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Isothermal Bunching of Electron Stream in Microwave Electron Devices

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Introduction

Amplitudinal characteristic of most modern microwave devices and frequency transducers has a shape [1] given in Fig. 1.

At a certain sufficiently small input signal power $P_{in\,\min}$ electronic amplifier or frequency transducer (further named a device) stops reproducing phase and spectrum of the input signal. The value $P_{in\,\min}$ near which complete absence of a control upon electron stream by the input signal takes place can be treated as a threshold power of the input signal. The threshold power and its dependence on parameters of microwave electronic devices are investigated up to now insufficiently. Existence of a finite value of $P_{in\,\min}$ is caused by fluctuation processes occurring in a device itself. It is usually considered that the threshold input signal power at a first approach is determined by total power of a noise reduced to the input of the device. From such point of view microwave amplifier or frequency transducer may be represented as a linear four-polar with constant gain having at its input additive mixture of the signal and the noise which obscures the signal.

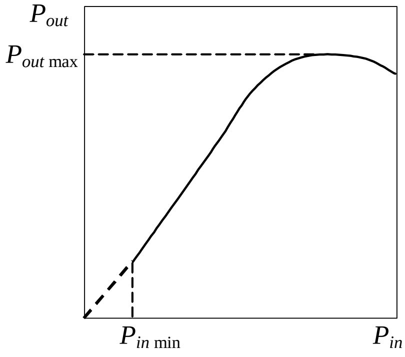

Fig. 1. Amplitudinal characteristic of microwave amplifiers and frequency transducers

Linearity of characteristics of the device in a regime of small amplitudes does not exclude, however, existence of nonlinear phenomena caused by statistic properties of electron stream. Mechanism of signal amplification in this regime does not remain invariable as well. Essential role may also be played by conditions of electron bunching in an area where electron stream, described, at the initial state, by certain function of distribution, begins to interact with a field of input signal.

Electron gas in power field of travelling wave

Consider one-dimensional problem of electron motion in a homogenous field of slow travelling wave with constant amplitude E_{xm} in a space, free from static fields. In a regime of synchronisation, when initial velocity of electrons v_0, determined without respect to the distribution of electron speeds, is equal to the phase velocity of slow travelling wave v_f, longitudinal component of a high frequency electric field of the wave

$$E_x = E_{xm} \sin \frac{2\pi x}{\Lambda} \tag{1}$$

in a mobile reference frame has a stationary character. Here Λ – length of slow-wave, travelling along the interaction space in x-direction.

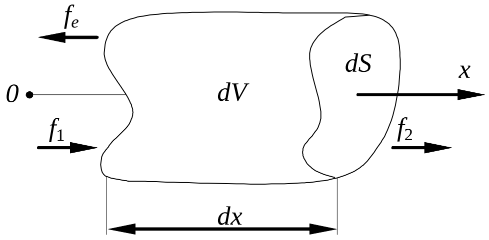

Fig. 2. Forces acting on volume element of electron stream in mobile reference frame

Let initial state of non-disturbed homogenous electron stream be characterised by initial concentration n_0 and absolute temperature T at Maxwellian distribution of electron speeds. Action of longitudinal electric field of slow travelling wave upon electron stream is similar to the influence of gravitational masses distributed periodically towards x-direction on molecules of a neutral gas finding themselves in such gravitational field. Signify n equilibrium concentration of electrons in a given field E_x. For simplicity ignore space charge fields as well. Then volume element of electron stream dV in Fig. 2 contains ndV electrons that experience action of the total electric force

$$f_e = ndV(-eE_x) = -nedSdxE_x \sin\frac{2\pi x}{\Lambda}, \qquad (2)$$

where $e = -1{,}6\cdot 10^{-19}$ C – charge of electron, dS – cross-section area of the stream element perpendicular to x-direction. On the other hand we know that pressure of a perfect gas p may be expressed through its concentration and absolute temperature: $p=nkT$, where $k=1{,}38\cdot 10^{-23}$ J/K – Boltzmanns's constant. Therefore, if the concentrations on both sides of the stream element are n and $n+dn$, forces acting from the left and right sides on this element are equal to

$$f_1 = (n+dn)kTdS \ \text{ and } \ f_2 = nkTdS. \qquad (3)$$

Condition of equilibrium for the stream element may be written now in such a shape

$$f_e = |f_1| + |f_2|. \qquad (4)$$

From (2), (3) and (4) it follows

$$\frac{dn}{dx} + n\frac{eE_{xm}}{kT}\sin\frac{2\pi x}{\Lambda} = 0. \qquad (5)$$

At small input signals the change in an electron temperature under influence of E_x may be ignored. Then solution of the equation (5) at $E_{xm}=const$ is

$$n(x) = n_1 \exp\left[-\frac{eE_{xm}\Lambda}{2\pi kT}\left(1-\cos\frac{2\pi x}{\Lambda}\right)\right], \qquad (6)$$

here n_1 – settled concentration of electrons at a centre of electron spoke (Fig. 3).

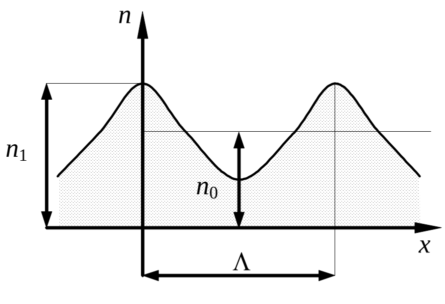

Fig. 3. Equilibrium and limit concentration of electrons in electron spokes

Obtained expression is similar to famous barometric formula, which describes distribution of molecules in a gravitational field of the Earth or another heaven body having own atmosphere. This equation describes a structure of equilibrium isothermal electronic atmosphere, which may be kept at limit conditions by the field of travelling wave with respect to electron distribution of speeds.

Dependence $n(x)$ according to (6) has a spatial period which equals length of the slow-wave Λ. The smaller the ratio $E_{xm}\Lambda/kT$ the lower the limit concentration of electron spokes and the higher the part of non-controlled electrons carrying out chaotic motions between areas of retarding and accelerating phases of the high frequency electric field of travelling wave.

Reduction of electron stream current

Here we shall calculate strength of electron stream current taking into account limit concentration of electron spokes.

Take a Fourier series expansion of electron equilibrium function $n(x)$ represented by (6):

$$n(x) = A_0 + \sum_{p=1}^{\infty} A_p \cos p\beta x. \qquad (7)$$

Here

$$\beta = \frac{2\pi}{\Lambda} = \frac{\omega}{v_f} \qquad (8)$$

– phase constant of the slow-wave; ω – frequency of the wave.

Constant component A_0 and amplitude of p-harmonic of electron concentration in accordance with (6) and (8) are equal to:

$$A_0 = \frac{2}{\Lambda} n_1 \exp\left(\frac{-eE_{xm}}{\beta kT}\right)\int_0^{\frac{\Lambda}{2}} \exp\left(\frac{eE_{xm}}{\beta kT}\right)\cos\beta x dx; \qquad (9)$$

$$A_p = \frac{4}{\Lambda} n_1 \exp\left(\frac{-eE_{xm}}{\beta kT}\right)\times$$

$$\times\int_0^{\frac{\Lambda}{2}} \exp\left[\left(\frac{eE_{xm}}{\beta kT}\right)\cos\beta x\right]\cos p\beta x dx. \qquad (10)$$

Integrals in (9) and (10) come to Bessel functions of imaginary argument of the p-order [2].

Confine further consideration with the constant component A_0 and amplitude of the first harmonic A_1:

$$A_0 = \frac{2n_1 J_0 eE_{xm}}{\beta k\Lambda T}\exp\left(\frac{-eE_{xm}}{\beta kT}\right); \qquad (11)$$

$$A_1 = \frac{4n_1 J_1 eE_{xm}}{\beta k\Lambda T}\exp\left(\frac{-eE_{xm}}{\beta kT}\right), \qquad (12)$$

where J_0 and J_1 – Bessel's functions of zero and first order.

Use the evident normalization $A_0=n_0$, where n_0 is non-disturbed concentration of electrons in the uniform electron stream at a given temperature T. From (11) and (12) one can obtain the amplitude of the first harmonic of elctron stream concentration in a final shape

$$A_1 = 2n_0 J_1(\alpha) / J_0(\alpha), \tag{13}$$

where $\alpha = eE_{xm} / \beta kT$.

At unrestricted vanishing of α amplitude A_1 approaches zero. Reduction of a limit electron spoke concentration at a finite temperature of electron gas may be described by the introduction of reduction coefficient of initial (undisturbed) electron concentration or reduction coefficient of a constant component of electron current W_i:

$$W_i = J_1(\alpha) / J_0(\alpha). \tag{14}$$

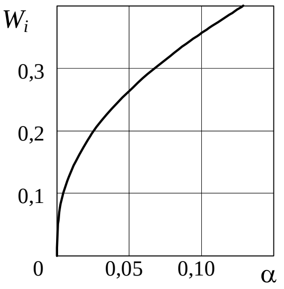

Fig. 4. Reduction coefficient of constant convection current component against parameter $\alpha = eE_{xm}\Lambda / 2\pi kT$

Fig. 4 represents graph of dependence between W_i and α. We can see from this graph, as well as from (13) and (14), that electron stream having constant component of convection current i_0 at given value of α can be substituted by equivalent electron stream i_{0eq} for which T=0:

$$i_{0eq} = i_0 W_i . \tag{15}$$

It is quite evident that real value of constant component of the convection current in all cases remains invariable. Decrease in i_{0eq} means only reduction of a part of convection current controlled by microwave input signal and rise in noise component of the current.

Equations (6), (13) and (15) correspond to the regime settled after finite time interval of relaxation. Therefore the reduction coefficient describes upper limit of isothermal bunching of electrons stream and upper limit of equivalent current i_{0eq} taking into account the finite temperature of electrons. Obtained relationships are valid not only for microwave electronic amplifiers, but also for frequency transducers because electron stream in an input cascade of cascade amplifier or frequency transducer does not “know” with which temporal harmonic of the convection current it will have to interact in a second cascade. Therefore it does not matter which a device is under consideration: in both cases $P_{in\min}$ remains the same.

Threshold input signal power

Equations (6), (13) and (15) let us evaluate a gain and minimum (threshold) input signal power of a travelling wave tube (TWT). For simplicity the finite temperature of electrons we introduce into linear theory of TWT through the reduction coefficient W_i.

In accordance with linear theory of TWT [1] parameter of amplification may be expressed by interaction impedance of slow-wave system on a level of electron stream R_e, permanent accelerating voltage U_0 and constant component of convection current i_0:

$$C = \sqrt[3]{i_0 R_e / 4U_0}\,. \tag{16}$$

The use of the coefficient W_i allows to introduce an effective parameter of amplification taking into account restrictions of the limit concentration of electrons (6) in the electron spokes:

$$C_{eff} = \sqrt[3]{i_0 R_e J_1(\alpha) / 4U_0 J_0(\alpha)} = C\sqrt[3]{W_i}\,. \tag{17}$$

Thus, the gain of TWT in a regime of small enough signals may depend on amplitude E_{xm} and input signal power P_{in} (or input signal power $P_{in\omega}$ at a frequency of fundamental temporal harmonic for TWT frequency transducers). Consider this type of nonlinearity caused by statistic properties of electron stream only.

The gain of TWT without taking into consideration a space charge and distributed losses of a slow-wave system in a regime of small amplitudes may be expressed as follows:

$$K = 10\log P_{out} / P_{in} = -a + bCN\sqrt[3]{W_i} - L, \tag{18}$$

where P_{out} – output signal power of TWT amplifier (output signal power $P_{out\ n\omega}$ at a frequency of working temporal harmonic for frequency transducers); a – parameter of initial losses; b – parameter of growing wave; L – losses in a local absorber; N – electrical length of the slow-wave system. The gain approaches zero, when

$$W_i = \left[(a + L) / bCN\right]^3. \tag{19}$$

It is important to emphasize that amplitude of the field E_{xm} in a slow-wave system of TWT, taking into account the three waves propagating in the system [1], remains practically constant along it.

From (18) and (19) one can see that there exists the limit value $W_{i\lim}$ and corresponding limit value of $\alpha_{\lim} = eE_{xm} / \beta kT$ at which TWT loses its ability of amplification.

At small amplitudes of a signal it is possible usage of asymptotic representation of modified Bessel’s functions $J_0(\alpha)$ and $J_1(\alpha)$ as follows [2]

$$J_0(\alpha) \approx 1;\ J_1(\alpha) \approx \alpha/2. \tag{20}$$

Then

$$E_{xm\lim} = eE_{xm}\Lambda / 4\pi kT. \tag{21}$$

From (19) and (21) we find the limit amplitude of the field

$$E_{xm\lim} = \frac{4\pi kT}{e}\left(\frac{a + L}{bCN}\right)^3. \tag{22}$$

The input signal power P_{in} is related to interaction impedance R_e and amplitude E_{xm} through well-known equation

$$R_e = \frac{E_{xm}^2}{2\beta^2 P_{in}}. \tag{23}$$

Substitution (22) into (23) gives us limit (threshold) input signal power

$$P_{in\lim} = \frac{2k^2T^2}{e^2R_e}\left[(a+L)/bCN\right]^6. \tag{24}$$

In a regime of synchronization we can assume that a=9,54 dB, b=47,3 dB. At L=0 and known values of k and e one can get:

$$P_{in\lim} = 1.01\cdot 10^{-12}\,\frac{T^2}{R_e(CN)^6}[\mathrm{W}]. \tag{25}$$

Quantity CN has its values in an interval from 1 to 2. Knowing the interaction impedance of the slow-wave system of TWT and absolute temperature allows us to appreciate the magnitude of $P_{in\lim}$. At $R_e = 10^2\,\Omega$, $T = 10^3\,K$ and CN=1,5 we obtain: $P_{in\lim} \cong 10^{-9}\,\mathrm{W}$.

Here we should take notice that equation (24) does not include frequency band of TWT in an expressed shape. In this respect equations (24) and (25) differ sufficiently from expression $P_{noise} = kT_{noise}\Delta f$, describing power of noise reduced to input of amplifier or frequency transducer which is sometimes used for estimation of the threshold input signal power. Here T_{noise} – noise temperature; Δf – absolute frequency band.

Conclusion

1. Appreciation of the threshold input signal power of microwave electronic amplifiers and frequency transducers is carried out, taking into consideration formation of electronic atmosphere and decrease in a useful part of convection current of electron stream, at a finite temperature of electron gas.
2. Obtained equations show that threshold input signal power must grow extremely at the increase of electronic temperature and at the decrease of interaction impedance of the slow-wave system.
3. Threshold input signal power of electronic microwave amplifiers and frequency transducers at a room temperature is of 10^{-9} W order.

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2. **Janke E., Emde F and Lösch F.** Tafeln Höherer Funktionen // Teubner Verlagsgesellshaft. – 1990. – P. 377.

J. O. Meilus, S. Gelžinis. Isothermal Bunching of Electron Stream in Microwave Electron Devices // Electronics and Electrical Engineering. – Kaunas: Technologija, 2008. – No. 2(82). – P. 13–16.

In order to evaluate the threshold input signal of microwave amplifiers and frequency transducers the performance of electron gas in a power field of travelling wave is investigated. Action of the longitudinal high frequency electric field of the slow travelling wave upon electron stream is likened to the influence of periodically distributed gravitational masses on the molecules of a neutral gas. Such representation allows to get an equation of distribution of electron concentration in an isothermal equilibrium electronic atmosphere which may be kept at limit conditions by the field of travelling wave. Equation for the threshold signal power shows that it grows extremely at the increase of electronic temperature and at the decrease of interaction impedance of the slow-wave system. It is shown as well that at the standard values of the main parameters represented by the equation and at a room temperature the threshold input signal power should be of 10^{-9} W order. Ill. 4, bibl. 2 (in English; summaries in English, Russian and Lithuanian).

И. О. Мейлус, С. Гельжинис. Изотермическая группировка электронного потока в электронных микроволновых приборах // Электроника и электротехника. – Каунас: Технология, 2008. – № 2(82). – С. 13–16.

Для оценки порогового входного сигнала микроволновых усилителей и преобразователей частоты сверхвысокочастотного сигнала рассматривается поведение электронов в силовом поле бегущей волны. Действие продольного электрического поля замедленной бегущей волны на электронный поток уподобляется влиянию периодически расположенных гравитационных масс на молекулы нейтрального газа. Такое представление позволяет получить уравнение, описывающее распределение концентрации равновесной изотермической электронной атмосферы, которую может в пределе удержать поле бегущей волны. Уравнение пороговой входной мощности показывает, что она резко возрастает при росте электронной температуры и при снижении сопротивления связи замедляющей системы. Показано также, что при стандартных значениях основных параметров, входящих в уравнение, и при комнатной температуре, пороговая входная мощность имеет величину порядка 10^{-9} Вт. Ил. 4, библ. 2 (на английском языке; рефераты на английском, русском и литовском яз.).

J. O. Meilus, S. Gelžinis. Izoterminis elektronų pluošto grupavimas elektroniniuose mikrobangų prietaisuose // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 2(82). P. 13–16.

Nagrinėjama elektronų elgesena mikrobangų stiprintuvų bei dažnio keitiklių bėgančiosios bangos jėgų lauke, siekiant įvertinti šių prietaisų minimalaus (slenkstinio) įėjimo signalo dydį. Lėtosios bėgančiosios bangos išilginio aukštadažnio lauko poveikis elektronų srautui tapatinamas su periodiškai išsidėsčiusių gravitacinių masių lauko įtaka neutraliųjų dujų molekulėms. Taikant tokią interpretaciją, gauta lygtis, aprašanti elektronų koncentracijos pasiskirstymą pusiausviroje izoterminėje elektroninėje atmosferoje, kurią ribiniu atveju gali išlaikyti bėgančiosios bangos laukas. Slenkstinės galios lygtis rodo, kad ši galia staigiai didėja, didėjant elektroninių dujų temperatūrai ir mažėjant lėtinimo sistemos ryšio varžai. Parodyta, kad, esant standartinėms pagrindinių parametrų, įeinančių į šią lygtį, vertėms, kambario temperatūroje slenkstinė įėjimo signalo galia yra 10^{-9} W eilės. Il. 4, bibl. 2 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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