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Specifics of Constant Envelope Digital Signals

I. Bilinskis, K. Sudars
Institute of electronics and computer science
14 Dzerbenes street, Riga LV-1006, Latvia, phone: +371 67555337, e-mail: bilinskis@edi.lv, sudars@edi.lv

Introduction

The digital signals discussed in this paper clearly are highly specific and, in this sense, unusual. First of all, their definition differs from the classical and traditionally used one. Timed sampling event sequences or, in other words, sampling point processes in time are considered as digital representations of the respective analog input signals rather than sequences of signal sample values as usual. Next, the sampling time instants under these conditions are signal-dependent and time intervals between them are nonuniform. On the other hand, these sampling intervals not necessarily are random. As they are related to the input signals, there might be periodicities present in the digital signals representing the original analog signals. Therefore overlapping of frequencies or aliasing might be expected. There may be also cross-interference between signal components typical for nonuniformly sampled signals. Actually the specific properties of the digital signals of this kind are not pre-determined, they strongly depend on the input signal and the used reference function.

Hence the considered digital signals indeed are highly specific and they have features unparalleled by other digital signals. It still has to be learned how to effectively process them under varying conditions. However there is a factor that draws attention and stimulates interest to this sampling approach. The point is that the digital signals obtained in the mentioned way have an outstanding positive feature. The envelope of the digital signal instantaneous value sequences, in the case of this kind of analog-to-digital conversions, remains constant no matter what is the spectral content of the respective analog signals. This fact represents a powerful advantage of the considered type of signal sampling as the constant envelope of various digital signals obtained under the mentioned conditions leads to various options in processing them. Some of these options, including complexity-reduced spectrum analysis and massive data acquisition, are briefly discussed in [1], [2]. Specific properties of the constant envelope digital signals, obtained in the case of this approach to sampling, are studied and described here. This paper actually is a follow-up to the paper [3] where the considered sampling scheme is discussed in some detail. The features of the digital signals obtained then are essentially specific. The feature of special interest is overlapping of frequencies or aliasing observed in the cases where sampling is performed in the mentioned way and the digital signal itself is a sampling point process or, in other words, a sequence of timed sampling events. Consideration of aliasing issues characterizing this type of digital signals follows.

Aliasing

To observe how frequencies are overlapping in the considered signal digitising case and how aliasing does impact processing of the obtained digital, DFT of a signal, containing only three components at frequencies f_1, f_2 and f_3, was performed. The sampling-specific complexity-reduced algorithm and the following equations, discussed in [1], [2], were used for calculations of the Fourier coefficient estimates at the frequency f_i. These estimates were calculated as follows:

$$\begin{cases} \hat{a}_i = \dfrac{A_r}{N}\sum_{k=0}^{N-1}\left(\sin 2\pi(f_r - f_i)t_k + \sin 2\pi(f_r + f_i)t_k\right) \\ \hat{b}_i = \dfrac{A_r}{N}\sum_{k=0}^{N-1}\left(\cos 2\pi(f_r - f_i)t_k - \cos 2\pi(f_r + f_i)t_k\right) \end{cases} \quad (1)$$

Evidently no multiplications of the signal and filtering function digital values have to be carried out in this case. This is a significant advantage of the discussed sampling approach as DFT based on calculations carried out according to these equations is exclusively applicable only for spectrum analysis of the digital signals obtained in result of sampling based on the reference sine-wave crossings.

The spectrogram obtained in this way is given in Figure 1. As can be seen, there are many other peaks in this spectrogram in addition to the peaks that might be considered as spurious noise. Actually the most pronounced additional peaks appear in the spectrogram in connection with the frequency overlapping or aliasing effect. In this specific sampling case, positions of the peaks related to aliasing depend on the frequencies related to the periodicity in the sampling point stream. This means that both the reference frequency and the mean sampling frequency play some role in that and are to be considered in this light.

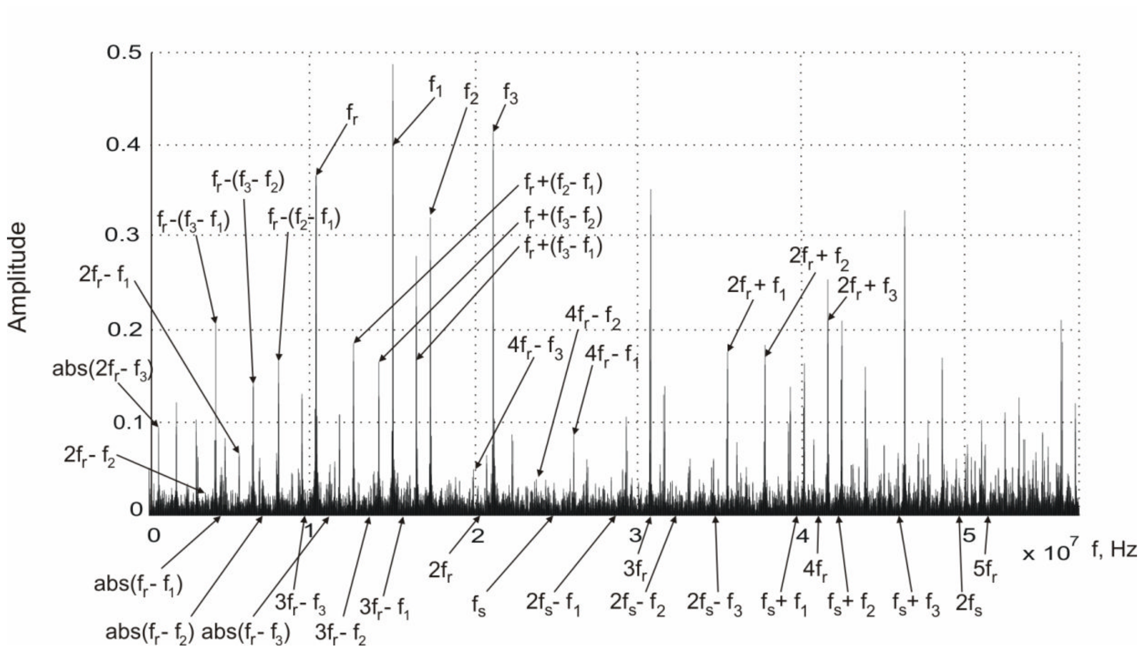

Fig. 1. Illustration of the aliasing frequency pattern.

Consider the spectrogram given in Figure 1. The frequencies f_i and f_n of the signal components, along with other typical frequencies such as the reference frequency f_r and the mean sampling rate f_s, are indicated in the spectrogram. Note that the frequencies of the reference function together with the mean sampling frequency define the positions of the aliasing frequencies displayed in it. The pattern of the peak positions on the frequency axis helps to reveal the essence of the mentioned relationships defining the aliasing conditions. Actually there are three rows of the expected aliasing frequencies. They are:

$$\begin{cases} f_i;\ f_r \pm f_i;\ 2f_r \pm f_i;\ 3f_r \pm f_i;\ \ldots; \quad i=1, 2, 3, \ldots \\ f_s \pm f_i;\ 2f_s \pm f_i;\ 3f_s \pm f_i;\ \ldots; \quad i=1, 2, 3, \ldots \\ f_r \pm (f_i \pm f_n);\ 2f_r \pm (f_i \pm f_n);\ 3f_r \pm (f_i \pm f_n);\ \ldots; \text{ for } i \neq n \text{ and } i=1, 2, 3, \ldots;\ n=1, 2, 3 \ldots \end{cases} \quad (2)$$

where f_i and f_n are frequencies of a signal components, f_r is the reference frequency and f_s is the mean sampling rate.

Peaks due to aliasing might be found at any frequency given in (2). Their magnitude depends on the specific conditions under which the crossings of the signal and the reference function occur. In general, the relationships defining the aliasing conditions in this case are more complicated than in the cases where the sampling process is pre-determined and does not depend on the signal. First, as can be seen from this spectrogram, there are more aliases than in the case of the conventional periodic sampling. Second, the aliasing process is suppressed. The peaks at frequencies of true signal components are much stronger then the aliases. That is due to the fact that the sampling process is both periodic and nonuniform. The non-uniformities of the sampling intervals lead to this effect of alias suppression. The fact that the aliases are to some extent suppressed is significant. This helps to separate them from the signal components. Third, the aliases related to the indicated in (2) three frequency rows might be not equally strong. For instance, while the aliases related to the mean sampling rate f_s are rather weak in the case illustrated by Figure 1, they are well pronounced in the diagram shown in Figure 2. This difference in aliasing can be traced to the differences in signal sampling conditions. Fourth, aliasing illustrated by Figure 1 occurs also at frequencies related to the reference frequency and signal component differences/sums $(f_i \pm f_n);\ \ldots;$ for $i \neq n$ and $i=1, 2, 3, \ldots;\ n=1, 2, 3 \ldots$ That is unusual.

Actually the question is arguable whether the aliasing effect does take place in the discussed case at all. Indeed, it is clear that there is no full-scale frequency overlapping. While peaks appear in the spectrogram at frequencies belonging to the row (2), they are significantly suppressed. This type of aliasing is considered as so-called fuzzy aliasing [1]. It seems that in this specific case it might be even assumed that there are cross-interference effects rather than aliasing while these effects are amplified at the indicated in (2) frequencies.

It is possible to check if this assumption is right. If the peaks in question do appear in result of the cross-interference due to the sampling irregularities, then it should be possible to take them out by adapting signal processing to the specific sampling non-uniformities. That was checked and the discussion of the obtained result follows in the next section.

Cross-interference between signal components

The considered sampling process is nonuniform and, consequently, some distortions of signal spectrograms due to the cross-interference between nonuniformly sampled signals are to be expected [1]. There are two aspects of this impact. There is a background noise with spurious frequency peaks and the mentioned interference actually distorts more or less the whole spectrogram. The spurious frequencies, reflecting the impact of the cross-interference between the signal components and present in the spectrogram, confirm this expectation. These spurious frequencies are not very noticeable in the particular spectrogram of Figure 1. They are more powerful in the spectrogram given in Figure 4. Under certain sampling conditions providing for small sampling point irregularities this kind of spectrum distortions might be negligible as it is shown in [2]. In other cases special signal processing procedures for adapting the sampled signal to the sampling non-uniformities has to be carried out in a way described in [1].

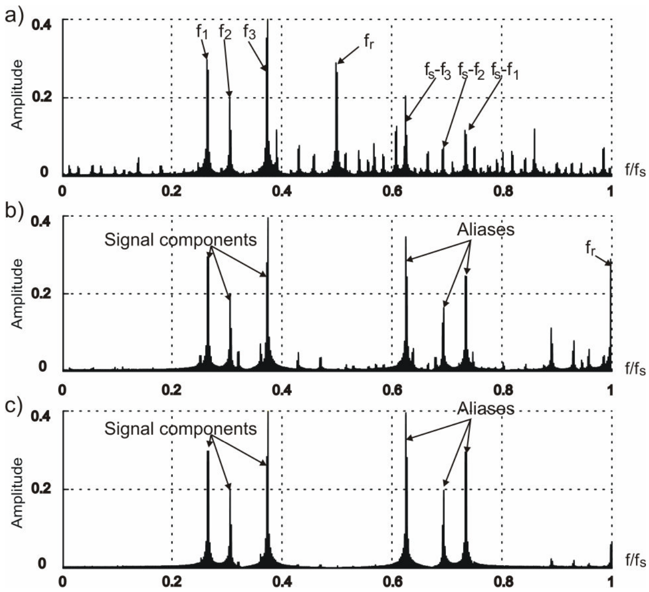

Fig. 2. Impact of sampling regularization on the aliasing conditions.

The impact of this cross-interference on the signal spectrograms directly depends on the signal sampling conditions. The spectrograms displayed in Figure 2 illustrate this. The signal in this particular case has only three components and the sampling conditions are varied. In result, the peaks in the spectrogram indicating the aliases and the power of the spurious frequencies vary as well. While all signal and the reference crossing events are taken into account in the first case illustrated by the spectrogram in Figure 2 (a), the sampling operation is enabled only during each sixth half-period of the reference function in the second case (Figure 2 (b)) and during each tenth half-period in the third case (Figure 2 (c)). As can be seen, this approach to sampling significantly impacts the features of the obtained digital signal. Introduction of the enabling function actually has the effect of sampling regularization. Increasing the interval during which the sampling operation is blocked results in smaller power of the introduced element of the randoness and that in turn leads to reduction of the effects induced by the cross-interference and to increasing of the peaks in the spectrogram due to aliasing.

Adapting digital signal processing to the sampling non-uniformities

Corruption of signal processing by aliasing and the cross-interference, of course, is not acceptable. Especially because the errors related to these effects might be even more significant than those shown in Figure 2. However there are at least two possible approaches to resolution of this problem. The first approach is based on the mentioned regularization of the sampling process. It is suggested in [1] and is considered in more detail in [2]. Suffice it to say that this kind of sampling regularization is based on introduction of a function enabling detection of the crossing events only during some time intervals when it is turned on. Typically the enabling function is activated during each *n*-th period for a half of that period of the reference function, where *n=1, 2, 3, ...* with *n* maximal values reaching up to about 25. In result of such sampling restrictions imposed by the enabling function the irregularities of the sampling intervals become much less harmful. The digital signals obtained under these conditions typically might be processed on the basis of the classical DSP algorithms with good results practically not corrupted by aliasing (if the sampling rate is high enough) and the cross-interference. The drawback of this approach is obvious. It could be used for signal digitising and the obtained digital signal processing in a limited frequency range.

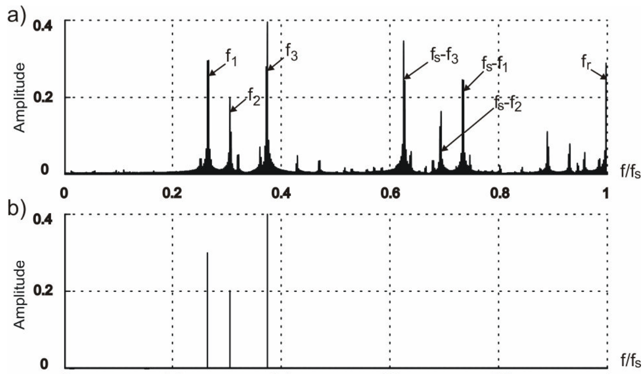

Figure 3. Results of the spectrum analysis adapted to the non-uniformities of sampling. (a) spectrogram obtained before such adapting; (b) spectrogram obtained by applying this type of adapting.

The second approach to the problem of achieving better results at processing digital signals obtained in the course of constant envelope sampling is based on adapting the signal processing to the specific non-uniformities of the involved sampling operation. The obtainable performance improvement is illustrated in Figures 3 and 4.

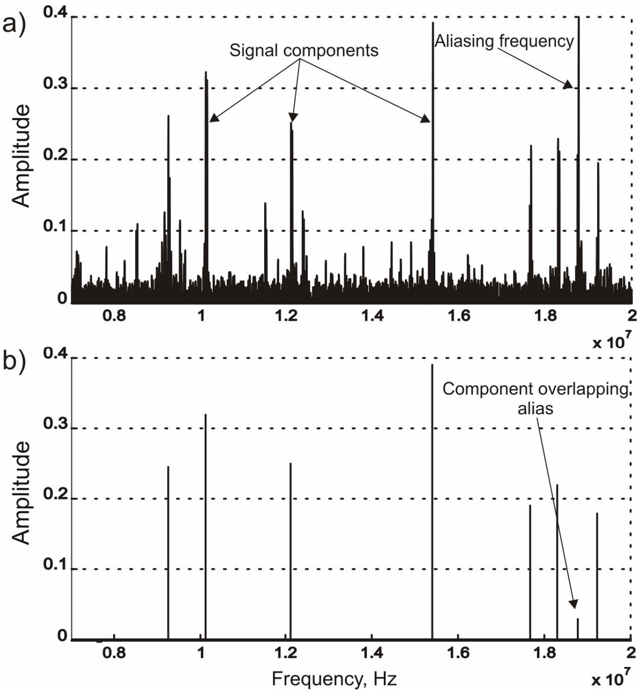

Figure 4. Illustration of separating overlapping signal true components and aliases.

As it was mentioned above, the peaks at frequencies of true signal components, in the spectrograms of the digital signals obtained in result of constant envelope sampling, typically are stronger then the aliases and that is useful. In other words, the non-uniformities of the sampling intervals lead to the alias suppression and this helps to take out the aliases as shown in Figure 3 or to separate them from the signal components. Figure 4 illustrates the latter aspect. In the illustrated case, a particular weak signal component overlaps a much stronger aliasing frequency (Figure 4 (a)).

When the signal processing was adapted to the sampling non-uniformities, the alias overlapping this signal component was taken out and the signal component was displayed as shown in Figure 4 (b). Adapting was carried out in a way described in [1]. As the discussed here sampling procedure is signal-dependent, it is not possible to prepare and use cross-interference coefficient matrix in this case. That of course represents a disadvantage.

Conclusions

To fully gain from the advantages of constant envelope sampling, it has to be learned how to effectively cope with the drawbacks related to the non-uniformity of the obtained digital signals. It was attempted to show in this paper that there are various techniques that might be used. Specifically, regularization of the sampling operation and adapting processing of the digital signals of this kind to the sampling non-uniformities are recommended as effective tools for this.

References

1. **Bilinskis I.** *Digital Alias-free Signal Processing*, John Wiley & Sons, Ltd (UK), 2007, 430 p.
2. **Bilinskis I., Sudars K.** *Processing of signals sampled at sine-wave crossing instants.* Proceedings of the “2007 Workshop on Digital Alias-free Signal Processing” (WDASP’07), 17 April 2007, London, UK, p. 45-50.
3. **Bilinskis I., Sudars K.** *Digital representation of analog signals by timed sequences of events*, “Electronics and Electrical Engineering”, 2008, No 3(83)

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Features of digital signals, obtained in the case where the signal sample values are taken at crossings of an original analog signal and a sinusoidal reference function, are discussed. It is shown that spectrum analysis of this kind of nonuniform digital signals, with constant envelope not depending on the signal parameters, might be performed without massive multiplication of multi-digit numbers. Observed aliasing and cross-interference effects are considered and adapting signal processing to the specific signal-dependent sampling non-uniformities is suggested, including adapted spectrum analysis and waveform reconstruction. Ill. 4, bibl. 3 (anglų k.; santraukos anglų, rusų ir lietuvių k.).

И. Билинскис, К. Сударс. Особенности цифровых сигналов с постоянной огибающей // Электроника и электротехника. – Каунас: Технология, 2008. – № 4(84). – С. 13–16.

Рассматриваются свойства цифровых сигналов, полученных в случае, когда дискретные отсчеты исходных сигналов берутся в моменты времени пересечения этими сигналами синусоидальной опорной функции. Показано, что спектральный анализ таких цифровых сигналов с неизменной огибающей можно осуществить без выполнения операций умножения многозначных чисел. Рассматриваются эффекты наложения частот и кросс-интерференции, а также адаптация обработки таких сигналов к специфическим нерегулярностям их дискретизации при спектральном анализе и восстановлении сигналов во временной области. Ил. 4, библ. 3 (на английском языке; рефераты на английском, русском и литовском яз.).

I. Bilinskis, K. Sudars. Skaitmeninių signalų su pastovia gaubtine tyrimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 4(84). – P. 13–16.

Nagrinėjamos skaitmeninių signalų ypatybės, tirtas atvejis, kada signalų pavyzdžių vertės yra paimtos nuo originalaus analoginio signalo. Parodoma, kad skaitmeninių signalų su pastovia gaubtine spektro analizė ne priklausomai nuo signalo parametrų, galėtų būti įvykdyta be masinės daugiaženklių skaičių daugybos. Išnagrinėti didelio dažnumo ir tarpusavio interferencijos efektai, kuriuos sudaro signalo apdirbimo adaptacija su specifiniais diskretizacijos nereguliarumais pritaikant spektro analizę ir bangos formos atstatymą laikinėse srityse. Ill. 4, bibl. 3 (in English; summaries in English, Russian and Lithuanian).

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