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Detecting Deterministic Structure from a High Noisy Pseudoperiodic Time Sseries

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Introduction

An accurate identification of the dynamics underlying a complex time series, is of crucial importance in understanding the corresponding physical process, and in turn affects the subsequent model development. In order to obtain the inherent properties of a system from the observed time series, a variety of methods have been proposed and are widely applied, such as surrogate tests [1], wavelets [2], Fourier transforms, and approaches based on time delay embedding [3]. Among these methods, approaches based on time delay embedding may be the most popular framework for analyzing chaotic time series. Based on Taken's embedding theory, some measures such as Lyapunov exponents [4-6] and correlation dimensions [7] have been proposed to characterize the global features of dynamical systems. Hovever, the presence of noise can greatly affect the analysis of the observed data from chaotic systems. Since the analysis of chaotic data in terms of dimensions, entropies, and Lyapunov exponents requires access to the small length scales (small-scale fluctuations of the signal), already a moderate amount of measurement noise on data is known to be destructive [8]. Recently introduced effective methods for distinguishing chaos from noise can deal with small or moderate amounts of noise [9, 10].

One class of time series – pseudoperiodic – has aroused great interest due to their close relation to some important natural and physiological systems. For high noisy pseudoperiodic time series (at signal-noise ratio SNR=0 dB) it is desirable to reduce the noise level. However, most noise reduction methods are designed for signals that can be treated by a linear model and fail to eliminate noise from a contaminated chaotic time series because the spectra of the chaotic signal and the noise overlap [3]. Noise reduction based on time delay embedding, which has been widely studied, may be the most promising way to filter the noisy chaotic data [11-13]. Several phase space projection methods, based on subspace decomposition, were proposed for application to the problem of additive noise reduction in the context of phase space analysis – the global projections method [13] and the local (nearest neighborhoods) phase spaces method [11-13]. A two step method is proposed to reduce colored noise [14]. These methods performed well with moderate amounts of noise. But I have not found any publication, devoted to detecting the deterministic structure from a high noisy pseudoperiodic time series, when the noise level reducing is desirable with expected to preserve the exponential divergence of nearest neighbors. At some level of the noise, due to the signal low frequency distortion and noise residual when nonlinear noise reduction is performed, we cannot reliable distinguish enhanced pseudoperiodic chaotic signal from enhanced noisy regular sinusoidal signal with Lyapunov exponent (either scale-dependent Lyapunov exponent [9]) calculated neither by Kantz [5] nor Rosenstein [6] algorithms – the divergence slope is not linear and has a similar behavior for both cases. The algorithm based on the correlation coefficient as a measure of the distance between overembedded vectors [15] also misclassifies filtered high noisy regular sinusoidal signal as deterministic chaos.

In this paper (i) the straightforward algorithm to detect chaos from pseudoperiodic time series is presented, which is robust for distortion of enhanced signal, (ii) it is demonstrated that the histograms of white vertical lines in recurrence plots (RP) are the powerfull tool for distinguish the filtered high-noisy chaotic pseudoperiodic data from filtered high-noisy periodic. The essentiality of the algorithm is as follows – the nearest k neighbors for every reference vector of reconstructed phase space in the primary neighborhood of radius ε are fixed and then the dynamics of number of nearest neighbors in the neighborhood of radius $\varepsilon + \Delta\varepsilon$ is monitored with time. The slope of dynamics averaged over all reference vectors of reconstructed phase space allows us to distinguish chaos from noisy regular signal.

Throughout the paper, the x component of the well-known Rossler system and an experimental laser dataset, both of which are chaotic and contain obvious periodic component, for illustration are used. The laser dataset is the record of the output power of the NH_3 laser available in Santa Fe Competition (Data Set A). The signals are contaminated by additive white Gaussian noise. Usually, neighborhoods of reconstructed phase space merges if all data are contaminated by large amounts of noise. Thus, it becomes a nontrivial problem to identify the correct neighbors. Therefore, for the noise reduction I have preferred the global projections method [13] to the local (nearest neighborhoods) phase spaces method [11]-[13].

The organization of this paper is as follows. In Sec. II, the algorithm based on divergence of nearby orbits in phase space for detecting chaos in pseudoperiodic time series is described. In Sec. III, the histograms of white vertical lines in RP for enhanced high noisy pseudoperiodic chaotic and high noisy regular signals are given. Finally, some discussions and conclusions are given in Sec. IV.

Algorithm based on divergence of nearby orbits in phase space

Let $\{z_i\}_{i=1}^{L}$ denote a filtered (by applying the global projections method [13] for noise reduction) time series with L samples. The phase points can be reconstructed by time delay embedding [4] – i.e., $\{\mathbf{z}_i\}_{i=1}^{L-(d-1)\tau}$:

$$\mathbf{z}_i = \left[z_i, z_{i+\tau}, z_{i+2\tau}, \cdots, z_{i+(d-1)\tau}\right]^T, \qquad (1)$$

where d – the embedding dimension and τ – time delay are chosen according to certain optimization criterion [3], [16] and $(\cdot)^T$ denotes the transpose of a real matrix. The near neighborhood of the reference point $\mathbf{z}_i$ is defined as

$$\mathbf{N}_i \stackrel{\Delta}{=} \left\{\mathbf{z}_j : \left\|\mathbf{z}_j - \mathbf{z}_i\right\| < \varepsilon, 1 \le j \le L-(d-1)\tau\right\}, \quad (2)$$

and arranged in ascending order of Euclidean distance between $\mathbf{z}_i$ and $\mathbf{z}_j$ as $\mathbf{N}_i = \{\mathbf{z}_{j_1}, \mathbf{z}_{j_2}, \cdots, \mathbf{z}_{j_N}\}$, where N is the number of neighbors and ε is the neighborhood radius. Then, only the k nearest neighbors for every $\mathbf{z}_i$ are picked.

For chaotic systems, the distance between two nearby vectors will increase exponentially over time due to the very nature of sensitivity to initial conditions. Therefore, the number of initially vector pairs N_k that satisfy the condition (2), is also expected to drop with the time. Since the distance between $\mathbf{z}_i$ and $\mathbf{z}_j$ is different, i. e. the distance of some pairs is near to 0, whereas the distance of other pairs is near to ε, for more exactly estimation of the divergence between a nearby trajectories (in order to evaluate the time, when distance between $\mathbf{z}_i$ and $\mathbf{z}_j$ exceeds the certain threshold) it is reasonable to introduce a shell (similar as [9]) $\varepsilon + \Delta\varepsilon$, where ε – the radius of the shell and $\Delta\varepsilon$ – the width of the shell. It is sufficient to introduce an additional condition,

$$|i - j| \ge (d-1)\tau, \qquad (3)$$

when finding pairs of vectors within shell. This means that, after taking a time comparable to the embedding window $(d-1)\tau$ it would be safe to assume that the initial separation has evolved to the most unstable direction of the motion [9]. Then the dynamic of amount of initial vector pairs, that satisfy the condition

$$\left\|\mathbf{z}_j - \mathbf{z}_i\right\| \le \varepsilon + \Delta\varepsilon, \qquad (4)$$

for every $\mathbf{z}_i$ is monitored with time and the averaged dependence $N_k(t)$ for all $\mathbf{z}_i$ (that have no less than k neighbors) versus time is calculated. The slope of this dependence indicates the average velocity of reduction of vector pairs that satisfy the condition (4), i. e approximately the average velocity of crossing the shell of width $\Delta\varepsilon$. The larger the $\Delta N_k / \Delta t$, the higher the level of chaos. Intuitively, this indicates that the slope is actually related to the largest Lyapunov exponent. So we can use $\Delta N_k / \Delta t$ as an indicator of chaos, which in analogous to [10] conditionally called *vector divergence rate* (VDR).

To evaluate the distinguishing capability of this approach let's consider a high-noisy x component of the Rossler system and a laser dataset, both contaminated with additive white Gaussian noise and enhanced by the nonlinear noise reduction method (global projections, [13]). The Rossler system is given by

$$\begin{cases} \dfrac{dx}{dt} = -(y+z), \\ \dfrac{dy}{dt} = x + a \cdot y, \\ \dfrac{dz}{dt} = b + z(x-c); \end{cases} \qquad (5)$$

with parameters a = 0,398, b = 2 and c = 4, [10].

Fig. 1 a) shows a plot of N_k versus time for x component of the Rossler system contaminated with additive white Gaussian noise of different levels and enhanced by the global projections method. Fig. 1 b) shows a plot of N_k versus time for regular sinusoidal signal also contaminated with additive white Gaussian noise of different levels and enhanced by the global projections method. The computations were done with 2000 points and $d = 4, \tau = 15$, $\varepsilon = 0{,}15; \Delta\varepsilon = 0{,}1$ (data are normalized in range from 0 to 1). Because the data set is relative small (about 30 periods of the signal), the first 4 nearest neighbors are used for each reference phase point. Otherwise there are not enough appropriate neighbors for the reference phase point. In each figure “Time” is used to denote the discrete evolution time step.

We observe that for the clean chaotic pseudoperiodic signal the $N_k(t)$ curves after a short transition are very similar (only the slope is negative) to the curves for largest Lyapunov exponents calculating [6] – there is a long near to linear region with approximately constant slope. For the contaminated by additive Gaussian noise with SNR up to 0 dB and filtered chaotic pseudoperiodic signals the curves of N_k versus time due to the distortion of the signal by nonlinear noise reduction fall faster and scaling region is not linear. But the common negative trend of the curves remains. For a filtered noisy periodic sinusoidal signal there are no such relations – the curves of N_k versus time generally remain flat (for SNR up to 5 dB) or oscillate around fixed value (for SNR up to 0 dB). That is, on average the nearest neighbors should neither diverge nor converge. This behavior allow us to distiguish noisy chaotic pseudoperiodic signal from noisy regular sinusoidal signal after noise reducing. Fig. 2 a) shows the curves of N_k versus time for another pseudoperiodic chaotic signal – laser dataset contaminated with additive white Gaussian noise and enhanced by the global projections method. The computations were done with 2000 points and $d = 5, \tau = 2$, $\varepsilon = 0{,}15;$ $\Delta\varepsilon = 0{,}1$ (data are normalized in range from 0 to 1). The results are similar to the Rossler system.

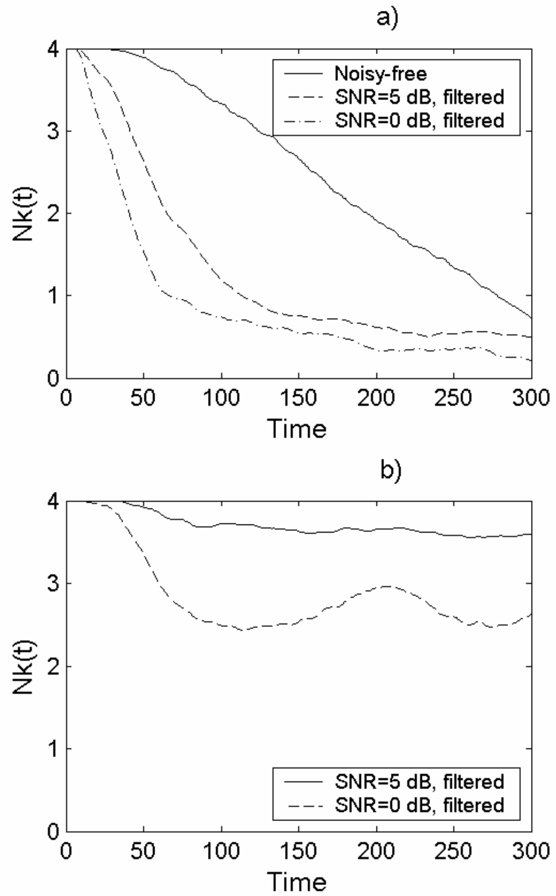

Fig. 1. Vector divergence rate for a) the noisy-free x component of the Rossler system and after reduction of additive Gaussian noise of different levels, b) the periodic sinusoidal signal after reduction of additive Gaussian noise of different levels

Fig. 2 b) shows the curves of N_k versus time for the two-torus quasiperiodic system. The corresponding time series, $z(i)$, was created by a superposition of two sinusoids with incommensurate frequencies [6]

$$z(i) = sin(2 \cdot \pi \cdot f_1 \cdot i \cdot \Delta t) + sin(2 \cdot \pi \cdot f_2 \cdot i \cdot \Delta t), \quad (6)$$

where $f_1 = 1{,}732051$; $f_2 = 2{,}236068$ and the sampling period was $\Delta t = 0{,}01$ s. The results are similar to the sinusoidal signal – the curves of N_k versus time generally remain flat (for noisy-free signal) or oscillate (for SNR up to 5 dB).

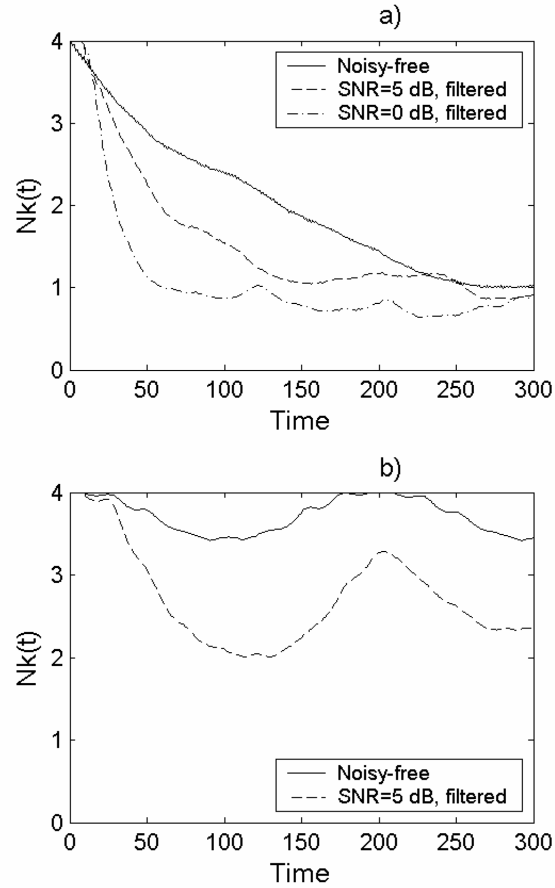

Fig. 2. Vector divergence rate for a) the noisy-free laser data set and after reduction of additive Gaussian noise of different levels, b) the noisy-free quasiperiodic signal and after reduction of additive Gaussian noise

Algorithm based on the recurrence properties in phase space

Recurrence plots (RPs) were originally introduced to visualize recurrences of trajectories of dynamical systems in phase space [17]. Suppose we have a dynamical system represented by the trajectory $\{\mathbf{z}_i\}$ for $i = 1, \cdots, M$ in a d-dimensional phase space. We then compute the binary matrix

$$\mathbf{R}_{ij} = \mathbf{\Theta}(\varepsilon - \|\mathbf{z}_i - \mathbf{z}_j\|), \quad i, j = 1, \cdots, M, \quad (7)$$

where ε is a predefined threshold, $\mathbf{\Theta}(\cdot)$ is the Heaviside function, and $\|\cdot\|$ is a norm defining the distance between two points. The graphical representation of $\mathbf{R}_{ij}$, called the “recurrence plot,” is obtained by encoding the value “one” by a black point, (i.e., the distance between the respective points is smaller than the predefined threshold ε) and “zero” by a white point (i.e., the distance between the respective points is larger than ε).

The recurrence time – i.e., the time that the trajectory needs to recur to the neighborhood of a previously visited state – corresponds to a white vertical line in an RP (the distance between diagonal lines). For a periodic motion of period *T*, the states recur at fixed time intervals and, hence, the corresponding RP consists of uninterrupted diagonal lines separated by the distance *T*. The RP of a chaotic system shows more intricate structures with many interrupted lines. The distance between diagonal lines is then not constant due to the multiple time scales present in the system and the interruption of the lines is due to the exponential divergence of nearby trajectories (more details [17], [18]). The corresponding histograms of white vertical lines [17] for the noisy-free x component of the Rossler system and for noisy signal with SNR=0 after nonlinear noise reduction are plotted in Figs. 3 a) and 3 b), respectively.

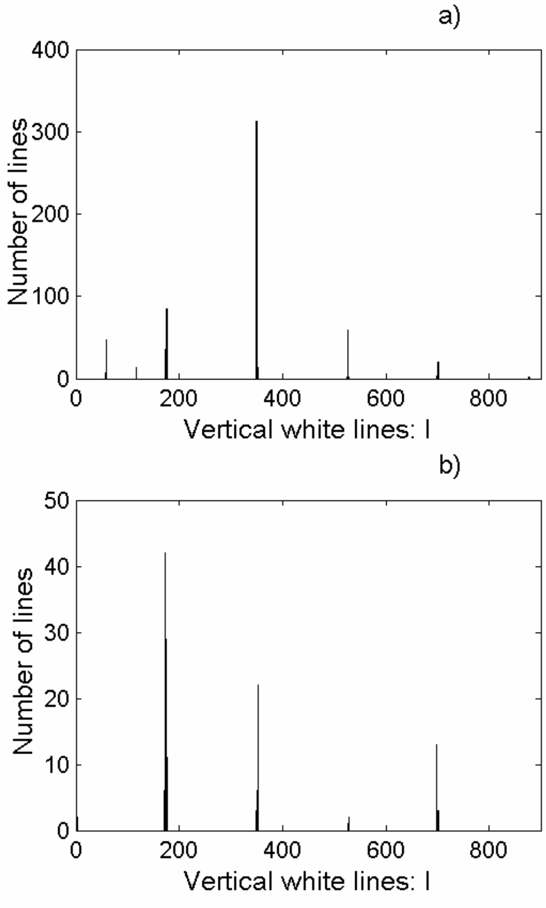

Fig. 3. The RPs histograms of the white vertical lines for the x component of the Rossler system (series length 2000): a) noisy-free, b) after additive Gaussian noise of SNR=0 dB reduction

The computations were done with 2000 points and $d = 4, \tau = 15$, and $\varepsilon = 0{,}4$. The histograms of white vertical lines for noisy sinusoidal signal with SNR=5 and SNR=0 after noise reduction are plotted in Fig 4. The computations were done with 2000 points and $d = 4, \tau = 15$, and $\varepsilon = 0{,}1$.

We observe that the histograms of white vertical lines in an RP for clean and enhanced time series measured from the *x* component of the Rössler system has similar behavior – the pseudoperiodic dynamics has several return times for a recurrence interval. These return times are multiple to the fundamental period of the Rössler system (about 60 discrete time points for this example).

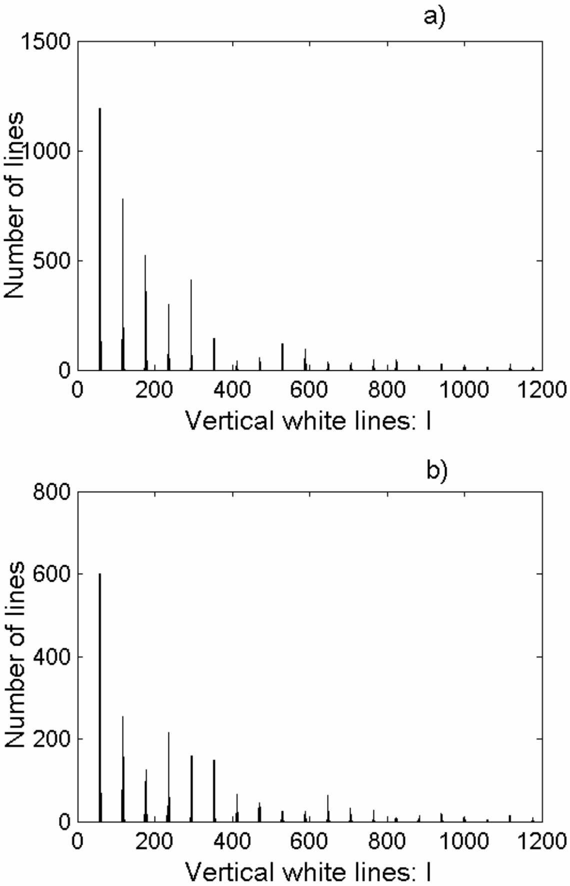

Fig. 4. The RPs histograms of the white vertical lines for the regular sinusoidal signal: a) after additive Gaussian noise of SNR=5 dB reduction, b) after additive Gaussian noise of SNR=0 dB reduction

Differently, the histogram of vertical white lines in an RP for enhanced regular sinusoidal signal has single dominant return time, which is equal to the period of the sinusoidal signal, and many return times multiple to the period and appeared less frequently. Those non basic return times are conditioned by distortion of the filtered sinusoidal signal and due to interruption of the lines in RP. Therefore, the histogram of white vertical lines in an RP successfully captures the recurrence properties and allow us to distinguish filtered chaotic signal from filtered regular one.

Discussion and conclusion

In this work, the distinguishing between high-noisy chaotic pseudoperiodic time series and high-noisy periodic or quasiperiodic time series enhanced by the global phase space projections method is investigated. The algorithm for detecting chaos is described, more robust for distortion of nonlinear noise reduction than widely used Lyapunov exponent. Similar to the Lyapunov exponent, the algorithm is based on the divergence of the nearest neighbors, but the averaged dynamic of amount of initial vector pairs, that satisfy the condition of the nearest neighbors, is calculated instead of the dynamic of distance between the vector pairs. The neighborhood radius ε must be chosen small with respect to the diameter of the reconstructed attractor [6] and the number k of the first nearest neighbors pairs of (i,j) should be possible large at defined ε, but in other hand enough appropriate neighbors for every reference phase point is required. Therefore, the number k depends also on the length of analyzed time series. To illustrate the robustness of this approach to the distortion of the filtered signal, algorithm was applied to the Rössler time series and experimental laser data, contaminated by additive white Gaussian noise and enhanced by nonlinear noise reduction method. We observe that the noisy-free chaotic motion is characterized by a nearly to linear $\langle N_k(t)\rangle \sim t$ curve. For enhanced chaotic pseudoperiodic time series due to the remaining distortion the curve $\langle N_k(t)\rangle \sim t$ drops nearly to exponentially and this process is irreversible. While for enhanced periodic and quasiperiodic signals with noise, there are no such relations – the curve shows a plateau or vary slowly around a certain value with small trend. This clear difference provides a direct method of distinguishing low-dimensional chaotic signal from a periodic signal with noise. By combining the proposed algorithm with primary nonlinear noise reduction methods we can distinguish between regular and chaotic signals contaminated by additive white Gaussian noise with SNR up to 0 dB.

Secondly, the numerical investigation of the recurrence properties of orbits from filtered x component of the Rossler system and filtered regular sinusoidal signal by using a two-dimensional visualization technique – the recurrence plot (RP) is also presented. It was founded that the histograms of white vertical lines in an RP successfully capture the recurrence properties of enhanced signals – the patterns in the RPs of enhanced pseudoperiodic chaotic and enhanced regular orbits remain qualitatively different similarly to the noisy-free signals. The pseudoperiodic dynamics has several return times for a recurrence interval and enhanced regular sinusoidal signal has single dominant return time, which is equal to the period of the sinusoidal signal. Based on the histogram of white vertical lines for data enhanced by nonlinear noise reduction methods one can conclude that the motion is regular or chaotic pseudoperiodic at relatively high level of additive white Gaussian noise – for SNR up to 0 dB.

References

1. **Small M., Tse C. K.** Detecting determinism in time series: the method of surrogate data // IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. – 2003. – Vol. 50, Iss. 5. P. 663 – 672.
2. **Chandre C., Wiggins S., Uzer T.** Time–frequency analysis of chaotic systems // Physica D. – 2003. – Vol. 181, Iss. 3.-4 – P. 171 – 196.
3. **Kantz H. Schreiber T.** Nonlinear Time Series Analysis // Cambridge University Press, Cambridge. – 2003. – P. 30 – 39.
4. **Wolf A. Swift J. B., Swinney H. L. Vastano J. A.** Determining Lyapunov exponents from a time series // Physica D. – 1985. – Vol. 16, Iss. 3. – P. 285 – 317.
5. **Kantz H.** A robust method to estimate the maximal Lyapunov exponent of a time series, // Phys. Lett. A – 1994. – Vol. 185, Iss. 1. – P. 77 – 87
6. **Rosenstein M. T., Collins J. J., De Luca C. J.** A practical method for calculating largest Lyapunov exponents from small data sets // Physica D. – 1993. – Vol. 65, Iss. 1-2. – P. 117 – 134.
7. **Farmer J. D., Ott E., Yorke J. A.** The dimension of chaotic attractors // Physica D. – 1983. Vol. 7, Iss. 1-3 – P. 153-180.
8. **Urbanowicz K., Kantz H.** Improvement of speech recognition by nonlinear noise reduction // Chaos. – 2007. – Vol.17, Iss.2. – P.023121-1 – 023121-6.
9. **Gao J. B., Hu J., Tung W. W., Cao Y. H.** Distinguishing chaos from noise by scale-dependent Lyapunov exponent // Phys. Rev. E. – 2006. – Vol. 74, No. 6. – P. 066204-1 – 066204 -9.
10. **Zhang J., Luo X., Small M.** Detecting chaos in pseudoperiodic time series without embedding // Phys. Rev. E. – 2006. – Vol. 73, No. 1. – P. 016216-1 – 016216-5.
11. **Grassberger P., Hegger R., Kanz H., Schaffrath C., Schreiber T.** On noise reduction methods for chaotic data // Chaos. – 1993. – Vol. 3, Iss.2. – P. 127 – 141.
12. **Hegger R., Kantz H., Matassini L.** Noise Reduction for Human Speech Signals by Local Projections in Embedding Spaces // IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. – 2001 – Vol. 48, No. 12. – P. 1454-1461.
13. **Johnson M. T., Povinelli R. J.** Generalized phase space projection for nonlinear noise reduction // Physica D. – 2005. – Vol. 201, Iss. 3-4. – P. 306-317.
14. **Sun J., Zhao Y., Zhang J., Luo X., Small M.** Reducing colored noise for chaotic time series in the local phase space // Phys. Rev. E. – 2007. – Vol. 76, No. 2. – P. 026211-1 – 026211-6.
15. **Pukenas K., Muckus K.** Algorithm for detecting deterministic chaos in pseudoperiodic time series. // Electronics and Electrical Engineering. – 2007. – No. 8 (80). – P. 53 – 56.
16. **Gao J, Zheng Z.** Direct dynamical test for deterministic chaos and optimal embedding of a chaotic time series// Phys. Rev. E. – 1994. – Vol. 49, No. 5. – P. 3807 – 3814.
17. **Zou Y., Pazó D., Romano M. C., Thiel M., Kurths J.** Distinguishing quasiperiodic dynamics from chaos in short-time series // Phys. Rev. E. – 2007. – Vol. 76, No. 1. – P. 016210-1 – 016210 -12.
18. **Zou Y., Thiel M., Romano M. C., Kurths J.** Characterization of stickiness by means of recurrence // Chaos. – 2007. – Vol.17, Iss.4. – P.043101-1 – 043101-8.

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The distinguishing between high-noisy chaotic pseudoperiodic time series and high-noisy periodic or quasiperiodic time series primarily enhanced by the nonlinear noise reduction methods is investigated. The different algorithm is described to detect deterministic structure from a pseudoperiodic time series enhanced by the singular value decomposition method. The algorithm is more robust for distortion of nonlinear noise reduction than widely used Lyapunov exponent. Similar to the Lyapunov exponent, the algorithm is based on the divergence of the nearest neighbors, but the averaged dynamic of amount of initial vector pairs, that satisfy the condition of the nearest neighbors, is calculated instead of the dynamic of distance between the vector pairs. By combining with nonlinear noise reduction methods the proposed algorithm can distinguish reliable between regular and chaotic pseudoperiodic time series, contaminated by additive white Gaussian noise with SNR up to 0 dB. Also, the difference of the recurrence properties between enhanced noisy pseudoperiodic chaotic and enhanced regular signals is analyzed. It is concluded, that the histograms of white vertical lines of the recurrency plots (RP) allow to distinguish chaotic signal enhanced by nonlinear noise reduction method from enhanced regular sinusoidal signal at a signal-additive white Gaussian noise ratio up to 0 dB. Ill 4, bibl. 18 (in English; summaries in English, Russian and Lithuanian).

К. Пукенас. Ообнаружение детерминистических структур в псевдопериодических временных рядах с высоким уровнем шумов // Электроника и электротехника. – Каунас: Технология, 2008. - № 4(84). – С. 75-80.

Исследуется возможность обнаружения детерминистического начала в псевдопериодических временных рядах с высоким уровнем шумов при использовании в качестве первичной фильтрации нелинейных методов. Описывается алгоритм для обнаружения хаоса в псевдопериодических временных рядах после применения первичной фильтрации методом декомпозиции сингулярного значения в глобальном фазовом пространстве, более устойчив к искажениям, обусловленным первичной нелинейной фильтрацией, чем широко применяемая экспонента Ляпунова. Как и экспонента Ляпунова алгоритм также основан на дивергенции векторов реконструированного фазового пространств, но вместо расчета динамики расстояния между векторами расчитывается усредненная зависимость количества ближайших векторов реконструированного фазового пространств от времени, являющаяся индикатором детерминистического хаоса. Показывается, что алгоритм позволяет детектировать хаос при использовании первичной нелинейной фильтрации при отношении сигнал-белый Гауссовый шум выше 0 дБ. Также показывается, что при использовании первичной нелинейной фильтрации гистограммы времени возврата, построенные на основании диаграмм повторения, позволяют отличать хаотическую природу псевдопериодических временных рядов от зашумленного синусоидального сигнала при отношении сигнал- белый Гауссовый шум выше 0 дБ. Ил. 4, библ. 18 (на английском языке; рефераты на английском, русском и литовском яз.).

K. Pukėnas. Deterministinių struktūrų detekcija didelio triukšmingumo pseudoperiodinėse laiko eilutėse // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 4(84). – P. 75-80.

Tiriama deterministinio chaoso detekcija pseudoperiodinėse laiko eilutėse esant aukšto lygio baltojo Gauso triukšmo pirminei filtracijai netiesiniais metodais. Aprašomas chaoso detekcijai singuliarinių reikšmių dekompozicijos globalinėje fazinėje erdvėje pagrindu filtruotuose signaluose algoritmas, atsparesnis pirminės netiesinės filtracijos sukeltiems iškraipymams, negu plačiai taikoma Liapunovo eksponentė. Kaip ir Liapunovo eksponentė, algoritmas remiasi artimiausių rekonstruotos fazinės erdvės vektorių trajektorijų divergencija, bet vietoj distancijos tarp vektorių dinamikos, yra skaičiuojama suvidurkinta artimiausių rekonstruotos fazinės erdvės vektorių kiekio priklausomybės nuo laiko kreivė, kurios nuožulnumas yra deterministinio chaoso indikatorius. Algoritmas įgalina detektuoti chaosą netiesiniais metodais filtruotose pseudoperiodinėse laiko eilutėse, kai pirminis signalo ir baltojo triukšmo santykis didesnis kaip 0 dB. Straipsnyje taip pat tiriami filtruotų pseudoperiodinių chaotinių laiko eilučių ir filtruotų reguliariųjų laiko eilučių pasikartojimo diagramų grįžimo laiko histogramų skirtumai. Parodoma, kad pasikartojimo diagramų grįžimo laiko histogramos įgalina patikimai atskirti filtruotas pseudoperiodines laiko eilutes nuo filtruoto sinusinio signalo, kai pirminis signalo ir baltojo triukšmo santykis didesnis kaip 0 dB. Il. 4, bibl. 18 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).