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Calculation of Parameters of Saccular Aneurysm

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Introduction

Recent investigations [1–3] show that mathematical modelling is the problem of today in medicine.

Aneurysm is an irregular sac formed by the dilatation of the wall of an artery. Parameters and location of aneurysm determine the treatment of a patient [4, 5].

Mathematically simply expressed surface is to be chosen for calculations of aneurysm parameters, i.e. the mathematical model was made. Medical practitioners try to approximate aneurysm to ellipsoid because it is close to shape of aneurysm. Such models are often made from one projection of aneurysm only [6, 7].

In this publication methodology for choosing ellipsoid parameters (center, directions and lengths of axes, volume) which approximates three-dimensional (3D) aneurysm image points (Fig. 1) was suggested. Also, spread of aneurysm points with respect to the ellipsoid surface and deviation of calculated volume were analysed.

Approximation by ellipsoid

The points of aneurysm surface, produced by processing 3D region of aneurysm image, were used as initial data. So, the coordinates and the volume in voxels of each surface point $P_i(x_i, y_i, z_i)$ were known (Fig. 2).

The approximation using ellipsoid was carried out in this sequence:

1. Calculation of mass center $P_c(x_c, y_c, z_c)$;
2. Calculation of the first ellipsoid axis d_1;
3. Projection of all surface points $Pi(x_i, y_i, z_i)$ onto the plain L, which is perpendicular to the first axis d_1;
4. Calculation of the second ellipsoid axis d_2;
5. Projection of points from the plain L to the line, perpendicular to the second axis d_2;
6. Calculation of the third ellipsoid axis d_3.

The center of ellipsoid was calculated as the mass center of all surface points.

Let Ω denote the set of these points:

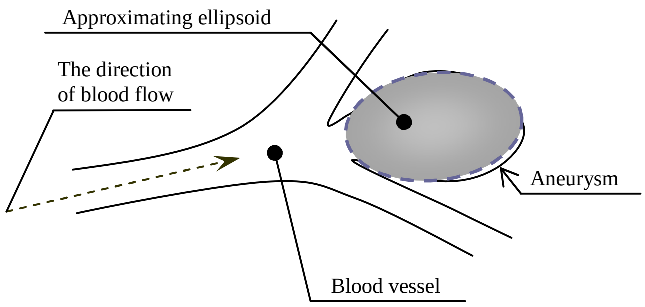

Fig. 1. Approximating aneurysm with ellipsoid

$$\Omega = \{P_i(x_i, y_i, z_i)\}, \tag{1}$$

where P_i – the points of aneurysm surface, x_i, y_i, z_i – the coordinates of these points, i = 1, 2, 3, ... N – current number of the point, N – number of points, used for modelling, where N = 106.

The formula for calculation of the mass center point P_c coordinates was as follows:

$$P_c(x_c, y_c, z_c) = \left(\frac{1}{N}\sum_{i=1}^{N} x_i, \frac{1}{N}\sum_{i=1}^{N} y_i, \frac{1}{N}\sum_{i=1}^{N} z_i \right), \tag{2}$$

where x_c, y_c, z_c – the coordinates of the mass center.

The point P_c was considered as an approximating ellipsoid center.

Further, it was purposeful to check if the points form a sphere. Therefore the Euclidean distance d_i between each point and the center was calculated as follows:

$$d_i = d(P_i, P_c) = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2}\ . \tag{3}$$

Later on the maximal and the minimal distances were chosen; their difference was compared with the chosen criteria ε by the following formula:

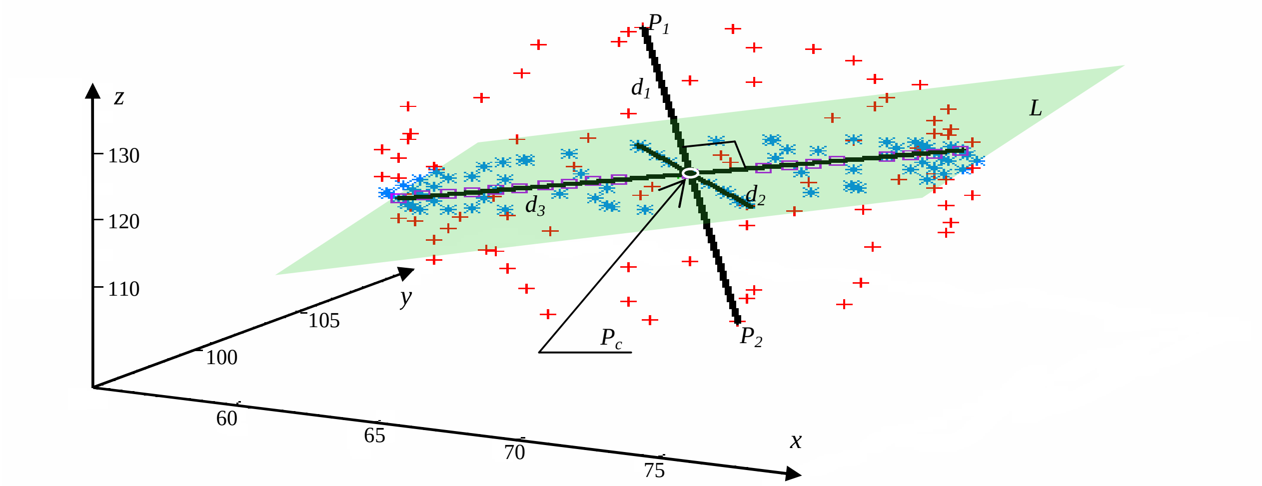

Fig. 2. Approximation of data points. Data points marked with “+”, the first projection marked with “*”, the second projection marked with “□”

$$\left| \max_i d_i - \min_i d_i \right| < \varepsilon \,. \tag{4}$$

If the condition (4) was satisfied, the aneurysm image points were approximated by a sphere. In this case, the volume of sphere was calculated. Then the parameters of the aneurysm were the volume and the center coordinates.

If the condition (4) was not satisfied, the approximation of the aneurysm by sphere was not allowed. Then the approximation was carried out with ellipsoid and the process started from the calculation of the axes.

The main axis of the ellipsoid was a line segment between two furthest points and lied on the center point P_c. The endpoints of the main axis were denoted $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$. The mentioned points were selected from Ω, therefore their coordinates were known. Then the length d_1 of the main axis was calculated as follows:

$$d_1 = d(P_1, P_2) = |P_2 - P_1| = \\ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\,, \tag{5}$$

where x_1, y_1, z_1 – the coordinates of the point P_1 and x_2, y_2, z_2 – the coordinates of the point P_2.

The length of the axis was one of the parameters necessary for the calculation of the ellipsoid volume. The current data describes the following parameters of the aneurysm: the center coordinates describe the spatial position, the main axis describes the orientation in 3D space and the maximal dimension. The orientation was described by the coefficients of the line equation:

$$\frac{x - x_c}{k} = \frac{y - y_c}{l} = \frac{z - z_c}{m}\,, \tag{6}$$

where $k = x_2 - x_1$, $l = y_2 - y_1$, $m = z_2 - z_1$ – the direction coefficients.

Two residual ellipsoid axes were calculated as described below. The equation of the plane L crossing the calculated center and perpendicular to the longest axis was made as follows:

$$kx - ly - mz - C = 0\,, \tag{7}$$

where $C = kx_c - ly_c - mz_c$ – the computable coefficient describing the center.

The second axis of the ellipsoid was obviously onto the plane L and all the current points were projected to this plane. Therefore a line segment perpendicular to the plane L was drawn through each point P_i and the crossing points of the line segment with the plane were searched. The equation of the mentioned line segment was formulated as the following system of equations:

$$\begin{cases} lx - ky = lx_i - ky_i\,, \\ mx - kz = mx_i - kz_i\,, \end{cases} \tag{8}$$

where k, l, m – the direction coefficients, x_i, y_i, z_i – the coordinates of any current point.

The system of equations (8) supplemented with the plane equation (7) forms the system below (9). The solution to the system was the searched projection of the point, calculated for each data point.

$$\begin{cases} lx - ky = A_i\,, \\ mx - kz = B_i\,, \\ kx - ly - mz = C, \end{cases} \tag{9}$$

where $A_i = lx_i - ky_i$, $B_i = mx_i - kz_i$. A_i and B_i – coefficients computed for each data point.

Thus, inserting the coordinates x_n, y_n, z_n of n point $P_n(x_n, y_n, z_n)$ to the system of equations (9), its solution was the coordinates x'_n, y'_n, z'_n of the projection point P'_n. The system (9) was solved according to Kramers formulas. The length of the second axis d_2 was calculated according to the formula (5).

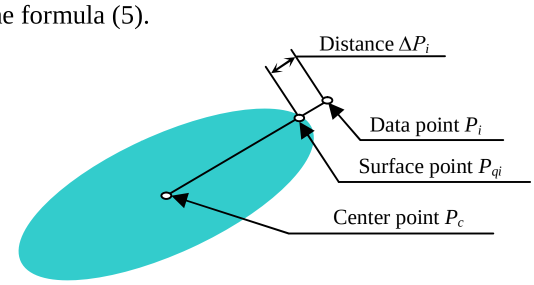

Fig. 3. Calculation of surface spread

The third axis of the ellipsoid d_3 was calculated as follows:

1. The points, previously projected onto plane L, were projected to the line segment perpendicular to the second axis d_2 of the ellipsoid;
2. In that new projection two furthest points were searched. Those points were considered to be the endpoints of the axis and the length was the distance between them.
3. Having calculated the third axis, one more parameter of the aneurysm, i.e. volume $\tilde{V}$, was calculated.

The volume $\tilde{V}$ was calculated according to the following formula:

$$\tilde{V} = \frac{4}{3}\pi \frac{d_1}{2}\frac{d_2}{2}\frac{d_3}{2}. \qquad (10)$$

Thus, the main parameters of the aneurysm were the spatial position of the ellipsoid center P_c (x_i, y_i, z_i) , the lengths of the axis d_1, d_2, d_3 , the directions k, l, m and the volume $\tilde{V}$.

Spread of aneurysm points

Spread of aneurysm points with respect to the chosen ellipsoid model was characterized by volume deviation and spread of the surface points.

The relative deviation e of aneurysm volume:

$$e = \left| \frac{V - \tilde{V}}{V} \right|. \qquad (11)$$

The evaluation of the spread of the surface points was more difficult. In this case the distance ΔP_i from each current point P_i to the ellipsoid surface point P_{qi} (Fig. 3) was calculated.

The distance ΔP_i was calculated by the following sequence:

1. A line segment was drawn through the ellipsoid center P_c crossing the chosen point P_i;
2. The coordinates of the point P_{qi} were calculated, where the mentioned line segment crossed the surface of the ellipsoid;
3. The distance between the surface point P_{qi} and the chosen point P_i was calculated.

The distance ΔP_i was calculated using the following system of equations:

$$\begin{cases} \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} - 1 = 0, \\ \dfrac{y - y_c}{y_i - y_c} = \dfrac{x - x_c}{x_p - x_c}, \\ \dfrac{z - z_c}{z_i - z_c} = \dfrac{x - x_c}{x_p - x_c}, \end{cases} \qquad (12)$$

where x, y, z – the coordinates of the searched point P_{qi}, a, b, c – the semi-axes of the approximating ellipsoid, $a = \frac{d_1}{2}$, $b = \frac{d_2}{2}$, $c = \frac{d_3}{2}$.

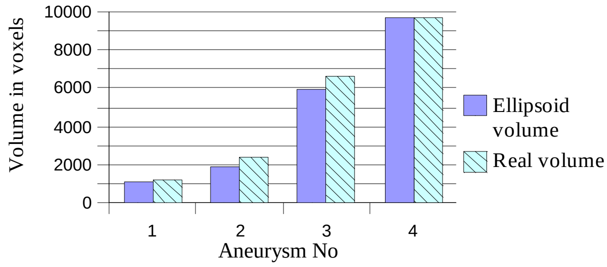

Fig. 4. Volume comparison

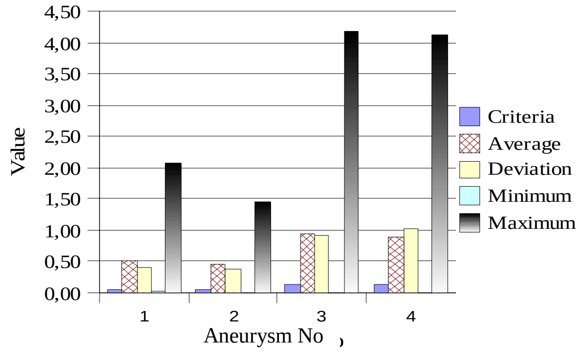

Fig. 5. Surface match criteria and parameters of distance ΔP_i

The first equation of the system (12) was the expression of the ellipsoid when its center and axes corresponded to the center and axes of the coordinates accordingly. The second and the third equations described the line segment crossing the points P_c and P_i. The system (12) was solved by expressing one variable from the second and the third equations and inserting the received expression into the first equation. Having solved the first equation with respect to the expressed variable, two solutions were received, i.e. two points where the line segment crossed the ellipsoid. The point P_{qi} that was closer to the data point P_i was chosen from the two points. The distance ΔP_i was calculated according to the formula (3).

Having calculated all distances to evaluate the spread of surface points, the Euclidean norm of the mismatch vector $\vec{r}_i$ was calculated:

$$\|\vec{r}\|_2 = \sqrt{\sum_{i=1}^{N} d^2(P_{qi}, P_i)}. \qquad (13)$$

The mismatch criteria of the surface was as follows:

$$\frac{1}{N}\|\vec{r}\|_2 < \tau , \qquad (14)$$

where τ – tolerance threshold.

If the condition (14) was satisfied, it was considered that the approximation of aneurysm surface points by ellipsoid was sufficiently good and suitable for further appliances.

Results

The aneurysm model in four different resolutions was investigated. The largest aneurysm number in the fourth and fifth figures corresponds to the largest aneurysm resolution.

Having compared the volumes of the approximating ellipsoids (Fig. 4) it is seen that in most cases the volume of the ellipsoid is smaller than that of the aneurysm, however, the deviation of the volume does not exceed 20 %.

Increasing volume of modelling aneurysm, the spread of surface points increases (Fig. 5). It is seen, that in larger resolutions (and with larger aneurysms) the distance ΔP_i deviation increases. The distance ΔP_i reaches maximum value 4 in two largest resolutions.

Conclusions

1. The ellipsoid is a suitable model for the human‘s intracranial aneurysm.
2. The position and the size of the aneurysm in human‘s brain are described by the following ellipsoid parameters: the coordinates of the center, the directions and lengths of the axes and the volume.
3. The deviation of the calculated volume does not exceed -20 % than that of real aneurysm.
4. When the volume of the modelling aneurysm is increasing, the deviation of the volume is decreasing. However, the spread of the surface points is increasing, due to the complex form of the aneurysm.

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The methodology of modelling 3D aneurysm image points was suggested in the article. The ellipsoid was used for the modelling of human‘s saccular aneurysm. The spread of aneurysm image points was analysed with respect to the chosen ellipsoid surface as well as the deviation of the calculated volume. The position and the size of the aneurysm in human‘s brain were described by the mass center point, the directions and the length of the ellipsoid axes as well as the ellipsoid volume. The mass center point was calculated as the mass center of aneurysm image points. The end points of the axes were found by choosing two furthest points of the aneurysm. The volume was calculated using the axes lengths of the ellipsoid. 3D aneurysm model at four different resolutions was used in this research. All the modelling stages were realised using programming language *C*. Ill. 5, bibl. 7 (in English; summaries in English, Russian and Lithuanian).

Л. Моцкус, Р. Мартавичюс, А. Ушинскас, М. Мейлунас. Расчет параметров мешкообразной аневризмы // Электроника и електротехника. – Каунас: Технология, 2008. – № 3(83). – С. 15–18.

Предложена методика моделирования совокупности точек изображения аневризмы. Для моделирования мешкообразной аневризмы применен эллипсоид, параметры которого приравниваются параметрам аневризмы. Проанализировано рассеивание точек изображения аневризмы относительно поверхности моделирующего эллипсоида и рассчитано отклонение объема. Положение и размер аневризмы в мозге человека характеризуют координаты центра эллипсоида, направление и длина его осей, а также объем эллипсоида. Координаты центра эллипсоида определяются как центр тяжести точек изображения аневризмы. Точки концов осей определяются путем выбора двух наиболее удаленных точек аневризмы. Объем аневризмы рассчитывается, используя длину осей. Исследования трёхмерной модели проводились, используя четыре различные резолюции. Все этапы моделирования реализованы на языке программирования *C*. Ил. 5, библ. 7 (на английском; рефераты на английском, русском и литовском яз.).

L. Mockus, R. Martavičius, A. Ušinskas, M. Meilūnas. Maišelinės aneurizmos parametrų skaičiavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 3(83). – P. 15–18.

Siūloma trimačio aneurizmos vaizdo taškų modelio sudarymo metodika. Žmogaus galvos kraujagyslių aneurizmai modeliuoti panaudotas elipsoidas, kurio parametrai prilyginami aneurizmos parametrams. Analizuojama aneurizmos vaizdo taškų sklaida pasirinkto elipsoido paviršiaus atžvilgiu ir apskaičiuoto tūrio nuokrypis. Aneurizmos padėtį žmogaus smegenyse ir jos dydį apibūdina aproksimuojančio elipsoido centro koordinatės, ašių kryptys, ašių ilgiai bei elipsoido tūris. Masės centro taškas apskaičiuojamas kaip aneurizmos vaizdo taškų masės centras. Ašių galai randami išrenkant du labiausiai nutolusius aneurizmos taškus. Tūris apskaičiuojamas iš rastų elipsoido ašių ilgių. Tyrime panaudotas trimatis aneurizmos modelis esant keturioms skirtingoms raiškoms. Visi modeliavimo etapai atlikti *C* programavimo kalba. Il. 5, bibl. 7 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).