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Stochastic Models of Quality Level of Mechatronic Products

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Introduction

Complex mechatronics products are defined in technical documentation as an entire series of parameters, the values of which determine the level of product quality. Parameters can be differentiated according to their importance regarding the implementation of purpose functions. International standard ISO-2859-0 [11] recommends to divide parameters into two – A, B – or three – A, B, C – classes (groups). Here A – most important or significant parameters, and B, C – secondary or less significant parameters. Such classification of parameters is convenient when analyzing problems of multiparametric product quality control [2–9]. When imitational modeling is applied, stochastic models of quality level are required for separate parameters, their groups and for entire product.

Assume, that product is characterized by l–dimensional random vector of independent parameters $X = (X_1, X_2, \ldots, X_l)$. Probability of defective product θ_i (in respect of the i-th parameter) is a random quantity (r.q.) with its density $g_i(\theta_i)$, distribution function $G_i(\theta_i)$ and main numerical characteristics: mean $E\theta_i = \mu_i$ and dispersion $V\theta_i = \sigma_i^2$, where $i = 1,2,\ldots,l$. For practical application we are going to use additional dispersion characteristics: standard deviation $\sigma_i = \sqrt{V\theta_i}$ and variation coefficient $v_i = \dfrac{\sigma_i}{\mu_i}$ [1].

Probability of good product η_i according to one parameter is

$$\eta_i = 1 - \theta_i . \tag{1}$$

R.q. η_i is characterized by density $\varphi_i(\eta_i)$, distribution function $\Phi_i(\eta_i)$, mean $E\eta_i = \overline{\mu}_i$ and dispersion $V\eta_i = V\theta_i = \sigma_i^2$. The following relation formulas are valid:

$$\begin{cases} \overline{\mu}_i = 1 - \mu_i, \\ g_i(\theta_i) = \varphi_i(1-\theta_i), \\ G_i(\theta_i) = 1 - \Phi_i(1-\theta_i). \end{cases} \tag{2}$$

The probability of defective product for the entire product r.q. θ and probability of good product r.q. η are calculated as [12]

$$\begin{cases} \eta = \prod_{i=1}^{l} \eta_i, \\ \theta = 1 - \eta = 1 - \prod_{i=1}^{l} (1-\theta_i). \end{cases} \tag{3}$$

Since η_i are inter-independent r.q., the following equation is valid

$$\varphi(\eta) = \prod_{i=1}^{l} \varphi_i(\eta_i) . \tag{4}$$

In general, the distribution function $\Phi(\eta)$ of the random multiplicative r.q. $\eta = \eta_1 \ldots \eta_l$ according to [1, 10] is defined by l-dimensional integral

$$\Phi(\eta) = \iint\limits_{D_\eta} \ldots \int \prod_{i=1}^{l} \varphi_1(\eta_1)\varphi_2(\eta_2)\ldots\varphi_l(\eta_l)\, d\eta_1 d\eta_2 \ldots d\eta_l , \tag{5}$$

here D_η – integration range. Then density $\varphi(\eta)$ is expressed according to (6)

$$\varphi(\eta) = \frac{d\Phi(\eta)}{d\eta} . \tag{6}$$

Using analogy with (2) for entire product

$$g(\theta) = \varphi(1-\theta),\ G(\theta) = 1 - \Phi(1-\theta) . \tag{7}$$

Means $E\eta$ and $E\theta$ of the r.q. η and θ are calculated as

$$\mathrm{E}\eta = \overline{\mu} = \prod_{i=1}^{l} \mu_i \text{ , } \mathrm{E}\theta = \mu = 1 - \overline{\mu} \text{ .} \tag{8}$$

Dispersion $V\theta = V\eta = \sigma^2$ is calculated by applying unifying formula of dispersions of two parameters σ_1^2 and σ_2^2 (9)

$$\sigma_{12}^2 = \sigma_1^2 \overline{\mu}_2^2 + \sigma_2^2 \overline{\mu}_1^2 + \sigma_1^2 \sigma_2^2, l = 2 \text{ .} \tag{9}$$

If $l > 2$, then we join σ_{12}^2 with σ_3^2, and receive σ_{13}^2 and so on. According to the analogy with dispersion characteristics of one parameter we will use the standard deviation $\sigma = \sqrt{V\theta}$ and variation coefficient $v = \frac{\sigma}{\mu}$. Further we will analyze particular cases, when functions $g_i(\theta_i)$ and $\varphi_i(\eta_i)$ are beta-distributions in respect of separate parameters.

Beta-distributions

We will provide the main formulas, required for the further analysis, when r.q. θ_i and also η_i are distributed according to the beta-distribution with shape parameters a_i, b_i and marking that: $\theta_i \sim Be(a_i, b_i)$, $\eta_i \sim Be(b_i, a_i)$ [1]:

$$\begin{cases} g_i(\theta_i) = B_i^{-1}(a_i, b_i)\theta_i^{a_i - 1}(1 - \theta_i)^{b_i - 1} \\ \varphi_i(\eta_i) = B_i^{-1}(a_i, b_i)\eta_i^{b_i - 1}(1 - \eta_i)^{a_i - 1} \end{cases}, \tag{10}$$

here $B_i(a_i, b_i) \equiv B_i = \frac{\Gamma(a_i)\Gamma(b_i)}{\Gamma(a_i + b_i)}$ – beta function, $\Gamma(z_i)$ – gamma function, $\Gamma(z_i) = (z_i - 1)\Gamma(z_i - 1)$ or $\Gamma(n) = (n-1)!$, when n is a whole number (h.n.), $i = 1 \ldots l$;

$$\begin{cases} \mu_i = \dfrac{a_i}{a_i + b_i}, \overline{\mu_i} = 1 - \mu_i = \dfrac{b_i}{a_i + b_i} \\ \sigma_i^2 = \dfrac{a_i b_i}{(a_i + b_i)^2 (a_i + b_i + 1)} = \dfrac{\mu_i \overline{\mu_i}}{a_i + b_i + 1} \end{cases}; \tag{11}$$

$$v_i = \frac{\mu_i}{\sigma_i} = \sqrt{\frac{a_i}{b_i}(a_i + b_i + 1)} \text{ .} \tag{12}$$

If $a_i = 1$ and μ_i is sufficiently small ($\mu_i < 0.03$), then $b_i >> 1$ and $v_i \approx 1$, i.e. $\sigma_i \approx \mu_i$.

Density $g_i(\theta_i)$ has maximum at the point θ_{iM} (mode) [8]

$$\theta_{iM} = \frac{a_i - 1}{a_i + b_i - 2} \text{ .} \tag{13}$$

Respectively the maximum of density $\varphi_i(\eta_i)$ is at the point $\eta_{iM} = 1 - \theta_{iM}$.

It is obvious, that (9)–(13) formulas may be applied for entire product, if r.q. $\theta \sim Be(a, b)$, or for separate groups A, B, C, if their defectivity levels θ_A, θ_B, θ_C are characterized by beta-distribution.

Biparametric mechatronics products

Product is characterized using two parameters $i = 1,2$, which are distributed according to beta-distribution ((10)–(13)). Then according to (3), equations $\eta = \eta_1 \eta_2$, $\theta = \theta_1 + \theta_2 - \theta_1 \theta_2$ are valid.

In order to avoid integration bands $\eta_i = 0$ we use dependency (14)

$$F(y) = P\{Y < y\} = 1 - P\{Y > y\} \tag{14}$$

here $F(y)$ is distribution function of r.q. Y, $P\{Y > y\}$ is probability, that $Y > y$. In this way, when $Y > y$, integration range D_η (5) is defined by hyperbola $\eta = \eta_1 \eta_2$ with upper variation interval limit $\eta_i = 1$ according to [10] (Fig. 1).

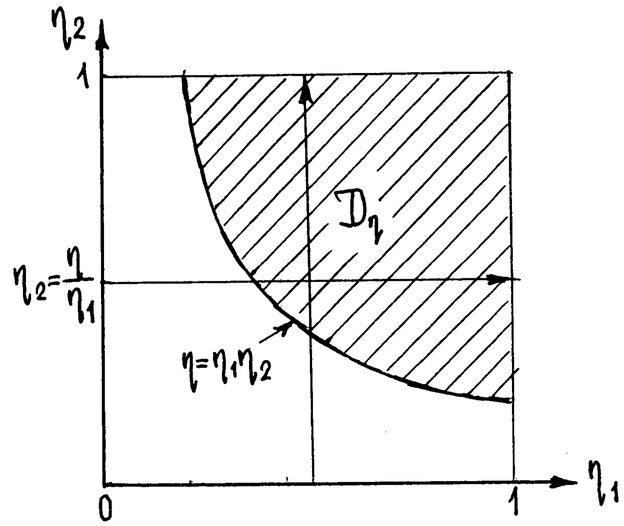

Fig. 1. Two-dimensional integration space D_η, $l = 2$

We receive

$$\Phi(\eta) = 1 - \int_{\eta}^{1} \varphi_1(\eta_1) d\eta_1 \int_{\eta/\eta_1}^{1} \varphi_2(\eta_2) d\eta_2 =$$
$$= 1 - (B_1 B_2)^{-1} \int_{\eta}^{1} \int_{\eta/\eta_1}^{1} \eta_1^{b_1 - 1} \eta_2^{b_2 - 1} \cdot \tag{15}$$
$$\cdot (1 - \eta_1)^{a_1 - 1} (1 - \eta_2)^{a_2 - 1} d\eta_1 d\eta_2$$

$$\varphi(\eta) = \frac{d\Phi(\eta)}{d\eta} = \int_{\eta}^{1} \frac{1}{\eta_1} \varphi_1(\eta_1) \varphi_2\left(\frac{\eta}{\eta_1}\right) d\eta_1 =$$
$$= 1 - (B_1 B_2)^{-1} \int_{\eta}^{1} \int_{\eta/\eta_1}^{1} \eta_1^{b_1 - 1} \eta_2^{b_2 - 1} \cdot \tag{16}$$

It can be proved, that with any positive values of a_i and b_i we receive $\theta \sim Be(a,b)$, i.e. density $g(\theta)$ is beta-distribution with parameters a, b:

$$\begin{cases} a = a_1 + a_2, \\ b = \begin{cases} b_2, then\ b_1 = b_2 + a_2, b_1 > b_2, \\ b_1, then\ b_2 = b_1 + a_1, b_1 < b_2. \end{cases} \end{cases} \quad (17)$$

It is obvious, that it is advisable to use such cases in modeling, since integration procedures are not needed any more. Thus the sufficiently abundant entirety of distributions $g(\theta)$ can be obtained with minimal whole-number values of a_i: $a_i = 1$ and $a_i = 2$. Then

$$\begin{cases} B_i^{-1}(1, b_i) = b_i,\ \ then\ a_i = 1; \\ B_i^{-1}(2, b_i) = b_i(b_i + 1),\ \ then\ a_i = 2. \end{cases} \quad (18)$$

When any positive values of a_i, b_i are present, density $g(\theta)$ can be approximated with sufficient precision using beta-density $g_\sigma(\theta)$ with parameters a^*, b^*:

$$\begin{cases} a^* = \mu(\frac{\mu\bar{\mu}}{\sigma^2} - 1),\ \ b^* = a\frac{\bar{\mu}}{\mu} = a(\frac{1}{\mu} - 1), \\ g_\sigma(\theta) = B^{-1}(a^*, b^*)\theta^{a^*-1}(1-\theta)^{b^*-1}. \end{cases} \quad (19)$$

here μ, $\bar{\mu}$ – according to (8), $\sigma^2 \equiv \sigma_{12}^2$ – according to (9), $B(a^*, b^*)$ – according to (10).

Relative approximation error δ is equal (in percent)

$$\delta = [\frac{g_\sigma(\theta)}{g(\theta)} - 1] \cdot 100\% \quad (20)$$

Assume, that $a_1 = a_2 = 1$, $b_1 = 3$, $b_2 = 2$ ($b_1 = b_2 + 1$).

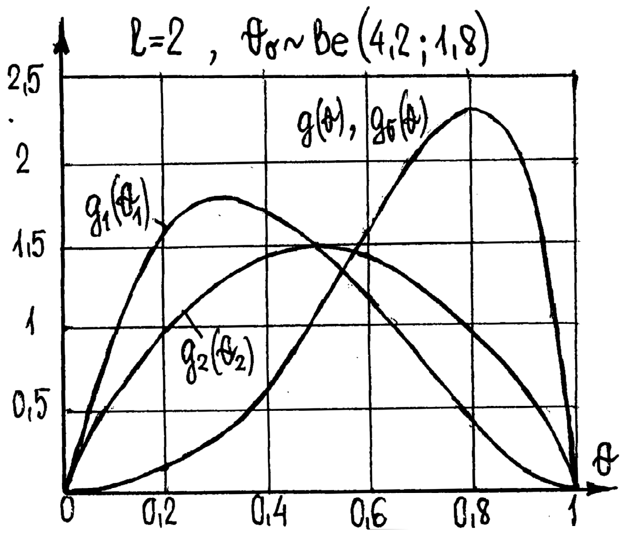

Fig. 2. Density $g(\theta)$: $a_1 = a_2 = 2$; $b_1 = 3$, $b_2 = 2$

We decrease parameter b_2 down to $b_2 = 1$. Receive $\theta \sim Be(4,1)$, i.e. $a = 4$, $b = 1$, when $a_i = 2$, $b_1 = 3$, $b_2 = 1$, $i = 1,2$ (see Fig. 3). If b_1 is increased up to $b_1 = 4$, when $a_i = b_2 = 2$, we receive beta-distribution $g(\theta)$ with $a = 4$, $b = 2$ (Fig. 4).

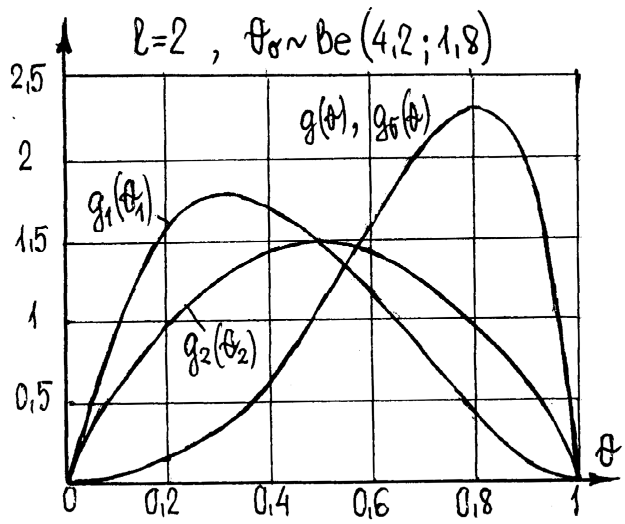

Fig. 3. Density $g(\theta)$: $a_1 = a_2 = 2$; $b_1 = 3$, $b_2 = 1$

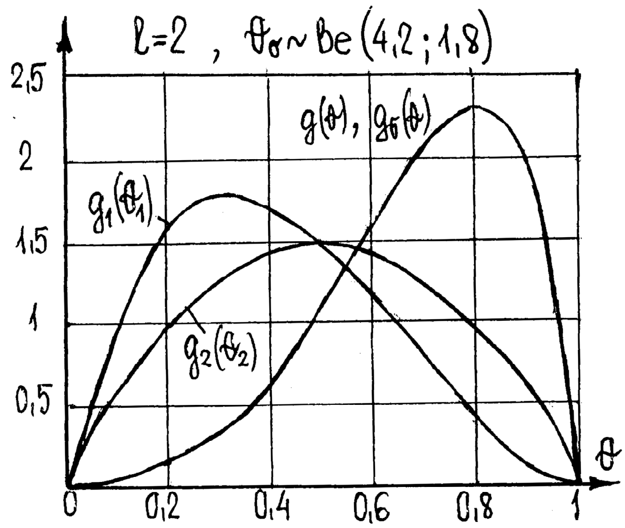

Fig. 4. Density $g(\theta)$: $a_1 = a_2 = 2$; $b_1 = 4$, $b_2 = 2$

Assume, that a_i and b_i values are transformed into non-whole number (rational) values: $a_1 = 1.6$, $a_2 = 2.3$, $b_1 = 4.4$, $b_2 = 3.7$. We approximate the exact expression of $g(\theta)$ using beta-density $g_\sigma(\theta)$ with parameters a^*, b^* (Fig. 5) and:

$$\mu_1 = 0{,}267,\ \mu_2 = 0{,}383,\ \mu_3 = 0{,}548,\ \sigma_1^2 = 0{,}028,$$
$$\sigma_2^2 = 0{,}034,\ \sigma^2 = 0{,}03;\ a^* = 4{,}02,\ b^* = 3{,}32;$$

$$g_1(\theta_1) = 13{,}25\theta_1^{0,6}(1-\theta_1)^{3,4};$$
$$g_\sigma(\theta) = 90{,}78\theta^{3,02}(1-\theta)^{2,32}. \quad (21)$$

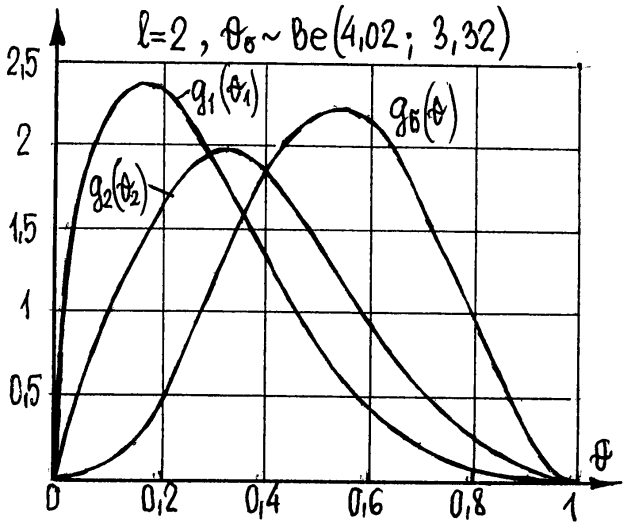

Fig. 5. Density $g(\theta)$: $a_1 = 1.6$, $a_2 = 2.3$; $b_1 = 4.4$, $b_2 = 3.7$

Three-parameter products

Three parameters ($l = 3$) are often used to characterize mechatronics products; then

$$\Phi(\eta) = 1 - (B_1 B_2 B_3)^{-1} \int_{\eta}^{1} \int_{\eta/\eta_1}^{1} \int_{\eta/\eta_1\eta_2}^{1} \eta_1^{b_1-1} \eta_2^{b_2-1} \eta_3^{b_3-1} \times$$

$$\times (1-\eta_1)^{a_1-1}(1-\eta_2)^{a_2-1}(1-\eta_3)^{a_3-1} d\eta_1 \, d\eta_2 \, d\eta_3$$

$$\varphi(\eta) = \frac{\eta^{b_3-1}}{B_1 B_2 B_3} \int_{\eta}^{1} \int_{\eta/\eta_1}^{1} \eta_1^{b_1-b_3-a_3} \eta_2^{b_2-b_3-a_3} (1-\eta_1)^{a_1-1} \times$$

$$\times (1-\eta_2)^{a_2-1} (\eta_1\eta_2 - \eta)^{a_3-1} d\eta_2 \, d\eta_1 \qquad (21)$$

With any positive values of a_i, b_i, $i = 1,2,3$ the approximation $g(\theta) \to g_\sigma(\theta)$ according to (20), when $\bar{\mu} = \bar{\mu}_1 \bar{\mu}_2 \bar{\mu}_3$, $\mu = 1 - \bar{\mu}$, $\sigma^2 = \sigma_{13}^2$ according to (9).

Let us have $a_i = 2$, $b_i = 3$, $l = 3$; receive (Fig.6)

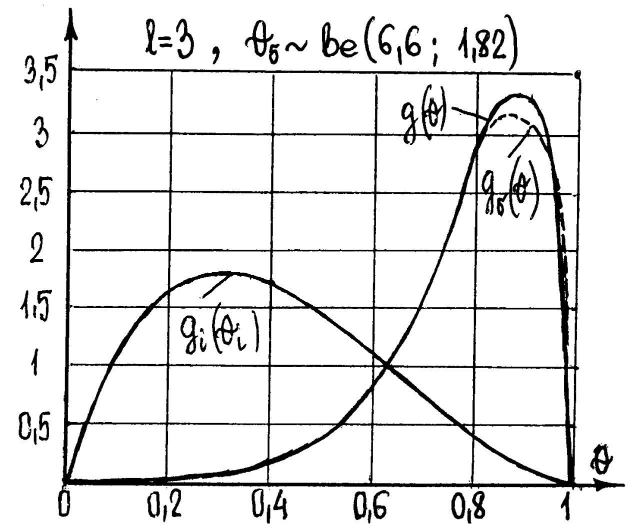

Fig. 6. Density three - parameter $a_i = 2$, $b_i = 3$, $i = 1,2,3$

When inserting μ, σ^2 values into (19), we obtain $a^* = 6.6$, $b^* = 1.82$ and $g_\sigma(\theta) = 36.82\theta^{5.6}(1-\theta)^{0.82}$ (see Table 4 and Fig. 6); $\delta = -5.13\%$ at the point $\theta_M = 0.87$.

Table 1. $l = 3$; $a_i = 2$, $b_i = 3$, $i = 1,2,3$; $\theta_\sigma \sim Be(6.6;1.82)$

θ	**0,2**	**0,4**	**0,6**	**0,8**	**0,9**	**0,95**	θ_M
$g(\theta)$	0,005	0,15	0,96	2,91	3,24	2,27	3,34
$g_\sigma(\theta)$	0,004	0,14	0,99	2,82	3,09	2,37	3,17
$\theta_M = 0{,}87$; $g(0) = g(1) = 0$							

We substitute uniform values $b_i = 3$ with the following values: $b_1 = 6$, $b_2 = 4$, $b_3 = 2$. Then we have: $\theta_1 \sim Be(2,6)$, $\theta_2 \sim Be(2,4)$, $\theta_3 \sim Be(2,2)$ and $\theta \sim Be(6,2)$ (Fig. 7), when

$$\mu_1 = \frac{1}{4}, \sigma_1^2 = 0{,}0208, g_1(\theta_1) = 42\theta(1-\theta)^5, \theta_{1M} = \frac{1}{6} \qquad (22)$$

$$\mu_2 = \frac{1}{3}, \sigma_2^2 = 0{,}0317, g_2(\theta_2) = 20\theta(1-\theta)^3, \theta_{1M} = \frac{1}{4} \qquad (23)$$

$$\mu_3 = 0{,}5, \sigma_3^2 = 0{,}05, g_3(\theta_3) = 6\theta(1-\theta), \theta_{3M} = 0{,}5 \qquad (24)$$

$$\mu = 0{,}75, \sigma^2 = 0{,}0208, g(\theta) = 42\theta^5(1-\theta), \theta_M = \frac{5}{6} \qquad (25)$$

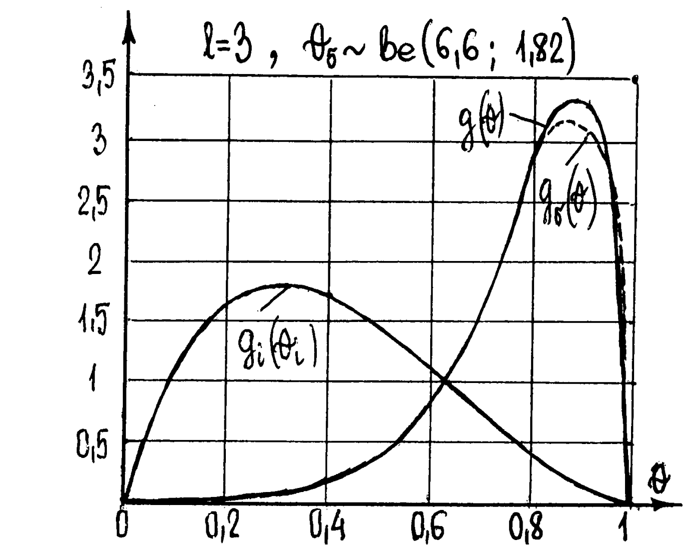

Fig. 7. Density three - parameter $a_i = 2$; $b_1 = 6$, $b_2 = 4$, $b_3 = 2$, $i = 1,2,3$

Practical applications

Two groups A, B; $\mu = 6\%$, $l = 4$, $r = s = 2$, $A \in i = 1,2$, $B \in i = 3,4 \equiv j$, $a_i = 1$, $a_j = 2$.

Beta-distribution modeling results are resented in Table 2.

Table 2. $l = 4$, $\mu = 6\%$, groups A, B; $r = s = 2$

i	a	b	$\overline{\mu}$	μ,%	σ,%	$\upsilon = \frac{\sigma}{\mu}$	θ_M,%
1	1	99	99/100	1	0,99	~1	0
2	1	98	98/99	1,01	1	~1	0
3	2	96	96/98	2,04	1,42	0,7	1,04
4	2	94	94/96	2,08	1,45	0,7	1,06
A	2	98	98/100	2	1,39	0,7	1,02
B	4	94	94/98	4,08	1,99	0,5	3,13
Σ	6	94	94/100	6	2,36	0,4	5,10

Densities $g(\theta) = \frac{(a+b-1)!}{(a-1)!(b-1)!}\theta^{a-1}(1-\theta)^{b-1}$, $0! = 1$.

Conclusions

1. For practical applications of stochastic defectivity modeling of multiparametric products it is advisable to use beta-densities with offered limitations, since in this case we avoid integration procedures and therefore do not obtain complex models of densities.
2. It is purposeful to use approximation of density $g(\theta)$ by beta-density $g_\sigma(\theta)$ when number of parameters $l < 5$, since when l value increases, the relative approximation error also increases.
3. Grouping of parameters according to their significance into classes (groups) A, B, C is recommended by ISO standards and it enables to highlight the influence of these groups onto the overall product defectivity level; it also enables modeling of situations when one group of parameters is eliminated or additional parameters are introduced.
4. In multiparametric case it is possible to perform the consistent joining of densities of separate parameters in pairs, thus narrowing the analysis down to biparametric models.

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Method is offered for synthesis of stochastic distributions of defectivity levels of multiparametric mechatronics products with interindependent parameters. This synthesis can be performed in groups of parameters or for entire product according to known distributions of defectivity levels of separate parameters. Synthesis models of density functions were presented, when defectivity levels of separate parameters are distributed according to beta law, with minimal whole-number values of one of the shape parameters. Limitations were determined, when defectivity levels are also described by beta-distribution according to parameter groups or for entire product. It was shown, that in such case models is considerably simplified, since multi-integration procedure is no longer needed, and also complex density models are not required any more. It was offered to use approximation by beta-density when any values of beta-distribution shape parameters are present. Although it was noted, that when the number of product parameters increases, approximation precision decreases. For practical applications it is advisable to differentiate average defectivity levels of separate parameters according to selected defectivity level of entire product, when ratio between defectivity levels in separate groups is selected or according to needed dispersion of parameters (selected variation coefficient). Ill. 7, bibl. 12 (In English; summaries in English, Russian and Lithuanian).

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Предложена методика синтеза вероятностных распределений уровня дефектности в отдельных группах параметров и для многопараметрических мехатронных изделий в целом по известным распределениям вероятностей уровней дефектности отдельных независимых параметров. Представлены модели синтеза плотностей вероятностей, когда уровни дефектности отдельных параметров описываются бета-распределением при минимальных целочисленных значениях одного из параметров формы. Определены ограничения, при выполнении которых уровни дефектности в отдельных группах и для изделия в целом также описываются бета-распределением. Показано, что в данном случае существенно упрощается моделирование, так как отпадает необходимость процедуры многократного интегрирования, а также не нужны весьма сложные модели плотностей вероятностей. Предложено при любых значениях параметров формы бета-распределения отдельных параметров для синтеза использовать аппроксимирующие бета-распределения. Для практических приложений рекомендуется средние значения уровней дефектности отдельных параметров дифференцировать согласно заданному уровню дефектности изделия в целом. Ил. 7, библ. 12 (на английском языке; рефераты на английском, русском и литовском яз.).

D. Eidukas, R. Kalnius. Daugiaparametrių mechatroninių gaminių kokybės lygio tikimybiniai modeliai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 3(83). – P. 43 – 48.

Pasiūlyta metodika daugiaparametrinių mechatroninių gaminių su nepriklausomais parametrais defektingumo lygių tikimybių skirstinių sintezei parametrų grupėse ir visam gaminiui pagal žinomų atskirų parametrų defektingumo lygių tikimybių skirstinių. Pateikti tankio funkcijų sintezės modeliai, kai atskirų parametrų defektingumo lygiai pasiskirstę pagal betą dėsnį, esant minimalioms sveikaskaitinėms vieno iš formos parametrų reikšmėms. Nustatyti apribojimai, kada defektingumo lygiai pagal parametrų grupes ir visam gaminiui taip pat aprašomi beta skirstiniu. Parodyta, kad tokiu atveju labai supaprastėja modeliavimas, kadangi nebereikalinga daugkartinio integravimo procedūra, o taip pat nebereikia sudėtingų tankių modelių. Pasiūlyta prie bet kokių beta skirstinio formos parametrų verčių sintezei taikyti aproksimaciją beta tankiu. Tačiau pažymėta, kad, augant gaminio parametrų skaičiui, mažėja aproksimacijos tikslumas. Praktiniams taikymams rekomenduojama atskirų parametrų vidutinius defektingumo lygius išdiferencijuoti pagal reikiamą parametrų išssklaidymą – pasirinktą variacijoskoeficientą. Il. 7, bibl. 12 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).