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Statistical Method Correction Possibilities

V. Plociņš
Institute of Electronics and Computer Science,
Dzerbenes st.14, Riga LV-1006, Latvia, phone +371-7-558115; e-mail krumins@edi.lv

The statistical method for discrete stroboscopic transform of signals was published in article [1]. In article [2], a correction for reduction of the systematic error of the method, which emerges in circumstances of a small number of samples, was found. This research is devoted to further improvement of signal transform precision.

Introduction

The principle of the statistical method is the following. Let us assume that with normally distributed noise, the momentary value of the masked signal at moment t_i is u_i. Due to the masking activity of the normally distributed grain noise of the stroboscopic converter's input level, actually the following value is observed:

$$U_i = u_i + \mathrm{X}, \tag{1}$$

where X is a normally distributed random value with a mean

$$E\mathrm{X} = 0 \tag{2}$$

and a standard deviation

$$DX = \sigma^2, \tag{3}$$

where σ is a constant value known for the specific device.

By performing standardization, further in the research we can assume that $\sigma = 1$. In accordance with the statistical method, at phase point t_i the value of the signal masked with noise is compared n times with a known threshold e_i. If, out of n times of comparison, U_i exceeds the threshold e_i n^+ times, then an estimation of the signal's momentary value is calculated

$$\hat{U}_i = \sigma\Phi^{-1}(\hat{P}_n) + e_i, \tag{4}$$

where Φ is the standard normal distribution function; Φ^{-1} – its inverse function; $\hat{P}_n = \dfrac{n^+ + \lambda(n^+)}{n}$ – estimation of the probability of exceeding the threshold; n^+ – the number of cases of exceeding the threshold; $\lambda(0) = 1$, $\lambda(n) = -1$ and $\lambda(n^+) = 0$ in all other cases. After calculation of $\hat{U}_i$, the next value of the threshold is set

$$e_{i+1} = e_i + \hat{U}_i \tag{5}$$

and, in an analogous way, the signal momentary value at the next phase point t_{i+1} is calculated. In case of weak (i.e. such signals, the amplitude of which is $A_1 \le \sigma$) and, furthermore, centred signals, a constant threshold equalling 0 may be used.

To speed up the locator, it is advisable to divide its mode into 2 stages: signal detection and exact registration modes. When the signal has been detected by using a small number of samples, for its exact registration the scanning can be repeated a sufficient number of times and averaging can be performed, until the necessary signal-noise ratio is obtained. However, it turns out that, with a small number of samples, after averaging we obtain signal estimation with a systematic error. This phenomenon was given the name A2 paradox [2]. The goal of this research is to reduce the error caused by the paradox.

With the help of parameter

$$p_i = P\{U_i > e_i\} = P\left\{\frac{X}{\sigma} > \frac{e_i - u_i}{\sigma}\right\} \tag{6}$$

we shall determine the random value Y_i, which assumes a value of 1 with a probability p_i and a value of 0 – with a probability $1 - p_i$. Then the value n^+ can be given as the sum of n independent random values distributed equally with Y_i. Then n^+ shall be the value of a binomially distributed random number with parameters n and p_i. Denoting this random value with X_i^n, we can write that

$$\pi_n(n^+) = P\left\{X_i^n = n^+\right\} = C_n^{n+} p_i^{n+} (1 - p_i)^{n-n^+}, \tag{7}$$

$$p_i = 1 - \Phi\left(\frac{e_i - u_i}{\sigma}\right) = \Phi\left(\frac{u_i - e_i}{\sigma}\right). \tag{8}$$

In our case, where $e_i = 0$ and $\sigma = 1$, we can write

$$\pi_n(j) = C_n^j \Phi(u_i)^j (1-\Phi(u_i))^{n-j} . \qquad (9)$$

As shown in [3], the average value of the measuring result of the corresponding A_1 is

$$\bar{A}_2 = \sum_{j=0}^{n} \pi_n(j) \Phi^{-1}(\frac{j+\lambda(j)}{n}) . \qquad (10)$$

After that, we shall change $\lambda(n^+)$ so as to reduce the signal estimation error. By introducing the denomination

$$\theta(j) = \Phi^{-1}(\frac{j+\lambda(j)}{n}) , \qquad (11)$$

we can write the average value of the modification result amplitude as

$$\bar{A}_2 = \sum_{j=0}^{n} \pi_n(j)\theta(j) . \qquad (12)$$

In this research, we shall use a superbroadband radiolocation signal model as a signal example – harmonic mono-oscillation with amplitude A_1. Due to the symmetry of the masking noise distribution function, we shall use the equation $\theta(j) = -\theta(n-j)$.

Apparently we can speak about correction only in a limited diapason of amplitudes A_1. Because of this we shall choose a range $0 < X < Y$, which we divide into r parts $0 < X = A_{11} < A_{12} < ... < A_{1r} = Y$. To minimise the systematic error of signal measuring, we are interested in the minimum of function

$$\sum_{i=1}^{r} (\frac{\bar{A}_{2i}}{A_{1i}} - 1)^2 \qquad (13)$$

and shall look for such correction parameters, at which the output signal values in the given diapason of amplitudes differ the least from the corresponding input signal values. Taking into account the continuity of $\bar{A}_2$ with respect to $\theta(j)$, it shall be in effect for the whole diapason, unless the division is too crude. Depending on the chosen parameters of correction, we shall obtain various methods of systematic error reduction.

"Multiplier" correction.

By using equation (12), at a fixed input signal amplitude A_1, number of samples n, the lowest discrete value $\varepsilon = \frac{\lambda(0)}{n}$ and introducing correction multiplier k (i.e., taking into account that $\theta(0) = k\Phi^{-1}(\varepsilon)$, $\theta(n) = -\theta(0)$ and $\theta(j) = k\Phi^{-1}(\frac{j}{n})$ in the remaining cases), the output signal amplitude average value $\bar{A}_2$ can be written as

$$\bar{A}_2(A_1, n, k, \varepsilon) =$$

$$= k(Q^n - P^n)\Phi^{-1}(\varepsilon) + \sum_{j=1}^{n-1} \pi_n(j) \Phi^{-1}(\frac{j}{n})) , \qquad (14)$$

where

$$0 < \varepsilon < 1 . \qquad (15)$$

Therefore, in each limited diapason of amplitudes, for the chosen $0 < A_{11} < A_{12} < ... < A_{1r}$ we can look for the minimum of function

$$\sum_{i=1}^{r} (\frac{\bar{A}_2(A_{1i}, n, k, \varepsilon)}{A_{1i}} - 1)^2 \qquad (16)$$

with respect to both k and ε .

With index i we shall denote the corresponding values s_i , t_i , Q_i , P_i , π_{ni} at amplitude A_{1i} . Then we must look for the minimum with respect to k and $\Phi^{-1}(\varepsilon)$ for function

$$B = \sum_{i=1}^{r} (\frac{k(s_i \Phi^{-1}(\varepsilon) + t_i)}{A_{1i}} - 1)^2 , \qquad (17)$$

where $s_i = Q_i^n - P_i^n$; $t_i = \sum_{j=1}^{n-1} \pi_{ni}(j) \Phi^{-1}(\frac{j}{n})$.

We shall denote $\Phi^{-1}(\varepsilon) = q$. To find the minimum, we will use the necessary condition of differentiable function extremum. For this purpose we will derive equation (17) with respect to k and q .

By expanding equation (17), we obtain

$$B = r + (\sum_{i=1}^{r} (\frac{s_i^2 k^2 q^2}{A_{1i}^2} + 2\frac{s_i t_i k^2 q}{A_{1i}^2} - 2\frac{s_i kq}{A_{1i}} + \frac{t_i^2 k^2}{A_{1i}^2} -$$

$$- 2\frac{t_i k}{A_{1i}})) . \qquad (18)$$

Both partial derivatives of equation (16) are:

$$(B)_k^` = \sum_{i=1}^{r} (2\frac{s_i^2 kq^2}{A_{1i}^2} + 4\frac{s_i t_i kq}{A_{1i}^2} - 2\frac{s_i q}{A_{1i}} + 2\frac{t_i^2 k}{A_{1i}^2} - 2\frac{t_i}{A_{1i}}) , \qquad (19)$$

$$(B)_q^` = \sum_{i=1}^{r} (2\frac{s_i^2 k^2 q}{A_{1i}^2} + 2\frac{s_i t_i k^2}{A_{i1}^2} - 2\frac{s_i k}{A_{1i}}) . \qquad (20)$$

By composing and solving system

$$\begin{cases} \sum_{i=1}^{r} (2\frac{s_i^2 kq^2}{A_{1i}^2} + 4\frac{s_i t_i kq}{A_{1i}^2} - 2\frac{s_i q}{A_{1i}} + 2\frac{t_i^2 k}{A_{1i}^2} - 2\frac{t_i}{A_{1i}}) = 0, \\ \sum_{i=1}^{r} (2\frac{s_i^2 k^2 q}{A_{1i}^2} + 2\frac{s_i t_i k^2}{A_{i1}^2} - 2\frac{s_i k}{A_{1i}}) = 0, \end{cases} \qquad (21)$$

with respect to k and q , we obtain:

$$k_1 = 0 , \qquad (22)$$

$$q_1 = -\frac{\sum_{i=1}^{r} \frac{t_i}{A_{1i}}}{\sum_{i=1}^{r} \frac{s_i}{A_{1i}}} \quad (23)$$

and

$$k_2 = \frac{-(\sum_{i=1}^{r} \frac{s_i t_i}{A_{1i}^2})(\sum_{i=1}^{r} \frac{s_i}{A_{1i}}) + (\sum_{i=1}^{r} \frac{t_i}{A_{1i}})(\sum_{i=1}^{r} \frac{s_i^2}{A_{1i}^2})}{(\sum_{i=1}^{r} \frac{s_i t_i}{A_{1i}^2})^2 + (\sum_{i=1}^{r} \frac{t_i^2}{A_{1i}^2})(\sum_{i=1}^{r} \frac{s_i^2}{A_{1i}^2})}, \quad (24)$$

$$q_2 = -\frac{(\sum_{i=1}^{r} \frac{s_i t_i}{A_{1i}^2})(\sum_{i=1}^{r} \frac{t_i}{A_{1i}}) - (\sum_{i=1}^{r} \frac{s_i}{A_{1i}})(\sum_{i=1}^{r} \frac{t_i^2}{A_{1i}^2})}{-(\sum_{i=1}^{r} \frac{s_i t_i}{A_{1i}^2})(\sum_{i=1}^{r} \frac{s_i}{A_{1i}}) + (\sum_{i=1}^{r} \frac{t_i}{A_{1i}})(\sum_{i=1}^{r} \frac{s_i^2}{A_{1i}^2})}. \quad (25)$$

By using the sufficient condition of the differentiable function extremum, it is easy to verify that the point with coordinates (22), (23) is not, but the point with coordinates (24), (25) is the minimum point.

As B is an continiusly differentiable function with respect to k and q, then the only local minimum shall also be the global minimum. Therefore we can calculate the coefficients of the ‘’multiplier’’ correction method in accordance with equations (24) and (25).

“Addend” correction.

For this correction method, we shall use parameters $\varepsilon = \frac{\lambda(0)}{n}$ and the addend c. Taking $\theta(0) = \Phi^{-1}(\varepsilon) + c$, $\theta(n) = -\theta(0)$ and $\theta(j) = sign(\Phi^{-1}(\frac{j}{n}))(|\Phi^{-1}(\frac{j}{n})| - c)$ in the remaining cases, we obtain equation (12) in the following form

$$\bar{A}_2(A_1, n, c, \varepsilon) = (Q^n - P^n)\Phi^{-1}(\varepsilon) + (Q^n - P^n)c + \sum_{j=1}^{n-1} \pi_n(j)\Phi^{-1}(\frac{j}{n}) + \sum_{j=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \pi_n(j)c - \sum_{j=\left\lfloor \frac{n+2}{2} \right\rfloor}^{n-1} \pi_n(j)c. \quad (26)$$

By using the previously introduced denomination $q = \Phi^{-1}(\varepsilon)$, equation

$$\Gamma = \sum_{i=1}^{r} (\frac{s_i q + w_i c + t_i}{A_{1i}} - 1)^2, \quad (27)$$

where

$s_i = Q_i^n - P_i^n$, $t_i = \sum_{j=1}^{n-1} \pi_{ni}(j)\Phi^{-1}(\frac{j}{n})$,

$w_i = \sum_{j=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \pi_{ni}(j) - \sum_{j=\left\lfloor \frac{n+2}{2} \right\rfloor}^{n-1} \pi_{ni}(j)$,

must be minimised with respect to q and c.

By expanding equation (27), we obtain:

$$\Gamma = r + (\sum_{i=1}^{r} (\frac{s_i^2 q^2}{A_{1i}^2} + 2\frac{s_i w_i qc}{A_{1i}^2} + 2\frac{s_i t_i q}{A_{1i}^2} - 2\frac{s_i q}{A_{1i}} + \frac{w_i^2 c^2}{A_{1i}^2} + 2\frac{w_i t_i c}{A_{1i}^2} - 2\frac{w_i c}{A_{1i}} + \frac{t_i^2}{A_{1i}^2} - 2\frac{t_i}{A_{1i}}). \quad (28)$$

Both partial derivatives of equation (28) with respect to q and c are

$$(\Gamma)'_q = \sum_{i=1}^{r} (2\frac{s_i^2 q}{A_{1i}^2} + 2\frac{s_i w_i c}{A_{1i}^2} + 2\frac{s_i t_i}{A_{1i}^2} - 2\frac{s_i}{A_{1i}}), \quad (29)$$

$$(\Gamma)'_c = \sum_{i=1}^{r} (2\frac{s_i w_i c}{A_{1i}^2} + 2\frac{w_i^2 c}{A_{1i}^2} + 2\frac{w_i t_i}{A_{1i}^2} - 2\frac{w_i}{A_{1i}}). \quad (30)$$

By composing and solving linear equation system

$$\begin{cases} \sum_{i=1}^{r} (2\frac{s_i^2 q}{A_{1i}^2} + 2\frac{s_i w_i c}{A_{1i}^2} + 2\frac{s_i t_i}{A_{1i}^2} - 2\frac{s_i}{A_{1i}}) = 0, \\ \sum_{i=1}^{r} (2\frac{s_i w_i c}{A_{1i}^2} + 2\frac{w_i^2 c}{A_{1i}^2} + 2\frac{w_i t_i}{A_{1i}^2} - 2\frac{w_i}{A_{1i}}) = 0, \end{cases} \quad (31)$$

with respect to q and c, we obtain:

$$q = \frac{F}{H}, \quad (32)$$

$$c = \frac{G}{H}, \quad (33)$$

where

$$F = (\sum_{i=1}^{r} \frac{w_i^2}{A_{1i}^2})(\sum_{i=1}^{r} \frac{s_i t_i}{A_{1i}^2}) - (\sum_{i=1}^{r} \frac{w_i^2}{A_{1i}^2})(\sum_{i=1}^{r} \frac{s_i}{A_{1i}}) - (\sum_{i=1}^{r} \frac{w_i t_i}{A_{1i}^2})(\sum_{i=1}^{r} \frac{s_i w_i}{A_{1i}^2}) + (\sum_{i=1}^{r} \frac{w_i}{A_{1i}})(\sum_{i=1}^{r} \frac{s_i w_i}{A_{1i}^2}), \quad (34)$$

$$G = (\sum_{i=1}^{r} \frac{s_i^2}{A_{1i}^2})(\sum_{i=1}^{r} \frac{w_i t_i}{A_{1i}^2}) - (\sum_{i=1}^{r} \frac{s_i^2}{A_{1i}^2})(\sum_{i=1}^{r} \frac{w_i}{A_{1i}}) - (\sum_{i=1}^{r} \frac{s_i t_i}{A_{1i}^2})(\sum_{i=1}^{r} \frac{s_i w_i}{A_{1i}^2}) + (\sum_{i=1}^{r} \frac{s_i}{A_{1i}})(\sum_{i=1}^{r} \frac{s_i w_i}{A_{1i}^2}), \quad (35)$$

$$H = (\sum_{i=1}^{r} \frac{s_i w_i}{A_{1i}^2})^2 - (\sum_{i=1}^{r} \frac{w_i^2}{A_{1i}^2})(\sum_{i=1}^{r} \frac{s_i^2}{A_{1i}^2}). \quad (36)$$

Therefore the only stationary point with coordinates (32) and (33) has been found. Taking into account that equation (27) is limited from the bottom, this point will also be the absolute minimum point.

“Combined” correction.

In order to further improve the precision of transform, we can use a combination of both previous corrections. In this case, at input signal amplitude A_1, multiplier k, lowest discrete value ε and addend c, i.e. taking into account that $\theta(0)=k(\Phi^{-1}(\varepsilon)+c)$, $\theta(n)=-\theta(0)$ and $\theta(j)=ksign(\Phi^{-1}(\frac{j}{n}))(|\Phi^{-1}(\frac{j}{n})|-c)$ in the remaining cases, we can write the output signal amplitude average value as

$$\bar{A}_2(A_1,n,k,c,\varepsilon)=k((Q^n-P^n)\Phi^{-1}(\varepsilon)+(Q^n-P^n)c+ \\ +\sum_{j=1}^{n-1}\pi_n(j)\Phi^{-1}(\frac{j}{n})+\sum_{j=1}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\pi_n(j)c-\sum_{j=\left\lfloor\frac{n+2}{2}\right\rfloor}^{n-1}\pi_n(j)c). \quad (37)$$

By using the previously introduced denomination $q=\Phi^{-1}(\varepsilon)$, the equation

$$\sum_{i=1}^{l}[\frac{k(a_{1i}q+a_{2i}c+a_{3i})}{A_{1i}}-1]^2, \quad (38)$$

where $a_{1i}=Q_i^n-P_i^n$; $a_{3i}=\sum_{j=1}^{n-1}\pi_{ni}(j)\Phi^{-1}(\frac{j}{n})$;

$$a_{2i}=a_{1i}+\sum_{j=1}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\pi_{ni}(j)-\sum_{j=\left\lfloor\frac{n+2}{2}\right\rfloor}^{n-1}\pi_{ni}(j)$$

must be minimised with respect to k, q and c.

By grouping monomials to kq, kc and k, we obtain b_1, b_2 and b_3 as coefficients, respectively. Then, in formula (38), we can write the equation in brackets as

$$(b_1kq+b_2ck+b_3k-1)^2=b_1^2k^2q^2+b_2^2c^2k^2+b_3^2k^2+ \\ +1+2b_1b_2ck^2q+2b_1b_3k^2q-2b_1kq+2b_2b_3ck^2- \\ -2b_2ck-2b_3k. \quad (39)$$

By expanding equation (38) and grouping monomials to k^2q^2, c^2k^2, ck^2q, k^2q, ck^2, k^2, kq, ck and k, we obtain $d_1,...,d_9$ as coefficients, respectively. Then, by partially deriving equation

$$d_1k^2q^2+d_2c^2k^2+d_3ck^2q+d_4k^2q+d_5ck^2+ \\ +d_6k^2+d_7kq+d_8ck+d_9k+r, \quad (40)$$

with respect to k, q and c, we obtain

$$(d_1k^2q^2+d_2c^2k^2+d_3ck^2q+d_4k^2q+d_5ck^2+d_6k^2+ \\ +d_7kq+d_8ck+d_9k+l)_k^{`}=2d_1kq^2+2d_2c^2k+2d_3ckq+ \\ +2d_4kq+2d_5ck+2d_6k+d_7q+d_8c+d_9 \quad (41)$$

$$(d_1k^2q^2+d_2c^2k^2+d_3ck^2q+d_4k^2q+d_5ck^2+d_6k^2+ \\ +d_7kq+d_8ck+d_9k+l)_q^{`}=2d_1k^2q+d_3ck^2+ \\ +d_4k^2+d_7k \quad (42)$$

$$(d_1k^2q^2+d_2c^2k^2+d_3ck^2q+d_4k^2q+d_5ck^2+d_6k^2+ \\ +d_7kq+d_8ck+d_9k+l)_c^{`}=2d_2ck^2+d_3k^2q+ \\ +d_5k^2+d_8k. \quad (43)$$

We compose an equation system

$$\begin{cases} 2d_1kq^2+2d_2c^2k+2d_3ckq+2d_4kq+2d_5ck+ \\ +2d_6k+d_7q+d_8c+d_9=0 \\ 2d_1k^2q+d_3ck^2+d_4k^2+d_7k=0 \\ 2d_2ck^2+d_3k^2q+d_5k^2+d_8k=0, \end{cases} \quad (44)$$

by solving of which we obtain $q=\frac{F}{G}$, $c=\frac{H}{I}$, $k=\frac{G}{I}$, where

$$F=4d_2d_6d_7-2d_2d_9d_4-d_7d_5^2-2d_6d_8d_3+d_9d_3d_5+ \\ +d_5d_8d_4, \quad (45)$$

$$G=4d_2d_9d_1-2d_2d_4d_7+d_7d_3d_5+d_3d_8d_4-d_9d_3^2- \\ -2d_5d_8d_1, \quad (46)$$

$$H=d_7d_5d_4-2d_7d_6d_3+4d_8d_1d_6+d_3d_9d_4-2d_5d_9d_1- \\ -d_8d_4^2, \quad (47)$$

$$I=2d_2d_4^2-8d_2d_1d_6+2d_1d_5^2-2d_3d_5d_4+2d_3^2d_6. \quad (48)$$

The solution of system (44) with certain q and c can also be $k=0$. By using the sufficient condition of the differentiable function extremum, it is easy to verify that in that case, the respective point is not the minimum. Therefore the point with coordinates *q*, *c* and *k* is the only possible point of extremum. As equation (40) is limited from the bottom, the only possible point of extremum found will be the minimum point. Therefore, in case of the ‘’combined’’ method, we can calculate the correction coefficients in accordance with equations of *q*, *c* and *k*.

Practical application of correction methods

As a practical example of application of these methods, we shall look at transform of superbroadband radiolocation signal masked with noise $\sigma=1$ at amplitude values $A_1=0.25$; $A_1=0.50$; $A_1=0.75$; $A_1=1.00$; $A_1=1.25$; $A_1=1.50$ and at a number of samples n within the range from $n=5$ to $n=30$. In a statistical modelling experiment, to obtain the average values of output amplitudes with sufficient precision, the number of numerical experiments is taken as $N=50000$. The obtained results are shown in tables 1-3. The theoretically calculated $\frac{A_2}{A_1}$ relations have been denoted with index 1, while index 2 denotes the experimentally obtained $\frac{A_2}{A_1}$ values.

Table 1. Results with the "multiplier" correction

n	0.25	0.50	0.75	1.00	1.25	1.50
5_1	0.980	0.999	1.018	1.023	1.007	0.970
5_2	0.979	1.008	1.019	1.027	1.005	0.969
10_1	0.987	0.996	1.008	1.015	1.009	0.984
10_2	0.989	0.992	1.013	1.016	1.009	0.982
15_1	0.989	0.995	1.004	1.012	1.010	0.990
15_2	0.986	0.999	1.011	1.013	1.008	0.990
20_1	0.990	0.995	1.002	1.010	1.010	0.993
20_2	0.995	0.998	1.005	1.011	1.011	0.992
25_1	0.991	0.995	1.001	1.008	1.010	0.995
25_2	0.987	0.999	1.001	1.005	1.011	0.997
30_1	0.992	0.995	1.000	1.007	1.010	0.996
30_2	0.997	0.997	1.006	1.008	1.010	0.996

Table 2. Results with the "addend" correction

n	0.25	0.50	0.75	1.00	1.25	1.50
5_1	0.978	1.001	1.022	1.026	1.007	0.965
5_2	0.967	1.005	1.027	1.023	1.003	0.964
10_1	0.977	1.000	1.021	1.027	1.011	0.972
10_2	0.978	1.001	1.023	1.026	1.010	0.971
15_1	0.975	1.000	1.022	1.030	1.016	0.978
15_2	0.979	1.005	1.026	1.030	1.017	0.978
20_1	0.977	0.999	1.019	1.030	1.020	0.983
20_2	0.982	1.004	1.024	1.031	1.020	0.983
25_1	0.979	0.999	1.017	1.028	1.022	0.987
25_2	0.980	1.004	1.019	1.027	1.021	0.988
30_1	0.981	0.999	1.014	1.025	1.023	0.990
30_2	0.981	1.002	1.014	1.025	1.022	0.991

Table 3. Results with the "combined" correction

n	0.25	0.50	0.75	1.00	1.25	1.50
5_1	1.004	0.996	0.996	1.003	1.007	0.995
5_2	0.997	0.996	0.990	1.003	1.009	0.996
10_1	1.003	0.996	0.996	1.003	1.007	0.995
10_2	1.005	0.995	1.001	1.004	1.003	0.992
15_1	1.003	1.000	0.996	1.002	1.005	0.996
15_2	1.001	1.003	0.997	1.006	1.005	0.996
20_1	1.003	0.997	0.996	1.002	1.005	0.997
20_2	1.009	1.000	0.996	1.002	1.005	0.996
25_1	1.003	0.997	0.996	1.001	1.005	0.998
25_2	1.004	0.997	0.996	0.999	1.004	0.997
30_1	1.003	0.998	0.996	1.001	1.005	0.998
30_2	0.999	0.999	0.999	1.001	1.005	0.998

From tables 1–3 it can be seen that:

1) the modelling results match the analytical calculations well;

2) all correction methods in the given diapason of amplitudes $A_1 = 0.25 - 1.5\sigma$ ensure quite high precision of measurements;

3) the highest precision is demonstrated by the "combined" method;

4) by increasing n, the systematic error of transform is reduced, and at sufficiently large n it can be disregarded.

For illustration, Fig. 1 displays a comparison of precision of the classic statistical method, the corrected method [2] and the "combined" method at $A_1 = 1$ and number of samples within a range from $n = 5$ to $n = 30$.

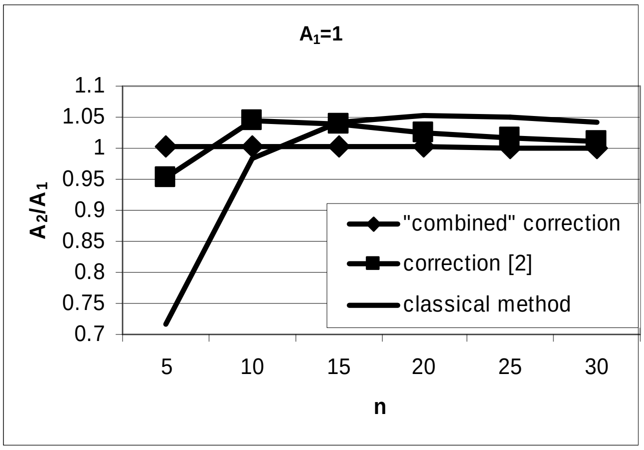

Fig. 1. Comparison of the "classic" statistical method, correction method [2] and "combined" correction method at $A_1 = 1$

It shows that the "combined" method ensures practically ideal compensation of the systematic error. It must also be pointed out that the correction method proposed in article [2] was designed for amplitude diapason up to $A_1 = 1$, while the results obtained in this research allow correcting the mistake up to $A_1 = 1.5$.

For illustration, Fig. 2 displays the result of modelling a transform of a superbroadband radiolocation signal with the "classic" (non-corrected) statistic method in the signal detection mode (one scan) at $A_1 = 1.5$, $\sigma = 1$ and $n = 5$. Fig. 3 displays the result of averaging the same input signal transform at $m = 300$ in the case of non-corrected method (thin line) and "combined" correction (thick line). As the modelling results show, in case of the "combined" method, the signal modification precision increase is quite significant.

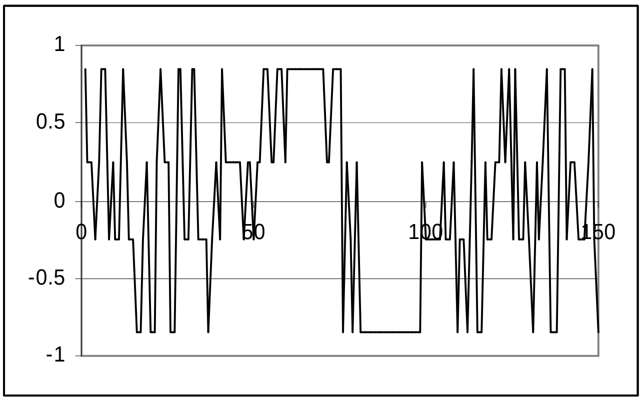

Fig. 2. Result of transform of a superbroadband radiolocation signal masked with noise with the "classic" statistic method in the signal detection mode (one scan) at $A_1 = 1.5$, $\sigma = 1$ and $n = 5$

In this research, the correction of the systematic error was designed for the variable signal amplitude diapason $A_1 = 0.25 - 1.5\sigma$. It is understandable that, at amplitude $A_1 = 0.25\sigma$ in the signal detection mode, the signal will be completely masked with noise. However, it must be taken into account that an actual superbroadband radiolocation signal is not a perfect mono-oscillation, but a quickly fading oscillation process with a certain bending. Because of that, in order to precisely register such actual signals with the averaging method, correction is necessary in a sufficiently broad diapason of amplitudes. In our case, it has been done in the diapason $A_1 = 0.25 - 1.5\sigma$.

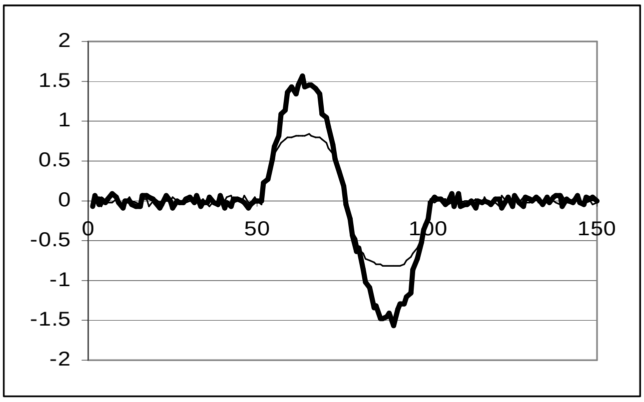

Fig. 3. Transform of a mono-oscillation masked with noise using the ''classic'' statistic method (thin line) and the ''combined'' statistic method (thick line) at $A_1 = 1.5$, $\sigma = 1, n = 5$ and $m = 300$

It must be pointed out that the "combined" correction method is not the most precise possible method in an absolute sense, because the minimising of the systematic error is performed only by 3 parameters. In the general case, the mistake should be minimised by $\lfloor n/2 \rfloor$ variables, which would be involved in a system of linear equations similar to (33). However it has no practical purpose, as the "combined" method of optimisation by three parameters is already displaying a sufficiently high precision of transform.

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2. **Kruminsh K., Lorencs A., and Plocinsh V.** A2 paradox in the statistical weak signal processing // Automatic Control and Computer Sciences. – Allerton Press, New York. – Vol. 41, No 1. – P. 3–14.
3. **Lorencs A.** Local properties of a statistical method for signal // Automatic Control and Computer Sciences. – Allerton Press, New York. – Vol. 36, No 5. – P. 3–17.

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V. Plociņš. Statistical Method Correction Possibilities // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 2(82). – P. 29–34.

The research is devoted to the problem of minimising the systematic error caused by the A2 paradox of signal processing statistical method. Two methods of minimisation of the error are offered, as well as a combination of both these methods for the signal amplitude diapason $A_1 = 0.25 - 1.5\sigma$. With examples of superbroadband radiolocation signal transform, it is shown that both offered methods and especially the combination of both methods offer a practically ideal correction of the systematic error. Ill. 3, bibl. 3 (in English; summaries in English, Russian and Lithuanian).

В. Плоцинш. Возможности корректирования статискических методов // Электроника и электротехника. – Каунас: Технология, 2008. – № 2(82). – С. 29–34.

Исследование посвящено проблеме уменьшения систематической погрешности, вызванной парадоксом A2 метода статистической обработки сигнала. Предлагаются два метода минимизации погрешности, так же комбинация обоих этих методов для диапазона амплитуды сигнала $A_1 = 0.25 - 1.5\sigma$. С примерами трансформации сигнала суперширокополосной радиолокации показано, что предлагаемые методы и особенно комбинация обоих методов фактически идеально исправляют систематическую погрешность. Ил. 3, библ. 3 (на английском языке; рефераты на английском, русском и литовском яз.).

V. Plociņš. Statistinių metodų koregavimo galimybės // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 2(82). – P. 29–34.

Tyrimas skirtas sisteminėms paklaidoms, gautoms taikant statistinį signalų apdorojimo metodą A2, minimizuoti. Pasiūlyti du paklaidos minimizavimo metodai bei abiejų metodų kombinacija. Metodai taikytini kai signalo amplitudės diapazonas $A_1 = 0.25 - 1.5\sigma$. Pateikti superplačiajuosčio radiolokacijos signalo transformacijų pavyzdžiai. Jais remiantis parodyta, jog abu siūlomi metodai ir ypač jų derinys užtikrina praktiškai idealią sisteminės paklaidos korekciją. Il. 3, bibl. 3 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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