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Primary Noise Reduction Efficiency for Detecting Chaos in High Noisy Pseudoperiodic Time Series

K. Pukėnas

Department of Sports Biomechanics, Information Science and Engineering, Lithuanian Academy of Physical Education, Sporto str. 6, LT-44221, Kaunas, Lithuania, phone: +370 37 302668; e-mail: k.pukenas@lkka.lt

Introduction

A long-standing fundamental issue in nonlinear time series analysis is to determine whether a complex time series is regular, deterministically chaotic, or random. An accurate identification of the dynamics underlying a complex time series, is of crucial importance in understanding the corresponding physical process, and in turn affects the subsequent model development. A steady stream of efforts has been made, and a number of effective methods have been proposed (also in the latest years – [1]-[3]) to tackle this difficult problem. The vast majority of these methods are based on attractor reconstruction from time series and such characteristics as largest Lyapunov exponents, K2 entropy, and correlation dimension calculation [4], [5]. Since the analysis of chaotic data in terms of dimensions, entropies, and Lyapunov exponents requires access to the small length scales (small-scale fluctuations of the signal), already a moderate amount of measurement noise on data is known to be destructive. One class of time series – pseudoperiodic – has aroused great interest due to their close relation to some important natural and physiological systems. Zhang *et all* [6] have proposed a method to detect deterministic structure from this certain class of chaotic time series, which can deal with small or moderate amounts of noise. But I have not found any publication, devoted to detecting the deterministic structure from a high noisy pseudoperiodic time series, when the noise level reducing is desirable with expected to preserve the exponential divergence of nearest neighbors. Noise reduction methods designed for signals that can be treated by a linear model fail to eliminate noise from a contaminated chaotic time series because the spectra of the chaotic signal and the noise overlap. Noise reduction based on time delay embedding, which has been widely studied, may be the most promising way to filter the noisy chaotic data [8]-[10]. Several phase space projection methods, based on subspace decomposition, were proposed for application to the problem of additive noise reduction in the context of phase space analysis – the global projections method [10] and the local (nearest neighborhoods) phase spaces method [7], [9], [10]. A two step method is proposed to reduce colored noise [11]. These methods performed well with moderate amounts of noise. Hovewer, in order to distinguish between regular and non-regular dynamics of time series, which exhibit pseudoperiodic behaviour, it is not necessary to eliminate the noise perfectly. Importantly, that signal distortion and noise residual on noise reduction would enable to detect the presence of chaos in a dynamical system by measuring the largest Lyapunov exponents or characteristics, related with the largest Lyapunov exponents.

The aim of present paper – to establish, which method fits best for a primary noise reduction of high noisy pseudoperiodic time series and evaluate primary noise reduction efficiency for detecting chaos in these time series. Also the straightforward and relative noisy resistant algorithm to detect chaos in pseudoperiodic time series by using the correlation coefficient as a measure of the distance [6] between vectors of reconstructed phase space is present. Proposed algorithm is principally based on the algorithm of Rosenstein [12] for largest Lyapunov exponent's calculation, but by over-embedding and an appropriately longer embedding window and by using the correlation coefficient as a measure of the distance instead of Euclidean distance. Throughout the paper, the *x* component of the well-known Rossler system for illustration, which is chaotic and contain obvious periodic component, is used.

The organization of this paper is as follows. In Sec. II, the principle of noise reduction for chaotic data in the global and local phase space is reviewed, and algorithm for detecting chaos in pseudoperiodic time series is described. In Sec. III, the results of calculations are given. Finally, some discussions and conclusions are given in Sec. IV.

Phase space projection methods of noise reduction for chaotic data

Let $z_n = s_n + w_n$ denote the time series contaminated by noise, where s_n is the clean data generated by a dynamical system and w_n is the additive noise. For a time series $\{z_n\}_{n=1}^{L}$ with L samples, the phase points can be reconstructed by time delay embedding [4], [5], i.e., $\{\mathbf{z}_n\}_{n=1}^{L-(d-1)\tau}$ and a reconstructed phase space (RPS) matrix $\mathbf{Z}$ with d rows and $L-(d-1)\tau$ columns (called a trajectory matrix) is defined by

$$\mathbf{Z} = \begin{bmatrix} z_1 & z_2 & \cdots & z_{L-(d-1)\tau} \\ z_{1+\tau} & z_{2+\tau} & \cdots & z_{L-(d-2)\tau} \\ \vdots & & \ddots & \\ z_{1+(d-1)\tau} & z_{2+(d-1)\tau} & & z_L \end{bmatrix}, \quad (1)$$

where d – the embedding dimension and τ – time delay. By applying the Principal Components Analysis (PCA), also known as Singular Systems Analysis per Broomhead and King [10], on the trajectory matrix, a projected trajectory matrix is computed via the equation:

$$\hat{\mathbf{Z}} = \mathbf{U}_1 \cdot \mathbf{U}_1^T \cdot \left(\mathbf{Z} - \overline{\mathbf{Z}}\right) + \overline{\mathbf{Z}}, \quad (2)$$

where $\mathbf{U}_1$ consists of the columns of the eigenvector matrix of the trajectory covariance $\mathbf{U}$ such that the corresponding singular values are greater than the noise level threshold, $\overline{\mathbf{Z}}$ is the mean over dimension $1,\cdots,d$ and $(\cdot)^T$ denotes the transpose of a real matrix.

The result is that the original attractor is projected onto the principal eigenvectors of the space. To implement this approach for noise reduction, the original time series is over-embedded, i.e. embedded into a dimension well over that required for attractor representation. As each element of the time series $\{z_n\}_{n=1}^{L}$ occurs as an entry of one of d successive phase vectors $\mathbf{z}_k$, $k = n-(d-1)\tau,\cdots,n$, there are d enhanced entries which may be different in values. An enhanced one-dimensional signal $\hat{z}_n$ is created from the new space, typically by time-aligning and weighted averaging the columns of the new trajectory matrix $\hat{\mathbf{Z}}$ [10], i. e.

$$\hat{\mathbf{Z}}_{al} = \begin{bmatrix} \hat{z}_1 & \cdots & \hat{z}_{1+(d-1)\tau} & \cdots & \hat{z}_{L-(d-1)\tau} & \\ & \cdots & \hat{z}_{1+(d-1)\tau} & \cdots & \hat{z}_{L-(d-1)\tau} & \cdots \\ & \ddots & \vdots & \ddots & \vdots & \ddots \\ & & \hat{z}_{1+(d-1)\tau} & \cdots & \hat{z}_{L-(d-1)\tau} & \hat{z}_L \end{bmatrix}. \quad (3)$$

The concept of projection within the reconstructed phase space can be easily adapted to apply to local neighborhoods within the space [9]-[11]. For these approaches, the time series is over-embedded as with the global method, and then each point in the space is individually transformed using a projection based only on its local neighborhood region. The near neighborhood of the reference point $\mathbf{z}_n$ is defined as

$$\mathbf{N}_n \overset{\Delta}{=} \left\{\mathbf{z}_k : \left\|\mathbf{z}_k - \mathbf{z}_n\right\| < \boldsymbol{\varepsilon}, \quad 1 \le k \le m\right\}, \quad (4)$$

where $\boldsymbol{\varepsilon}$ – the size of the neighborhood, $m = L-(d-1)\tau$.

Similar to global phase space, the local projection (LP) method assumes that the local phase space, i.e., the neighborhood $\mathbf{N}_n$ of the reference point $\mathbf{z}_n$, can be divided into an M-dimensional signal subspace and a (d–M)-dimensional white noise subspace, where M is the minimum embedding dimension of the dynamical system. For a preset M, the standard eigenvalue decomposition for the covariance matrix of the windowed neighborhood data matrix $\mathbf{N}_n$ is performed, i.e.,

$$\mathbf{C}_n \cdot \mathbf{u}_i - \boldsymbol{\lambda}_i \cdot \mathbf{u}_i = 0. \quad (5)$$

The matrix $\mathbf{C}_n$ is defined as $\mathbf{C}_n = \frac{1}{N}\sum_{z_k \in N_n} \mathbf{X}_k \cdot \mathbf{X}_k^T$ with notation $\mathbf{x}_k = \mathbf{z}_k - \overline{\mathbf{z}}_n$, where $\overline{\mathbf{z}}_n$ – the center of the neighborhood, i.e., $\overline{\mathbf{z}}_n = \frac{1}{N}\sum_{z_k \in N_n} \mathbf{z}_k$, and N – the number of neighbors in $\mathbf{N}_n$. Sorting the eigenvalues $\boldsymbol{\Lambda} = diag(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \cdots, \boldsymbol{\lambda}_d)$ in descending order, the eigenvectors $\mathbf{U}_1 = [\mathbf{u}_1, \cdots, \mathbf{u}_M]$, associated with the M largest eigenvalues, span the signal subspace, and the eigenvectors $\mathbf{U}_2 = [\mathbf{u}_{M+1}, \cdots, \mathbf{u}_d]$, corresponding to the $(d-M)$ smallest eigenvalues, span the noise subspace, respectively. Then the phase vector $\mathbf{z}_n$ can be decomposed as

$$\mathbf{z}_n = \overline{\mathbf{z}}_n + \mathbf{U}_1 \cdot \mathbf{U}_1^T\left(\mathbf{z}_n - \overline{\mathbf{z}}_n\right) + \mathbf{U}_2 \cdot \mathbf{U}_2^T\left(\mathbf{z}_n - \overline{\mathbf{z}}_n\right) \quad (6)$$

in the local phase space, where $\mathbf{U}_1 \cdot \mathbf{U}_1^T\left(\mathbf{z}_n - \overline{\mathbf{z}}_n\right)$, $\mathbf{U}_2 \cdot \mathbf{U}_2^T\left(\mathbf{z}_n - \overline{\mathbf{z}}_n\right)$ – the projections of $\left(\mathbf{z}_n - \overline{\mathbf{z}}_n\right)$ in the signal subspace and the noise subspace, respectively.

Eliminating $\mathbf{U}_2 \cdot \mathbf{U}_2^T\left(\mathbf{z}_n - \overline{\mathbf{z}}_n\right)$, we obtain the enhanced signal vector

$$\hat{\mathbf{z}}_n = \overline{\mathbf{z}}_n + \mathbf{U}_1 \cdot \mathbf{U}_1^T\left(\mathbf{z}_n - \overline{\mathbf{z}}_n\right). \quad (7)$$

As each element of the time series $\{z_n\}_{n=1}^{L}$ occurs as an entry of one of d successive phase vectors $\mathbf{z}_k$, $k = n-(d-1)\tau,\cdots,n$, there are d enhanced entries which may be different in values. The arithmetic weighted mean over these values is taken as the enhanced element $\hat{s}_n$. More details about the phase projection methods can be found in Refs. [9]-[10].

Usually, neighborhoods merge if all data are contaminated by large amounts of noise. Thus, it becomes a nontrivial problem to identify the correct neighbors. In some degree the high dimension of the embedding space helps to identify neighbors also for rather high noise levels. Furthermore, the correlation coefficient as a measure of the distance between certain vectors of reconstructed phase space instead of Euclidean distance is used in both cases – by prefiltering and diverging slope between neighbors vectors calculation. This approach for pseudoperiodic signal is proposed by Zhang *et al* [6], although the execution of the idea in this paper is different. Differently from method proposed by Zhang *et al*, where the pseudoperiodic time series are segmented into consecutive (no overlapping) cycles according to the local minimum (or maximum) without embedding, I have applied this measure for phase-space vectors of reconstructed dynamics.

The exponential divergence of initially close state-space trajectories, as an indicator of chaos, is calculated according slightly modified Rosenstein method for largest Lyapunov exponent's calculation [12]:

1) For a given vector $\mathbf{z}_i$ with time delay $\tau = 1$ (defined approximately as one period T_p of pseudoperiodic time series) of reconstructed phase space matrix (1) with d rows and m colums, $i = 1, 2, \cdots, m/2 - T_p - T_d$ (T_d is explained below) the correlation coefficient ρ_{ij} as the distance between each pair of vectors $\mathbf{z}_i$ and $\mathbf{z}_j$ for $|j - i| \geq T_p$ is calculated. The constraint $|j - i| \geq T_p$ is necessary to exclude temporally correlated points. The correlation coefficient characterizes the similarity between vectors $\mathbf{z}_i$ and $\mathbf{z}_j$. Considering the continuity and smoothness of the vector fields of deterministic systems, two vectors with a larger ρ_{ij} will also be close in the phase space, i. e., for the relation between vectors describing, the correlation coefficient can be used equivalently as the phase-space distance [6].
2) The search of the most similar vectors is executed using a sliding overlapping window of constant length $T = m/2$ for all i. For a given i the values of lag j changes from 1 to $m/2$.
3) The algorithm locates most similar ij^{th} pair of vectors (with maximum correlation coefficient ρ_{mi}) of each point i. Like Rosenstein algorithm [12], the averaged divergence $\rho_m(k)$ between two nearby vectors at time steps k ($k = 1, 2, \cdots, T_d$) is calculated

$$\rho_m(k) = \frac{1}{\Delta t}\left\langle \ln \rho_{m_i}(k) \right\rangle, \qquad (8)$$

where $\langle \cdots \rangle$ denotes the average over all values of i, Δt – the sampling period of the time series.
4) For chaotic systems, the distance between two nearby vectors will increase exponentially over time due to the very nature of sensitivity to initial conditions. Therefore, the correlation between two nearby vectors, which decreases smoothly and monotonously with the distance between vectors, is also expected to drop exponentially with the step k [6]. The semilogarithmic plot $\ln(\rho_m(k)) \sim k$ (or versus time $t = k \cdot \Delta t$) thus appears to be a line nearby straight, whose slope is actually related to the largest Lyapunov exponent. The larger the $|\Delta \ln \rho_m(k)/\Delta k|$, the higher the level of chaos. Since $\rho_m(k)$ is close to 1, $\ln(\rho_m(k)) \approx \rho_m(k) - 1$ and we can estimate slope as $|\Delta \rho_m(k)/\Delta k|$. The curve saturates at longer times since the system is bounded in phase space and the average divergence cannot exceed the "length" of the attractor (in sense of the Euclidean distance, since the absolute value of correlation coefficient cannot be less than zero) and T_d is defined normally only for the slope region.

Numerical Results

Both prefiltering methods and diverging slope calculating approach are applied to time series measured from the Rossler system, which is defined by

$$\begin{cases} \dfrac{dx}{dt} = -(y + z), \\ \dfrac{dy}{dt} = x + a \cdot y, \\ \dfrac{dz}{dt} = b + z(x - c) \end{cases} \qquad (9)$$

with parameters a = 0.2, b = 0.2 and c = 5.7. It has been argued that the LP method can obtain better results by over-embedding with time delay $\tau = 1$ and an appropriately longer embedding window [9]-[11]. While the embedding window cannot be set too long, otherwise there are not enough appropriate neighbors for the reference phase point (here the appropriate neighbors mean that the wave forms of the data segments covered by the neighbors are well matched that of the reference phase point). Thus a tradeoff of the length of the embedding window should be made [11]. In this paper, I have set the initial embedding dimension $d = 60$ and $\tau = 1$ and the first 10 nearest neighbors were used for each reference phase point. The results were obtained with relative small data sets – the series length for Rossler system is 1600 points.

The final projection dimensions for noise-reduction algorithms are 2 dominant dimensions. Also a single filtering iteration was performed.

The performance of both prefiltering methods at various level of additive white Gaussian noise, i.e., measurement or instrumentation noise, was investigated. The divergence slope was calculated by using newly created embedding vectors from enhanced one-dimensional signal $\hat{z}_n$. Fig. 1 shows a plot of $\langle \rho_{mi}(k) \rangle$ versus k (in each figure "<Divergence>" and "Iteration" are used to denote $\langle \rho_{mi}(k) \rangle$ and k, respectively) for the x component of the Rossler system with additive white Gaussian noise of different levels after primary reduction of noise.

Fig. 2 shows a plot of $\langle \rho_{mi}(k) \rangle$ versus k for the regular sinusoidal signal with additive white Gaussian noise of different levels after primary reduction of noise.

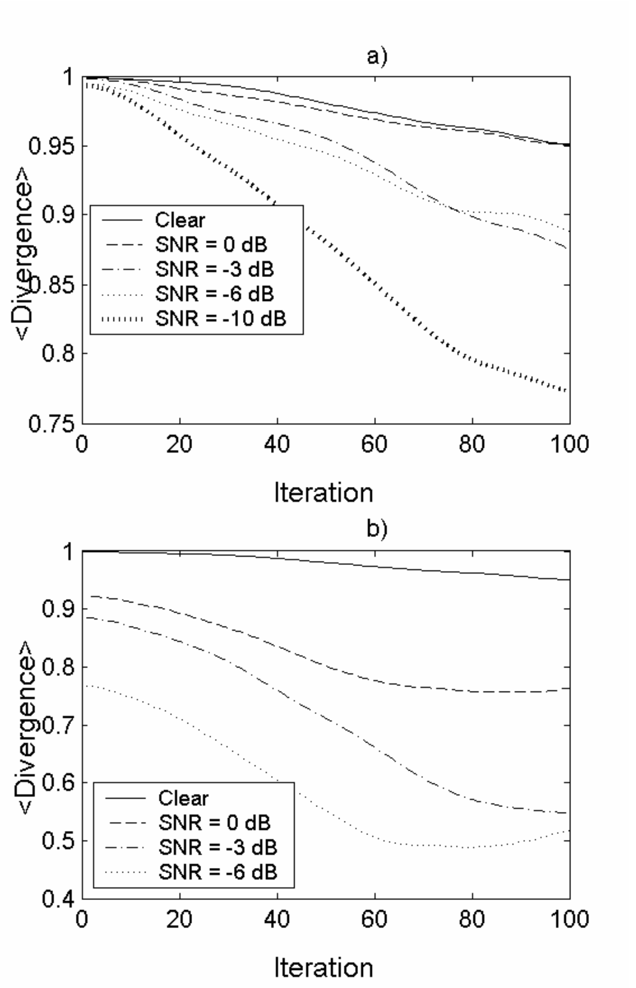

Fig. 1. Plot of <Divergence> versus iteration for the x component of the Rossler system (series length 1600) with additive Gaussian noise of different levels after primary noise reduction; a – with global space projection method, b – with local space projection method

In Fig.1, we can see that more preferable is global space algorithm. After prefiltering the truly chaotic noisy pseudoperiodic Rossler signal with global space algorithm the slope of nearby straight lines indicates deterministically chaotic behavior for signal-noise ratio (SNR) up to –10 dB. With SNR decreasing the divergence lines remain nearby straight, although the negative slope of lines increases. Whereas, after prefiltering the truly chaotic noisy pseudoperiodic Rossler signal with local space algorithm the divergence lines lose straight shape at SNR near to zero.

But in Fig. 2, we can see that for a periodic sinusoidal signal with noise the curves of $\langle \rho_{mi}(k) \rangle$ versus k remain relatively flat only for SNR up to 6 dB after prefiltering the noisy sinusoidal signal with global space algorithm.

With lower SNR the negative slopes $|\Delta\rho_m(k)/\Delta k|$ are larger and the regular sinusoidal signal due to low frequency distortion of amplitude on filtering is wrongly detected as chaotic. After prefiltering the noisy periodic sinusoidal signal with local space algorithm, the divergence lines indicate wrongly deterministically chaotic behavior at similar SNR.

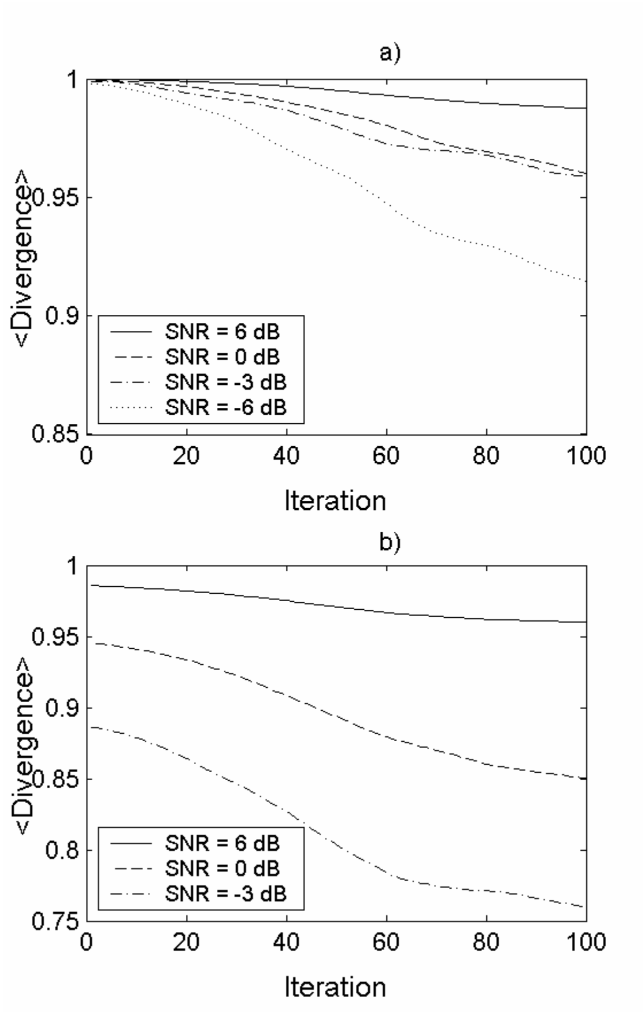

Fig. 2. Plot of <Divergence> versus iteration for the regular sinusoidal signal (series length 1600) with additive Gaussian noise of different levels after primary noise reduction; a – with global space projection method, b – with local space projection method

Discussion and conclusion

The obtained results allow to conclude, that noise reduction, based on phase space projection methods (the global projection method and local neighborhood projection method), marginally enlarges the possibility to distinguish chaos from regular signal in noisy pseudoperiodic time series by further applying algorithms for largest Lyapunov exponent's calculation or similar methods, which track the exponential divergence of nearest neighbors, for enhanced signal. The noise resistant algorithm for chaos detecting described above, based on to calculating the averaged divergence $\rho_m(k)$ at time steps k between two nearby vectors of reconstructed phase space, performed reasonably well for SNR up to 10 dB without prefiltering for time series data exhibiting strong pseudoperiodic behavior. For the noisy chaotic time series with SNR up to 10 dB, correlation coefficient $\langle \rho_{mi}(k) \rangle$ will decrease with k, and a scaling region is present in the plot of $\langle \ln \rho_{mi}(k) \rangle \sim k$ or $\langle \rho_{mi}(k) \rangle \sim k$. The clear presence of a negative slope given the qualitative confirmation of a exponential divergence of initially close state-space trajectories and systems chaotic behavior. While for contaminated with noise periodic signals for SNR up to 10 dB, there are no such relations (Fig. 3).

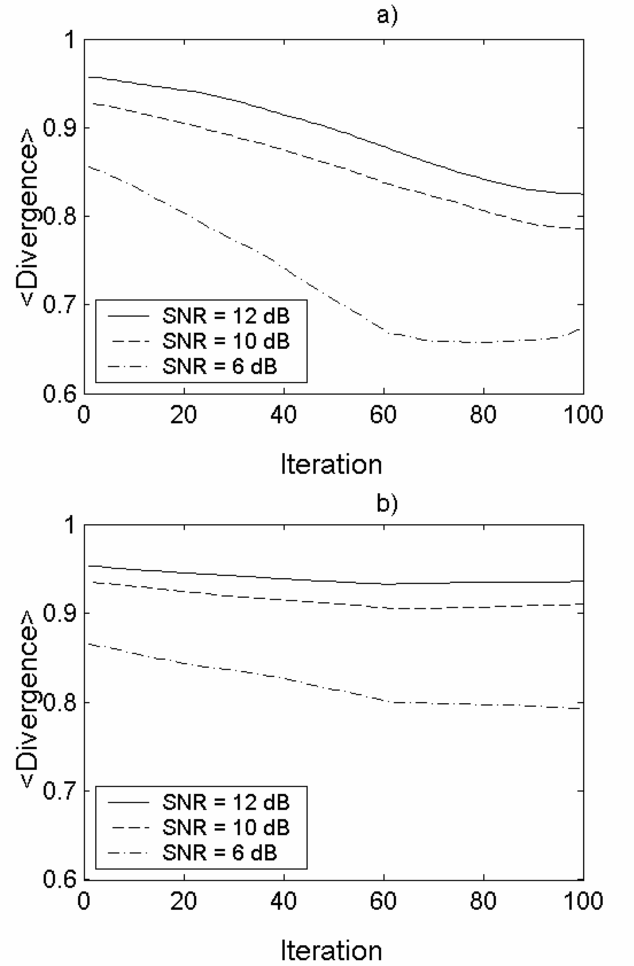

Fig. 3. Plot of <Divergence> versus iteration by applying algorithm for chaos detecting without primary noise reduction; a – for the x component of the Rossler system (series length 1600) with additive Gaussian noise of different levels, b – for the regular sinusoidal signal with additive Gaussian noise of different levels

That is, on average the nearest neighbors should neither diverge nor converge. Multiple averaging of distance measuring (correlation coefficient) between two nearby high-dimensional vectors (the original time series is over-embedded) allows reducing noticeably the influence of random high-dimensional noise. The traditional universal algorithms for calculating largest Lyapunov exponents [12], [13] cannot reliably estimate the largest Lyapunov exponents at noise level about SNR = 10 dB.

The overall error on filtering is given by a combination of the signal distortion and noise residual. Although by applying global space projection method the signal is cleared from high-frequency noise (by applying local space projection method the level of noise is still high after first iteration), however high noisy signal is distorted. Truly chaotic behavior of pseudoperiodic signal is masked and the regular sinusoidal signal is wrongly detected as chaotic by applying described algorithm for chaos detecting.

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K. Pukėnas. Primary Noise Reduction Efficiency for Detecting Chaos in High Noisy Pseudoperiodic Time Series // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 2(82). – P. 83–88.

The deterministic structure from a high noisy pseudoperiodic time series is detecting by using the different noise reduction methods based on principal component analysis for a primary noise reduction and algorithm for detecting chaos is described. The correlation coefficient as a measure of the distance between closest embedding vectors is used and averaged diverging slope, as an indicator of chaos, is calculated according modifying well known Rosenstein method for largest Lyapunov exponents calculating. It is concluded, that the global phase space singular value decomposition method gives better results for pseudoperiodic signal with high level of additive white noise (the noise is comparable with signals) than local phase space projection method. But commonly the prefiltering of high noisy pseudoperiodic time series gives limited advantage – by using global phase space method for noisy signal prefiltering we can reliably distinguish chaos from regular sinusoidal signal in the presence of additive white noise at a signal-noise ratio up to 6 dB. The proposed algorithm for chaos detecting performed reasonably well for SNR up to 10 dB without prefiltering for time series data exhibiting strong pseudoperiodic behavior. Ill 3, bibl. 13 (in English; summaries in English, Russian and Lithuanian).

К. Пукенас. Эффективность первичной фильтрации при обнаружении хаоса в псевдопериодических временных рядах при наличии шумов высокого уровня // Электроника и электротехника. – Каунас: Технология, 2008. – № 2(82). – С. 83–88.

Исследована возможность обнаружения детерминистического начала в псевдопериодических временных рядах с высоким уровнем шумов при использовании в качестве первичной фильтрации методов, основанных на анализе принципиальных компонент, описан простой алгоритм для обнаружения хаоса. В качестве меры дистанции между определенными векторами реконструированного фазового пространства применен коэффициент корреляции, на основании которого рассчитывается усредненная наклонная дивергенции между ближайшими векторами, являющаяся индикатором детерминистического хаоса. Усредненная зависимость расхождения между ближайшими векторами от времени рассчитывается по методу Розенштейна для определения максимальной экспоненты Ляпунова. Показано, что метод, основан на декомпозиции сингулярного значения в глобальном фазовом пространстве, является более предпочтительным для первичной фильтрации псевдопериодических временных рядов с высоким уровнем белого шума (сравнимого с сигналом) чем метод, основан на проекции в локальном фазовом пространстве. Но в общем случае, первичная фильтрация псевдопериодических временных рядах с высоким уровнем шумов дает ограниченное преимущество – при использовании метода, основанного на декомпозиции сингулярного значения в глобальном фазовом пространстве, можно обнаруживать хаотическую природу псевдопериодических временных рядов при наличии белого шума при отношении сигнал–шум выше 6 дБ. В то время предлагаемый алгоритм способен детектировать хаос при отношении сигнал-шум выше 10 дБ без первичной фильтрации Ил. 3, библ. 13 (на английском языке; рефераты на английском, русском и литовском яз.).

K. Pukėnas. Pirminio filtravimo efektyvumas deterministinio chaoso detekcijai pseudoperiodinėse laiko eilutėse, esant aukšto lygio triukšmams // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 2(82). – P. 83–88.

Tiriama deterministinio chaoso detekcija pseudoperiodinėse laiko eilutėse esant aukšto lygio triukšmams pirminei filtracijai naudojant metodus, pagrįstus esminių komponenčių analize, aprašomas chaoso detekcijos algoritmas. Naudojant koreliacijos koeficientą, kaip distancijos tarp tam tikrų rekonstruotos fazinės ervės vektorių matą, apskaičiuojama suvidurkinta divergencijos tarp artimiausių vektorių priklausomybės nuo laiko kreivė, kurios nuožulnumas yra deterministinio chaoso indikatorius. Suvidurkintai divergencijai tarp artimiausių vektorių skaičiuoti panaudotas Rozenšteino metodas didžiausiajai Liapunovo eksponentei apskaičiuoti. Straipsnyje parodoma, kad pirminei pseudoperiodinių laiko eilučių filtracijai esant aukšto lygio baltam triukšmui, labiau tinkamas metodas, pagrįstas singuliariųjų reikšmių dekompozicija globalinėje fazinėje erdvėje, negu algoritmas, pagrįstas projekcija lokalinėje fazinėje erdvėje. Tačiau apskritai pirminės pseudoperiodinių laiko eilučių esant aukštam triukšmų lygiui filtracijos taikymas duoda ribotą efektą – pirminei filtracijai taikant metodą singuliariųjų reikšmių dekompozicijos globalinėje fazinėje erdvėje pagrindu galima detektuoti chaosą pseudoperiodinėse laiko eilutėse, kai signalo ir balto triukšmo santykis viršija 6 dB. Tuo tarpu straipsnyje aprašomas algoritmas užtikrina chaoso detekciją be pirminės filtracijos, kai signalo ir triukšmo santykis viršija 10 dB. Il. 3, bibl. 13 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).