

Structure and Features of Sigma-Delta Modulators

V. Puidokas, A. J. Marcinkevicius

Vilnius Gediminas Technical University, Computer Engineering Department, Naugarduko str. 41, LT-03227 Vilnius, Lithuania. Ph. +370 67590792, e-mail: Vytenis.Puidokas@el.vgtu.lt, Albinas.Marcinkevicius@el.vgtu.lt

Introduction

Sigma-Delta ($\Sigma\Delta$) modulators ($\Sigma\Delta$ M) are widely used in analogue-to-digital and digital-to-analogue converters (ADCs and DACs). Their main advantage lies in high linearity of a binary code generation mechanism, which is necessary for a good quality 24 or greater bit resolution audio equipment.

$\Sigma\Delta$ DACs are ascribable to oversampling DACs as they help to avoid problems that tend to arise in case of usual analogue-to-digital (A/D) and digital-to-analogue (D/A) Nyquist-rate converters [1, 2]. Nyquist-rate converters are characterized by a high speed (10-1500 MHz), however, with a rather small resolution (up to 14-16 bits). In order to realize them, precise analogue components in filters and conversion circuits are needed. Moreover, the latter circuits are sensitive to noise and interference. Sampling frequency of Nyquist's converters must be twice as high as the highest frequency of a signal. The main technical characteristics of DACs are presented in Table 1.

Table 1. Main technical characteristics of DACs

Parameters	DAC types			
	R-String	R-2R	I-Steering	$\Sigma\Delta$
Frequency, MHz	0,1 – 5	0,1 – 15	10 – 1500	0,01 – 2
Resolution, bits	8 – 16	8 – 16	6 – 14; 16	8 – 24; 32
Settling time, μ s	0,2 – 10	0,06 – 10	0,0006 – 0,1	10^2 – 0,5
Integral non linearity, %	0.01 – 0.2	0.001 – 0.2	0.01 – 0.2	10^{-5} – 0.015
Power cons., mW	0,5 – 120	1 – 500	50 – 2000	0.05 – 100

$\Sigma\Delta$ DACs are characterized by a high level of precision (24-32 bits), but a low speed (0.2-2 MHz). Such converters are most often applied in high-resolution digital signal conversion circuits of audio systems used for various purposes. Although $\Sigma\Delta$ M are applied in high quality digital audio systems, scientific literature does not provide much discussion on methods of analysis and synthesis of their features and patterning algorithms.

The present paper analyzes the structure of Sigma-Delta modulators, presents variants of DAC circuits, approaches the analysis of quantization noise characteristics as well as discusses topics of application in power D/A converters.

Structure and Quantization Noise of Sigma-Delta DAC

A 1-bit Sigma-Delta DAC structure (fig. 1) consists of an interpolation filter, a $\Sigma\Delta$ modulator which, regarding a signal, works as a low frequency filter, whereas regarding quantization noise – as a high frequency filter, and a 1-bit D/A converter whose output is being switched over between positive and negative reference voltage of equal magnitude. The signal received is being filtered by way of an output analog low frequency filter.

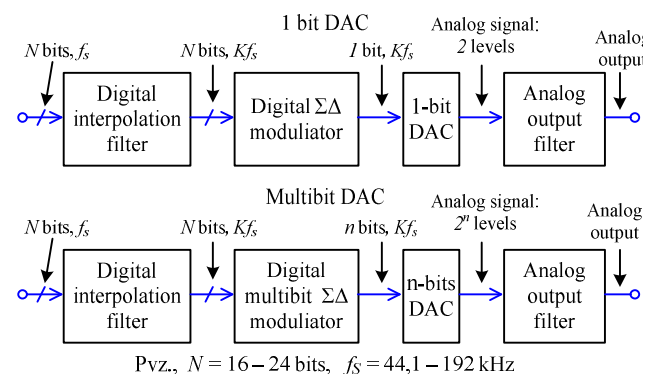


Fig. 1. Sigma-Delta D/A converter: one-bit and multibit

A multibit Sigma-Delta converter employs a multibit Sigma-Delta modulator and an n-bit D/A converter. Converters of both types share a similar principle of operation. Usually in $\Sigma\Delta$ DAC modulators $n < N$, in ADC modulators – it is vice versa.

One-bit $\Sigma\Delta$ modulators are widely used in analogue-to-digital and digital-to-analogue converters (ADCs and DACs). Their main advantage is a great number of bits and linearity of conversion characteristics which is suitable for audio systems with 24 or greater bit resolution under 192 kHz sampling frequency, as well as for high precision measurement engineering.

The generalized scheme of a Sigma-Delta D/A converter's modulator is presented in figure 2 [3]. It consists of a digital input signal register, an integrator, a quantizer and a feedback loop.

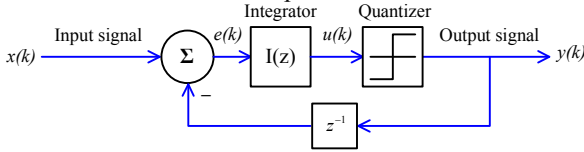


Fig. 2. General structure of a discrete time $\Sigma\Delta$ modulator circuit

The mathematical description of the generalized modulator scheme is complicated. Having changed a quantizer (fig. 2) with a white noise source, as it is shown in figure 3, frequency transient characteristic can be described by way of discrete time theory equations. Figure 3 shows a 1st order Sigma-Delta modulator's linear quantizer model with the quantization noise source Q .

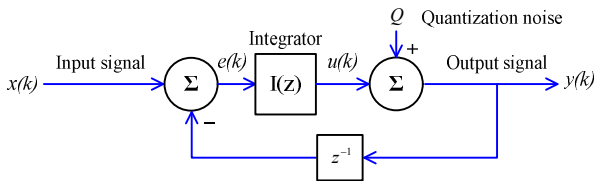


Fig. 3. Linear quantizer model

Applying a method of discrete time systems analysis, an output signal equation can be written down as follows:

$$Y(z) = Q(z) + I(z)[X(z) - z^{-1}Y(z)]. \quad (1)$$

Having obtained $Y(z)$ from the latter equation, we arrive at:

$$Y(z) = X(z) \frac{I(z)}{1 + I(z)z^{-1}} + Q(z) \frac{1}{1 + I(z)z^{-1}}. \quad (2)$$

As the ideal integrator is expressed in the following way:

$$I(z) = \frac{1}{1 - z^{-1}}, \quad (3)$$

thus a 1st order $\Sigma\Delta$ modulator's output is described:

$$Y(z) = X(z) + (1 - z^{-1})Q(z). \quad (4)$$

Now fig. 3 can be supplied as it is shown in fig. 4.

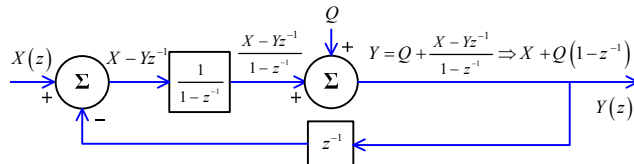


Fig. 4. A supplemented linear quantizer model

Taken that the quantizer's noise is random, a differentiator $(1 - z^{-1})$ in equation (4) doubles the quantization noise, but shapes it into high frequencies.

In case of using converters of the Nyquist-rate (either ADCs or DACs), a relatively high level of noise is being obtained (fig. 5). If used oversampling converters (interpolation, Delta, Sigma-Delta, etc.), lower level of noise is being obtained (fig. 6) as the quantization noise energy spreads over larger area of frequencies, thus the total level falls down. In Sigma-Delta converters quantization noise is active shaped into the area of higher frequencies (fig. 7). Upon the application of a low frequency filter, the required signal is being produced with a minimal amount of noise. Thus, it comes as no surprise that namely Sigma-Delta converters are being applied in cases when high converter resolution is needed.

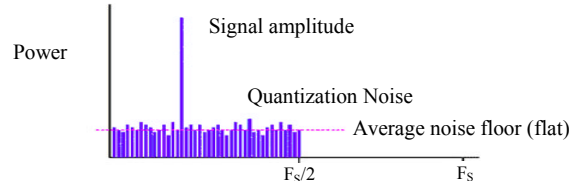


Fig. 5. Level of quantization noise in Nyquist-rate converters

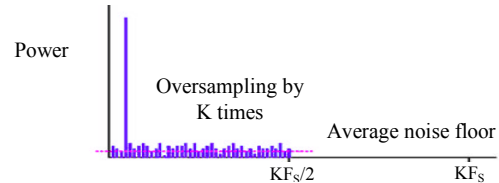


Fig. 6. Level of quantization noise after oversampling by $K > 1$

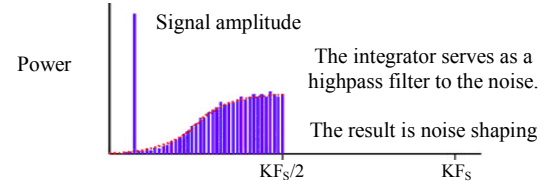
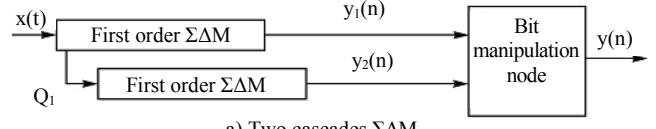


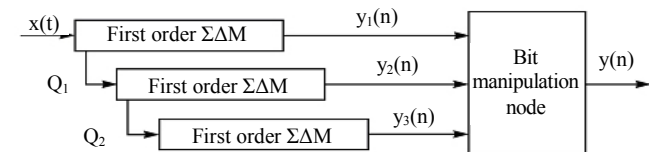
Fig. 7. Level of quantization noise after noise shaping

Clearer Noise Shaping

Cascaded modulators are most easily subject to approximate analytical analysis. Though they should not be confused with higher order one-cascade modulators, the majority of authors do not conform to the strict distinction in methodological literature. In the ideal case a noise spectrum of the 3rd order one-cascade and the 1st order three-cascaded modulators is the same. One of the ways to achieve clearer noise shaping is to join in cascades several 1st order Sigma-Delta modulators (fig. 8).



a) Two cascades $\Sigma\Delta$



b) Three cascades $\Sigma\Delta$

Fig. 8. Two- and three-cascades Sigma-Delta modulators

From the latter modulator into the succeeding the difference between outputs of an integrator and a quantizer is transferred. If signals of the two- and three-cascaded inputs are Q_1 and Q_2 respectively, then the output of the 2nd cascade quantizer is:

$$Y_2(z) = Q_1(z) + (1 - z^{-1})Q_2(z). \quad (5)$$

Then the output of a two-cascaded $\Sigma\Delta$ modulator is:

$$Y(z) = X(z) + (1 - z^{-1})^2 Q_2(z). \quad (6)$$

Analogically for a three-cascaded $\Sigma\Delta$ modulator:

$$Y_3(z) = Q_2(z) + (1 - z^{-1})Q_3(z), \quad (7)$$

$$Y(z) = X(z) + (1 - z^{-1})^3 Q_3(z). \quad (8)$$

Noise shaping is far more manifested in two- and three-cascaded modulators (fig. 9). The more steps (or the higher the order of a modulator), the higher the order of a high frequency filter which affects quantization noise.

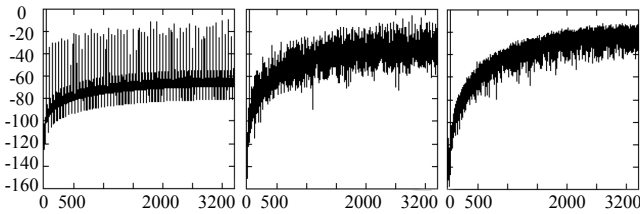


Fig. 9. A spectrum of a one-, two- and three-cascaded $\Sigma\Delta$

There is no definite answer on whether one-cascaded high order modulators are better than multi-cascaded 1st order modulators. It is almost the same as asking „what is better: many one-cascaded amplifiers or one multi-cascaded?“. It is advisable to analyze each case separately. However, the best solution usually is an intermediary variant.

Application of Sigma-Delta Modulators in Power DACs

Application of Sigma-Delta modulators in powerful DACs, i.e. a class D digital power audio amplifier, has its own nuances. Usually they are not manifested in ordinary weak converters. The majority of means that suit the latter [4-5] absolutely do not fit in this case.

Low (1-3) order one-bit DACs. Advantages:

1. High level of linearity – low nonlinear distortions;
2. Stability – the lower the order, the greater stability;
3. Good output key realization.

Drawback: low signal-to-noise ratio (SNR).

Variant 1. Oversampling ratio (OSR) can be increased in order to increase SNR. Then the modulator will have to work in higher frequency, which will lead to the increased number of output key switches per time (see fig. 1 – a 1-bit DAC). Despite other drawbacks, this will have a great negative effect on the efficiency coefficient. It worth noting that high coefficient of efficiency constitutes the main advantage of class D amplifiers.

Variant 2. SNR can be increased by way of heightening modulator's order. Fig. 10 [6] shows the effect of this upon the peak signal-to-noise ratio (PSNR). However, the increase of modulator's order dramatically decreases its stability. Thus, this makes it extremely difficult to design such modulators. It often occurs that modeling in the high level (with MATLAB, SIMULINK) produces incorrect results. There has been designed a 7th order modulator which according to modeling results (according to [7-8], input signal value not exceeding the limit value) must work steadily. However, after realization in the Field Programmable Gate Array, the modulator tended to constantly actuate. If a modulator actuates from time to time, practice of resetting an actuated modulator and making it continue its work is widely applied.

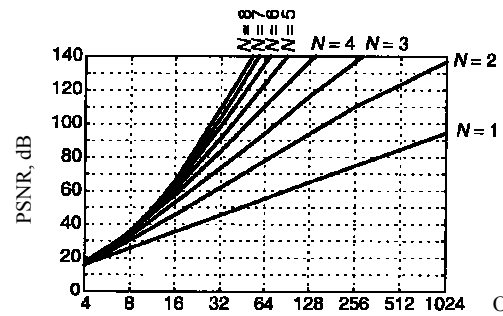


Fig. 10. Empirical limit for PSNR of a one-bit N-order modulator

Variant 3. Having changed a one-bit quantizer with a multibit one, we can substantially increase signal-noise ratio without bringing any changes upon OSR. However, here we observe additional nonlinearity which causes nonlinear distortions. Without introducing various corrections, they become very evident. All in all, the foremost drawback lies in extreme difficulty to realize a qualitative output key in power converters. Small converters avoid this kind of problem, thus such a variant is widely used in their realization.

Variant 4. As it has been mentioned, in the ideal case, a noise form of a three-cascaded 1st order modulator is analogous to the one of a one-cascaded 3rd order modulator. Application of a multi-cascaded structure can help to avoid instability which is characteristics of high order modulators. In this way without much effort is a very high SNR, including very good linearity. However, designing is much more difficult, and transfer functions must be accurately matched, whereas when applying modulators of higher order (>1), a multibit quantizer is needed.

Variant 5. When a Sigma-Delta modulator is applied along with modulations of other kind, e.i. Pulse Width Modulation [9]. At present this is the most widely applied variant in power converters (nearly all of industrial). However, this is beyond the limits of the current article.

In recent years enrichments of a modulator still have been taking place, resulting in its features being evidently improved [10-11] or itself being applied for some specific purposes. The aim of the authors of the present article is to modify a modulator to such an extent as to make a class D power amplifier competitive with widely used class AB amplifiers in terms of its features.

Conclusions

Despite the fact that industry provides other kinds of converters, converters based on the Sigma-Delta modulation principle have been deeply entrenched in the field of audio equipment. Their advantages: high linearity level of the binary code generation mechanism and high resolution. A $\Sigma\Delta$ modulator's effect upon a signal is that as of a low frequency filter, while upon quantization noise – as of a high frequency filter. Still 1st order noise shaping is lightly manifested and it is usually not satisfactory – the SNR is too low. It can be increased by several means: by way of increasing oversampling ratio, heightening the order, increasing the number of cascades as well as by using multibit quantizers. Each of the mentioned ways brings about certain drawbacks, thus in order to choose the best one each separate case shall be taken into account. There are many problems of the using $\Sigma\Delta$ modulators as output stage in powerful DAC (D class). Thus many are preferring to use PWM, but not digital $\Sigma\Delta$ modulators.

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V. Puidokas, A. J. Marcinkevicius. Structure and features of Sigma-Delta Modulators // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 4(84). – P. 39–42.

The article highlights the main advantages of Sigma-Delta ($\Sigma\Delta$) converters comparing to Nyquist-rate converters. For the sake of comparison there are presented the main technical characteristics of industrial digital-to-analogue converters (DACs), the description of an $\Sigma\Delta$ DAC structure in case of a 1-bit and multibit quantizer, the analytical description of operation of a 1st order modulator as well as presentation of noise shaping. Also, the analysis of clearer noise shaping was given. Some discussion on application of modulators in power DACs accompanied with its won nuances was presented. There were given several variants on how to increase signal-to-noise ratio (through increasing of oversampling ratio, heightening the order, increasing the number of steps and using multibit quantizers), as well as some problems which arise and aren't solved in powerful $\Sigma\Delta$ DACs. Ill. 10, bibl. 11 (in English; summaries in English, Russian and Lithuanian).

V. Пуйдокас, А. Й. Марцинкявичюс. Структура и характеристики Сигма-Делта модуляторов // Электроника и электротехника. – Каунас: Технология, 2008. – № 4(84). – С. 39–42.

В статье проанализированы основные преимущества Сигма-Делта преобразователей сигналов по сравнению с преобразователями Найквиста. Рассмотрены параметры и технические характеристики новейших типов цифроаналоговых преобразователей выпускаемых промышленностью. Описаны новые Сигма-Делта структуры, которые созданы на основе однобитового и мультибитового квантатора. Предложено аналитическое описание работы модулятора первого порядка с учётом формирования широкополосных помех. Рассмотрены вопросы применения модуляторов в мощных ЦАП. Предложены способы улучшения отношения сигнал/шум методами передискретизации, увеличением числа ступеней и применением мультибитовых квантаторов. Сформулированы технические проблемы, возникающие при разработке мощных ЦАП для декодирования звука в цифровых аудиосистемах, а также с какими не решёнными проблемами встречаемся при разработке $\Sigma\Delta$ ЦАПю. Ил. 10, библи. 11 (на английском языке; рефераты на английском, русском и литовском яз.).

V. Puidokas, A. J. Marcinkevičius. Sigma-delta moduliatorių sandara ir charakteristikos // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 4(84). – P. 39–42.

Straipsnyje nurodyti pagrindiniai Sigma-Delta ($\Sigma\Delta$) keitiklių pranašumai, palyginti su Naikvisto kriterijaus keitikliais. Palyginimui pateiktos pagrindinės, pramonėje gaminamų skaitmeniniu-analoginiu keitiklių (SAK), techninės charakteristikos. Aprašyta $\Sigma\Delta$ SAK sandara, esant vieno bito bei multibitiniam kvantatoriui; analitiškai aprašytas 1-os eilės moduliatoriaus veikimas bei pavaizduotas triukšmo formavimas. Aptartas moduliatorių taikymas galingose SAK, kas turi savų niuansų. Pateikta keletas variantų kaip didinamas signalo-triukšmo santykis (didinant perdiskretizavimo santykį, aukštinant eilę, didinant pakopų skaičių bei naudojant multibitinius kvantatorius) ir su kokiais neišspręstomis problemomis tenka susidurti naudojant galinguose $\Sigma\Delta$ SAK. Il. 10, bibl. 11 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).