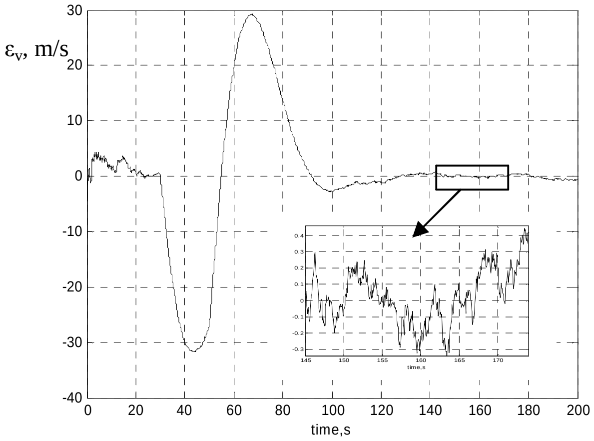

Fig. 2. Mobile object velocity estimation error when acceleration a_x=5m/s^2 take place from t=30s till t=50s and with small system noise ($q_d = q_v = q_a$=10^{-6})

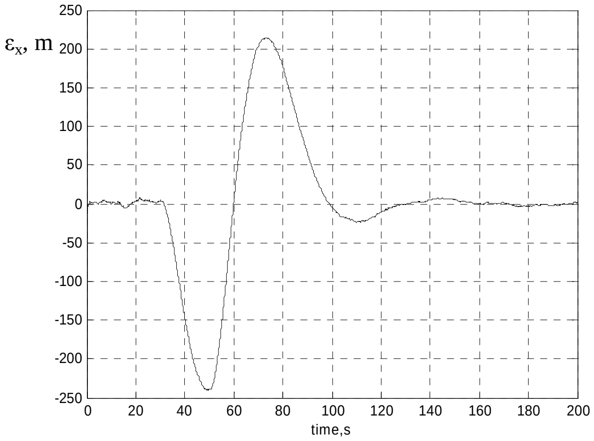

Fig. 3. Mobile object x-coordinate estimation error when acceleration a_x=10m/s^2 take place from t=30s till t=50s and with small system noise ($q_d = q_v = q_a$=10^{-6})

As we can see change of acceleration seriously affects Kalman algorithm performance characteristic. And with bigger change of acceleration, estimation error will increase (Fig.4). There are exists several techniques to decrease estimation error [2]. One of them is to add fictitious system noise [2, 4]. Modeling results for the case with additional fictitious system noise are presented in Fig.5, 6 (corresponding diagonal elements of system noise covariance matrix are increased to 0.01).

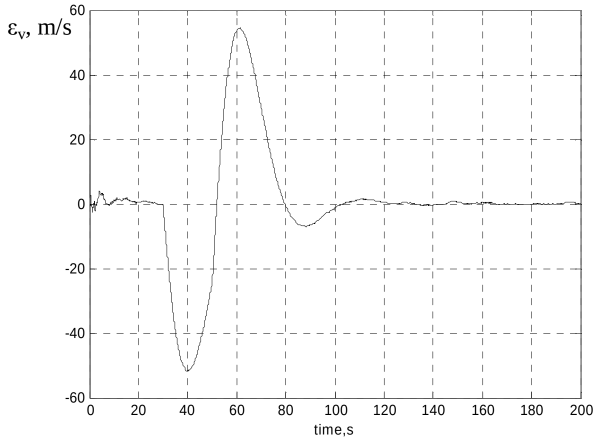

Fig. 4. Mobile object velocity estimation error when acceleration a_x=10m/s^2 take place from t=30s till t=50s and with small system noise ($q_d = q_v = q_a$=10^{-6})

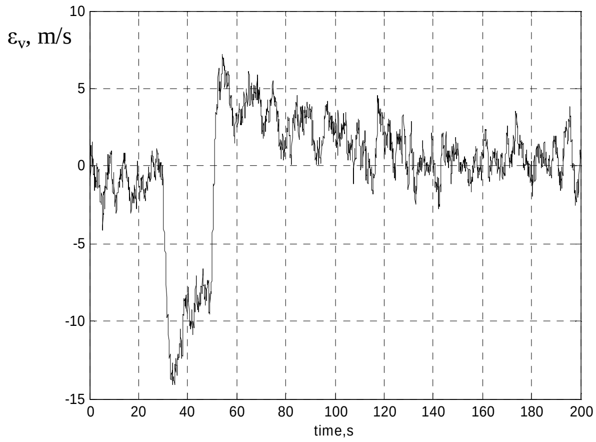

Fig. 5. Mobile object velocity estimation error when acceleration a_x=10m/s^2 take place from t=30s till t=50s (q_v =0.01)

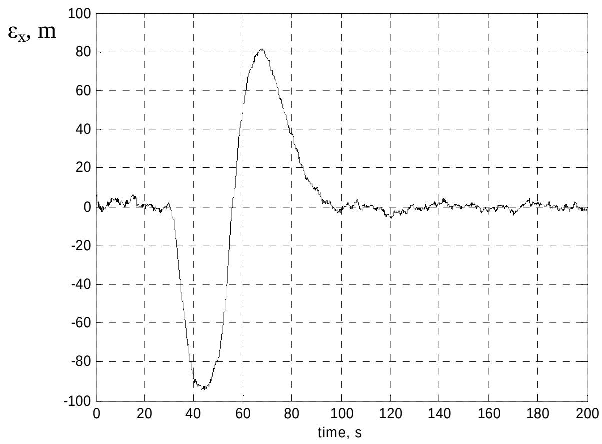

Fig. 6. Mobile object x-coordinate estimation error when acceleration a_x=10m/s^2 take place from t=30s till t=50s (q_d =0.01)

The modeling results prove that performance of standard Kalman algorithm is poor during sudden vehicle acceleration change.

Adaptive Kalman algorithm

Kalman gain matrix correction algorithm (KGCA) can be used to decrease system state variables (position, velocity) estimation errors. The algorithm steps are :

Step 1. To detect time epoch when acceleration start to change;

Step 2. To add special function' values to Kalman gain matrix diagonal elements so to improve algorithm performance during vehicle velocity change and post change period. Therefore equations for Kalman gain matrix updating and correcting are following:

$$\begin{aligned} &\mathbf{K}_k = \mathbf{P}_{k\,(predicted)} \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k\,(predicted)} \mathbf{H}_k^T + \mathbf{R}_k), \\ &\mathbf{F}_k = \underset{\ell}{diag}\{f_\ell\}, \\ &\mathbf{K}_{k(corrected)} = \mathbf{K}_k + \mathbf{F}_k , \end{aligned} \tag{4}$$

where f –special function's values at time epoch k, ℓ - number of state variables in the dynamic system model. Three functions f was tested. These functions' general forms are shown in Fig.7. The modeling results (using these functions for correction in (4)) for position estimation error when acceleration a_x=10m/s^2 take place from t=30s till t=50s are shown in the Table 1.

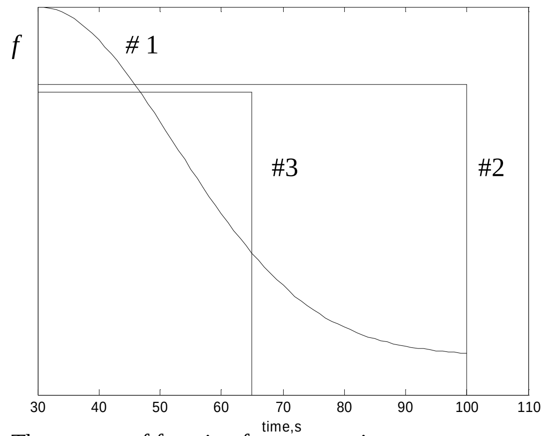

Fig. 7. Three types of function f representation

Table 1. Three types of function

MSE estimating error decrease using KGCA and three types of function			
function	#1	#2	#3
MSE	26	300	2569

Simulating results show that function #1, used for Kalman gain matrix correcting allow reducing system parameter estimation error in a greater degree. The reason of this can be that function #1 has smooth transition and hence Kalman gain correction is conducted softly without extra disturbance just after acceleration has changed. Function #1 mathematical description is following:

$$f(t_k) = Ae^{-B(t_k - t_D)^2}, \tag{5}$$

where A,B –fixed values are decided by user or designer, t_D –time epoch, when acceleration has changed.

The KGCA algorithm modeling results are shown in Fig. 8, 9 (diagonal elements of system noise covariance matrix are equal to 10^{-6} and function (5) is used for Kalman gain correction).

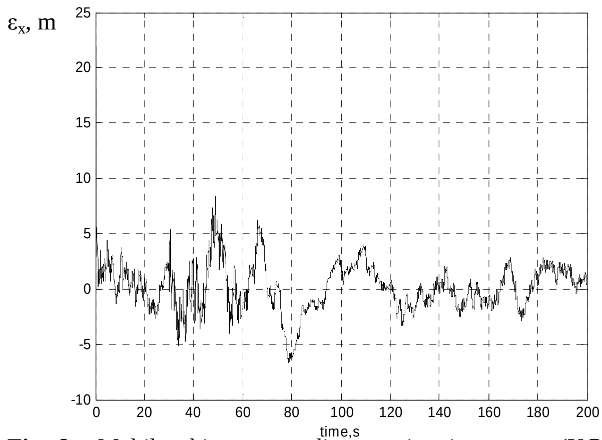

Fig. 8. Mobile object x-coordinate estimation error (KGCA) when acceleration a_x=10m/s^2 take place from t=30s till t=50s and with small system noise (q_d = q_v = q_a=10^{-6})

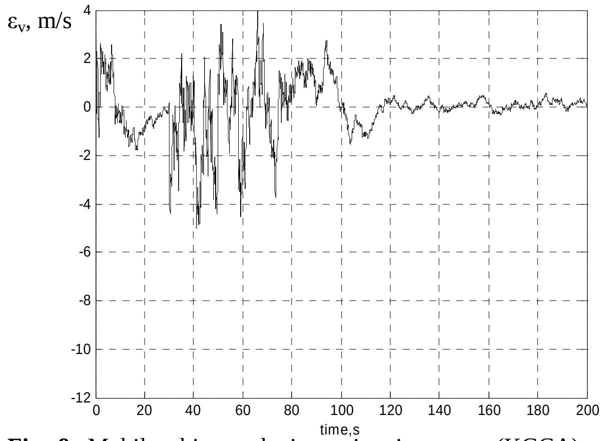

Fig. 9. Mobile object velocity estimation error (KGCA) when acceleration a_x=10m/s^2 take place from t=30s till t=50s and with small system noise (q_d = q_v = q_a=10^{-6})

As can be noticed from Fig.8, 9, there's considerable estimation error decrease if adaptive algorithm is used. But also can be noticed that at time of Kalman gain correction, there are increased variations of estimation error, comparing with time, when correction's values became very small and no system dynamic change. These variations are due to effect of Kalman gain increase.

The standard and adaptive Kalman algorithm (KGCA) modeling results are presented in Table 2, 3 (MSE_v-velocity estimation MSE; MSE_d-position estimation MSE along x-coordinate). The system noise matrix diagonal elements are equal to 10^{-6} and acceleration a_x take place from t=30s till t=50s.

Table 2. MSE estimating error

MSE estimating error, a_x=10m/s^2, r_d =225m^2, r_v=25(m/s)^2		
Standard Kalman algorithm	MSE_v=579	MSE_d=7689
Adaptive Kalman algorithm (KGCA)	MSE_v=1.82	MSE_d=26

Table 3. MSE estimating error

MSE estimating error, a_x=5m/s^2, r_d =225m^2, r_v=25(m/s)^2		
Standard Kalman algorithm	MSE_v=145	MSE_d=1913
Adaptive Kalman algorithm (KGCA)	MSE_v=1.66	MSE_d=23

Conclusions

In the current paper, adaptive Kalman algorithm with the filter gain correction for the case of fast dynamics change is presented. In the adaptive algorithm, the detected dynamics changes are regarded with acceleration/velocity change. The developed adaptive KF is applied to the integrated navigation system for velocity and x-coordinate estimating. In case of fast acceleration change, the performance of the integrated system is examined. The presented algorithm can be used also for the case of 2-D acceleration model to decrease estimation error during vehicle maneuvering.

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Submitted for publication 2008 02 17

V. Bistrovs. Analyse of Kalman Algorithm for Different Movement Modes of Land Mobile Object // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 6(86). – P. 89–92.

INS and GPS information complex processing is based mainly on Kalman filtering algorithm. During navigation tasks solving (estimation of velocity and position), tracked object dynamics change occurs quite often. For these cases conventional Kalman algorithm can not be used, as it’s well known that standard Kalman algorithm estimation error increases in cases of quick changes of estimated state parameters. Different methods are being developed to overcome this drawback. The presented Kalman gain correction algorithm (KGCA) decreases estimation error that occurs, during object dynamics alterations. The modeling results of adaptive KGCA algorithm are presented and compared with conventional one. The MSE error reducing benefits for the case of adaptive algorithm use are shown as well. Ill. 9, bibl. 4 (in English; summaries in English, Russian and Lithuanian).

В. Быстров. Анализ алгоритма Калмана для различных типов движения наземного объекта // Электроника и электротехника. – Каунас: Технология, 2008. – № 6(86). – С. 89–92.

Комплексная обработка INS и GPS информации базируется, в основном, на использовании алгоритма Калмана. Во время слежения за объектом для решения навигационных задач (оценка скорости и положения), очень часто наблюдается изменение динамики движения объекта. Для данного случая стандартный алгоритм Калмана не может быть использован, так как ошибка оценивания стандартного алгоритма Калмана значительно увеличивается при быстром изменении параметров состояния системы и становится неприемлемой. Различные методы разрабатываются для уменьшения влияния этого недостатка фильтра Калмана. Представленный адаптивный алгоритм Калмана с коррекцией коэффициента усиления позволяет уменьшить ошибку оценивания, когда динамика объекта меняется. Результаты моделирования данного алгоритма представлены и показано уменьшение ошибки оценивания в сравнении со стандартным алгоритмом Калмана. Ил. 9, библ. 4 (на английском языке; рефераты на литовском, английском и русском).

V. Bistrovs. Kalmano algoritmo, skirto įvairių tipų antžeminio objekto judėjimui tirti, analizė // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 6(86). – P. 89–92.

INS ir GPS informacijos komplekso apdorojimas remiasi Kalmano filtravimo algoritmu. Atliekant navigacines užduotis sekamo objekto dinamika pasikeičia gana dažnai (greičio ir padėties įvertinimas). Šiais atvejais tradicinis Kalmano algoritmas negali būti taikomas, nes standartinė Kalmano algoritmo įvertinimo klaida padidėja. Šiai problemai spręsti kuriami įvairūs metodai. Pristatyti Kalmano korekcijos algoritmai (KGCA), kurie sumažina klaidą, kuri įvyksta dėl objekto dinamikos pokyčių. Pateikti adaptyvaus KGCA algoritmo modeliavimo rezultatai ir palyginti su tradiciniu būdu gautais rezultatais. Il. 9, bibl. 4 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

DOI: 10.5755/j02.eie.11192