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Digital Representation of Analog Signals by Timed Sequences of Events

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Introduction

To convert an analog signal into digital, sample values of the original analog signal usually are taken at some specifically defined sampling time instants and the digital signal is formed as a sequence of these sample values. As the definition of the sampling time instants of the signal sample value taking might differ, analog signals actually might be converted into their digital counterparts in various ways. The most popular approach is based on sampling signals periodically but it certainly is not exclusive. The sampling process might as well be also nonuniform. A specific signal digitising technique suggested in [1] is considered in some detail in this paper. It is rarely used so far. According to it, signal sample values are taken at time instants when the signal crosses a sinusoidal reference function. Such digitising of signals, based on their crossings of a given constant parameter reference sinusoid, has features unparalleled by other digital signals. For a wide class of input signals, the envelope of their digital counterparts is invariable. And that leads to a remarkable method for representing the analog signals. They might be fully digitally represented just by sampling time instant sequences rather than by the sequences of their sample values as usual. Consequently, the conditions for processing this type of digital signals are essentially specific as well. Therefore adding the described digital signal to the collection of other more conventional digital signal types widens the variety of the signal processing and the signal processing system design options. When development of a specific application is planned, this approach to signal digitising is well worth considering. Basics of this signal digitising approach are further discussed.

Sampling based on signal and reference crossings

Consider sampling of an analog input signal $x(t)$ as a process of taking sample values of this signal $x(t_k)$ at the time instants $\{t_k\}$when crossings of the signal $x(t)$ and a reference sine wave u_r occur. At these time instants the following equality holds:

$$x(t_k) = u_r(t_k) = A_r \sin(2\pi f_r t_k + \varphi_r),\ k=1, 2, 3,\ldots \quad (1)$$

where A_r, f_r and φ_r are the amplitude, frequency and phase angle of the reference sinusoid u_r respectively.

The time instants t_k apparently are given as

$$t_k = kT_r + \frac{T_r}{2\pi}\arcsin\frac{u_r(t_k)}{A_r} \quad (2)$$

where T_r is the period of the reference sine-wave function.

Signal sample value taking at the time instants $\{t_k\}$, satisfying equation (1), represents the basic model of the considered sampling process. Fig. 1 illustrates it.

Diagrams in Fig. 1 clearly show that the considered scheme of signal sampling is signal-dependent and essentially nonuniform. As is evident from them, the envelopes of the two obtained distinctly different digital signals are exactly the same. This is a significant positive fact leading to far reaching consequences for processing of this type of digital signals. Algorithms for processing such signals often are less complicated than the widely used conventional ones. While consideration of them is out of the scope for this paper, spectrum analysis and waveform reconstruction based on this type of algorithms are discussed.

The mentioned fact that different digital signals of the considered kind have invariable envelope also means that the sampling point processes, given in Fig. 2 (a) and (b), actually fully describes the respective sampled signals as the envelopes of the sample value sequences are invariable and pre-determined. Indeed, if a sampling time instant t_k is given, the equation (1) provides for recovery of the respective signal sample value $x(t_k)$.

Therefore this type of point processes actually could be considered as digital signals fully representing the respective original analog signals in the digital domain.

An essential informative parameter characterizing this kind of sampling is the ratio σ/μ, where σ is the standard deviation of the sampling intervals and μ is the mean value of them. This parameter, for the sampled signals in Fig. 1 (a) and (b), is $\sigma/\mu=0.4863$ and $\sigma/\mu=0.9055$, respectively.

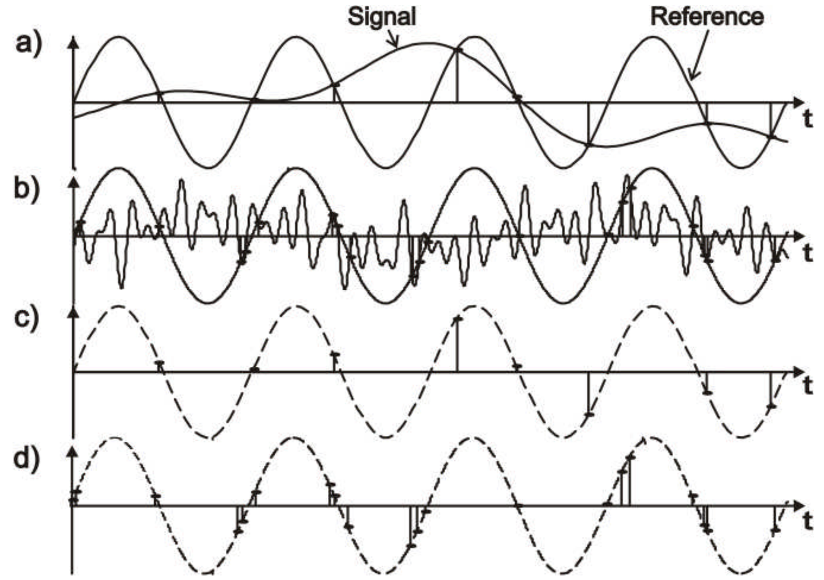

Fig. 1. Illustration of a constant envelope sampling process based on a signal and a sinusoidal reference function crossings. (a), (b) diagrams illustrating sampling of two signals having components at differing frequencies; (c) signal digital sample value sequence of the signal shown in (a); (d) signal digital sample value sequence of the signal shown in (b)

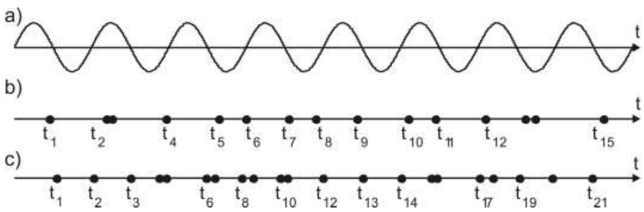

Fig. 2. Sampling point processes fully representing the respective analog signals in the digital domain. (a) reference function; (b) point process representing the sampled signal shown in Figure 1 (a); (c) point process representing the sampled signal given in Fig. 1 (b)

Empirical distributions of the sampling intervals in both of these cases are given in Fig. 3. Evidently they strongly differ. The first one illustrates a sampling case that is much better than the second one. First of all it is better in the sense that it is more regular. The deviations of the sampling interval values from their mean value are less pronounced. Second, only a small number of the sampling intervals in that case are very short. That is not the case with the empirical distribution given in the Fig. 3 (b). In that case, a considerable number of crossings occur very closely in time and that represents a problem for processing the respective signal sample values. It might be said that the nonuniformity of sampling in the second illustrated case is much stronger. And that is clearly shown in the given empirical distributions and follows from the numerical value of the ratio σ/μ.

The displayed two cases of two differing signal sampling reveal the fact that this approach to sampling and to analog signal digitising leads to digital signals that could be given either as sequences of signal sample values with envelopes defined by the used reference function or just as sequences of sampling (crossing) time instants $\{t_k\}$. Both types of the digital signals represent the respective analog signals equally well. The quality of this kind of digital signals, obtained by detecting the original signal crossings with a reference sinusoid, remains to be found out. This issue is discussed in [2] and to some extent in the following sections.

Trading-off the mean sampling rate against the time-resolution

Features of the digital signal formed in result of the analog input signal and the reference sinusoid crossings obviously depend on both of the involved processes. As the crossing point process might be considered as the digital signal representing the original analog signal, the basic features of concern are the regularity of the intervals between the sampling (crossing) points and the accuracy of the indicated crossing time instants. Both of these features to some extent depend on the frequency of the used reference sinusoid. Therefore the question arises how to select the appropriate value of this parameter of the reference function.

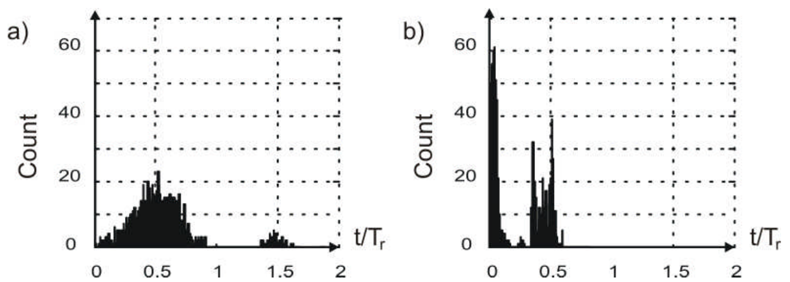

Fig. 3. Empirical distributions of the sampling interval duration normalized to the reference period: (a) for the sampled signal shown in Fig. 1(a); (b) empirical distribution for the sampled signal shown in Fig. 1 (b)

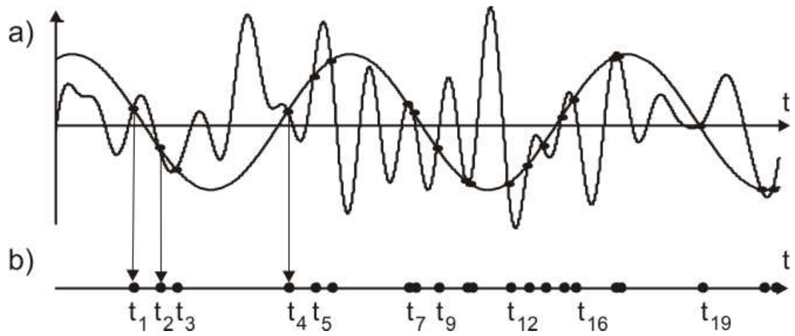

Fig. 4. Digital signal (b) in the case where a signal crosses a low frequency reference function as shown in (a)

There are some considerations that might be taken into account. In general, at low reference frequencies the relative resolution with which the crossing events are fixed in time is better than at high reference frequencies. From that point of view, low reference frequency seems to be preferable. On the other hand, at low reference frequencies, the crossings typically occur as it is shown in Fig. 4 (a). Then the crossing point pattern in time basically depends on the signal frequency content as the signal typically crosses the reference function a number of times during each period of the reference and the sampling point process then is characterized by relatively high values of the ratio σ/μ as it is rather irregular. Consequently, distortions of the sampled signal processing due to the cross-interference phenomenon might be expected then.

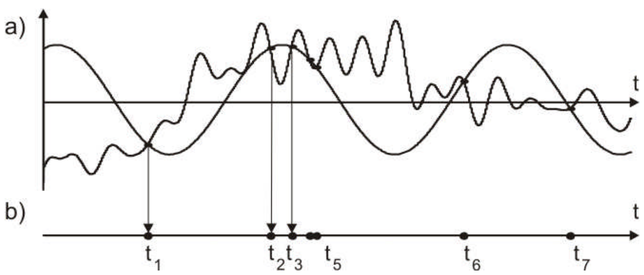

Fig. 5. Illustration of a sampling case where the reference sinusoid is not crossing the signal within relatively large signal segments. (a) crossings of the signal and the reference function; (b) obtained digital signal

Another drawback typical for the cases where low frequency reference functions are used is illustrated in Fig. 5. As can be seen, there are relatively large gaps between the detected crossing points. In result, essential information is lost during these time intervals. The formed digital signal, namely, the sampling (crossing) point process, given in Fig. 5(b), again is nonuniform and it is characterized by large values of the ratio σ/μ.

The digital signal, obtained in the cases where relatively high frequency reference function is used, typically is more regular as the reference function basically imposes the crossings.

A typical sampling process of this kind is given in Fig. 6. As can be seen, the formed digital signal indeed is more regular in this case.

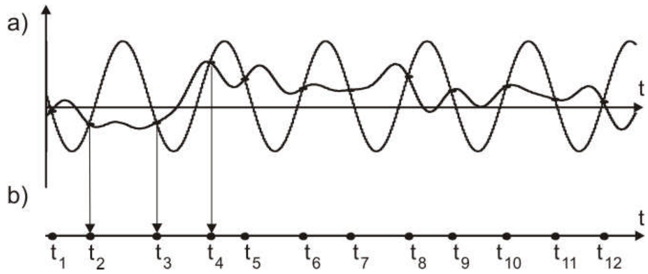

Fig. 6. Illustration of the digital signal forming in the case where relatively high frequent reference function is used. (a) sampling process; (b) formed digital signal

As the given illustrations show, usually it is preferable to use reference sinusoids at relatively high frequencies. However even then the irregularities of the obtained digital signal might lead to significant distortions of the signal processing results. These distortions are typical for any kind of nonuniform sampling and they are caused by the cross-interference between the nonuniformly sampled signal spectral components. While more often than not it is desirable to work at higher reference frequencies, increasing of the reference frequency is limited. And the consideration of the time-resolution basically is the dominating factor setting up this limit. Therefore the mean sampling rate, directly depending on the reference frequency, often has to be traded-off against the achievable time-resolution.

Conclusions

The discussed signal digitising technique, based on signal sample value taking at time instants when the signal crosses a sinusoidal reference function, certainly is specific and it has unusual positive and not so positive features. Probably the most valuable property of this kind of nonuniform digital signals is their constant envelope not depending on the original analog signal frequency content. That makes it possible to develop algorithms for processing them without massive multiplication of multi-digit numbers. The specific features of the described digital signals given as sequences of timed sampling events are discussed in [2]. It seems that this approach to analog signal digitising has high application potential. In particular, it is well suited for massive data acquisition from a large quantity of signal sources [1, 3].

References

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A specific signal digitising technique, based on signal sample value taking at time instants when the signal crosses a sinusoidal reference function, is considered. The various digital representations of the original signals obtained in this way have invariable envelope and that leads to specific features of the digital signals unparalleled by other digital signals. The timed sampling event sequences then actually could be considered as digital signals fully representing the respective original analog signals in the digital domain. Adding these signals to the collection of other more conventional digital signal types widen the application range for digital signal processing and provides for specific application options. Ill. 6, bibl. 3 (in English; summaries in English, Russian and Lithuanian).

И. Билинскис, К. Сударс. Дискретные представления аналоговых сигналов точечными потоками событий // Электроника и электротехника. – Каунас: Технология, 2008. – № 3(83). – С. 89–92.

Рассмотрен особый подход к представлению сигналов в дискретном виде, основанный на взятии отсчетов сигнала в моменты времени пересечения сигнала с синусоидальной опорной функцией. Различные дискретные представления сигналов, полученные при таком подходе, имеют одну и ту же огибающую, и это придает этим цифровым сигналам особые свойства. Потоки событий в моменты времени пересечения можно рассматривать как особый вид цифровых сигналов. Пополнение коллекции разнообразных видов цифровых сигналов этой конкретной разновидностью расширяет область применения цифровой обработки сигналов, увеличивая возможности выбора. Ил. 6, библ. 3 (на английском языке; рефераты на английском, русском и литовском яз.).

I. Bilinskis, K. Sudars. Skaitmeninis analoginių signalų vaizdavimas naudojant įvykių laike srautus // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 3(83). – P. 89–92.

Nagrinėjamas savitas signalų skaitmenizavimo metodas, kurio esmė yra ta, jog signalo atskaita paimama, kai signalas kerta atraminę sinuso funkciją. Taip gauti įvairūs skaitmeniniai pradinių signalų vaizdai, pasižymintys nekintančia gaubtine. Tai leidžia išskirti savitas skaitmeninių signalų savybes, skiriančias juos nuo kitų skaitmeninių signalų. Signalo atskaitos registravimo įvykių sekos gali būti laikomos skaitmeniniais signalais, išsamiai apibūdinančiais atitinkamus pradinius analoginius signalus skaitmeninėje erdvėje. Šiais signalais papildžius tipinių skaitmeninių signalų rinkinius, galima būtų praplėsti esamas skaitmeninio signalų apdorojimo pritaikymo sritis bei sukurti naujas pritaikymo galimybes. Il. 6, bibl. 3 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).