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Marginal Distribution Density of Free Reflection Simplex Search Algorithm when Target Function is Square

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Introduction

Search optimization of real objects is always carried under conditions of disturbances, which emerge because of imprecise measurement, influence of unknown factors on object functioning indicators, object characteristic change and other reasons. Random disturbances create step direction errors, false view of extreme position and so decrease effectiveness of search. Because of that, when creating optimization algorithms, applied for solving stochastic problems, it is necessary to perform search system research working under disturbances conditions. Precisely enough simplex search characteristics in the stage of correction, under influence of disturbances, can be described using classical methods of probability theory according to the technique created in [1]. This technique also was used for example in [5].

When solving stochastic problem, search process can be considered as consisting of two stages. In the first stage simplex, when moving towards the target is moving from the initial point till the area of extreme is reached, in the second stage simplex movement is similar to random wondering in the target area. These stages can be respectively called climbing (or descending when looking for a minimum) and correction (localization of extreme). Far from the target search the direction should be stabilized and after attaining extreme – destabilized. So in the stage of climbing there can be used simplex search algorithm with suppressed return, and after attaining target area, better results can be obtained using simplex search algorithm with free reflection of vertexes [1].

In this work the marginal distribution density of free reflection simplex search algorithm was found when working under condition of normally distributed disturbances, with square function target. Simplex state probabilities were calculated using Monte Carlo method [2, 3].

Target Function and the orientations of the simplex

When seeking for statistical characteristics of simplex search in the stage of correction, it is possible to apply the following mathematical model of the object.

$$y(\mathbf{x},\varepsilon) = Q(\mathbf{x}) + \varepsilon, \tag{1}$$

in which the observed (measured) value y of the optimized indicator is equal to the sum of the index value $Q(\mathbf{x})$ and random disturbance ε.

The object in the zone of extreme may be described by the following function:

$$Q = -a_0 \sum_{i=1}^{k} x_i^2, \quad a_0 > 0, \tag{2}$$

here a_0 – coefficient, k – number of factors x.

In three-dimensional space it is the second range surface – paraboloid. In two-dimensional factor space ($k = 2$) a regular simplex is an equilateral triangle. Suppose, that in the search process the simplex has four orientations with equal probabilities in respect of gradient of $Q(\mathbf{x})$ [1]:

1) gradient is pointed from center to single vertex;
2) gradient is pointed from vertex to center;
3) gradient is collinear to the edge, when a correct step is pointed to the right;
4) gradient is collinear to the edge, when a correct step is pointed to the left.

These ways of orientation are presented in Fig 1.

Considering the chosen ways of simplex orientation, when executing search of extreme, a correct step provides the increase of the function (directional cosine of the angle between the vector of step and $grad\,Q$ is greater than zero) and conversely, an incorrect step provides the decrease of the function $Q(\mathbf{x})$. Also for orientations 3 and 4 there exists a zero step, after which $Q(\mathbf{x})$ does not change. Correct, incorrect and zero steps in Fig 1 are indicated respectively by symbols "+", "–" and "0".

The value

$$A = |grad\ Q| L_{n-1}, \tag{3}$$

is introduced which describes useful signal for orientations 3 and 4 and the value $A_0 = Ah_0$, equal to the useful signal having 1 or 2 ways of orientation (L_{n-1} is the length of the edge of the simplex after the search step; h_0 - altitude of the simplex, when $L=1$).

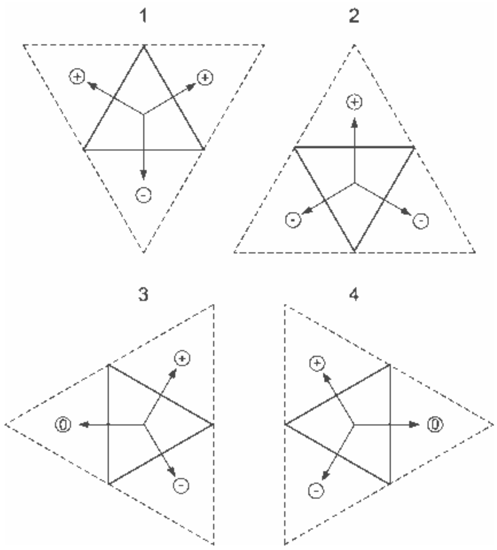

Fig. 1. Ways of orientation of the simplex

Incorrect step will be executed in case if disturbance in the vertex with the biggest value of the function Q together with A_0 is less then the disturbances in other vertexes. The probability if such an event is calculated using technique presented in [1].

Probabilities of the movement of the simplex in stage of correction

Close to the extreme each step of the search depends not only on the simplex orientation, but also on the distance of its centre from the target. In this case, the states a, b, c,..., of the given system to which the distances from the target ρ_a, ρ_b, ρ_c, …, correspond, as well as some ways of orientation of the simplex according to $grad\, Q$ are shown in Fig 2.

The sets of these simple states, which are allowed by the rules of the algorithm, form complex states of the multilink Markov chain. Stochastic diagram of two link Markov chain for investigation of search (when $k=2$), using algorithm with free reflection of vertexes, is shown in Fig 3.

For instance, complex state 2 is the presence of the centre of the simplex in point b (Fig 2.), if it is known, that before that it was in point a.

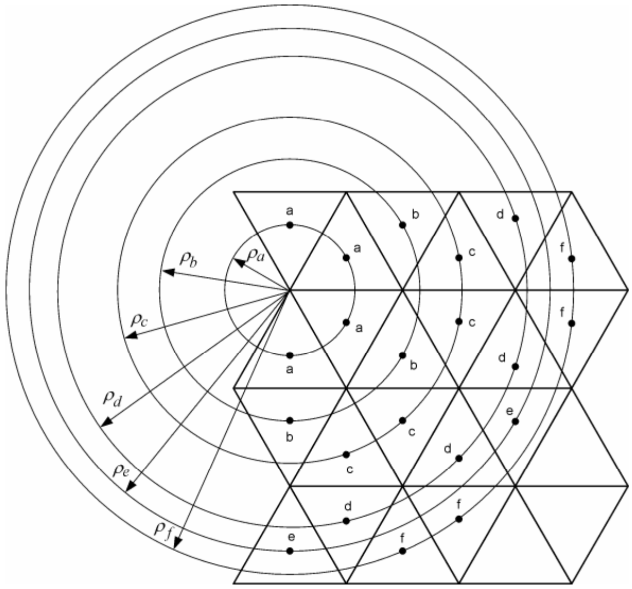

Fig 2. Ways of orientation of the simplex in the stage of correction

When seeking for transitional probabilities, the technique described in [1] is applied, yet in this case the value of the useful signal corresponding to the simplex orientation must be calculated. For instance, transitional probability p_{12} is equal to probability of event P.

$$p_{12} = P\{(\varepsilon_1 + A < \varepsilon_3) \cap (\varepsilon_1 + A < \varepsilon_2) \mid (\varepsilon_0 - A < \varepsilon_1)\}. \quad (4)$$

In order for it to occur the following conditions are necessary:

1. In the previous step of search, the difference between disturbance in simplex vertex with index 0 and useful signal (3) in this vertex has to be less than the disturbance in vertex with index 1.
2. If this condition of prehistory is satisfied, the values of disturbance in vertexes of the current step are verified. The sum of the disturbance in vertex 1 and the useful signal in it has to be less then disturbances in other vertexes of the simplex.

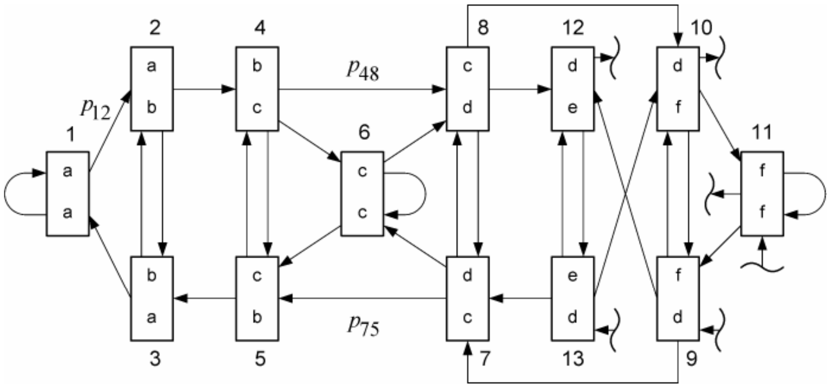

Fig. 3. Stochastic diagram of Markov chain

Marginal distribution density of the simplex wandering in the environment of the target

One of the approaches to calculate transitional and marginal probabilities of Markov chains is Monte Carlo method [2, 3]. This approach is simple, precise enough and convenient when calculating digitally. In case when target function is square, change of the useful signal A for each transition probability can be found as follows. First the distances of the vertexes from the extreme and values of the target function in each vertex of the simplex are found [4].

The change of the useful signal A for each transitional probability is calculated as follows [4] (here the indexes near the signal A indicate the number of the vertex):

p_{12} $aa \to ab$
$A_{01} = Q_1 - Q_0 = 1 - 0 = 1;$
$A_{12} = A_{13} = Q_1 - Q_0 = 1 - 0 = 1,$

p_{23} $ab \to ba$
$A_{01} = Q_1 - Q_0 = 1 - 0 = 1;$
$A_{32} = A_{31} = Q_2 - Q_1 = 3 - 1 = 2$ etc.

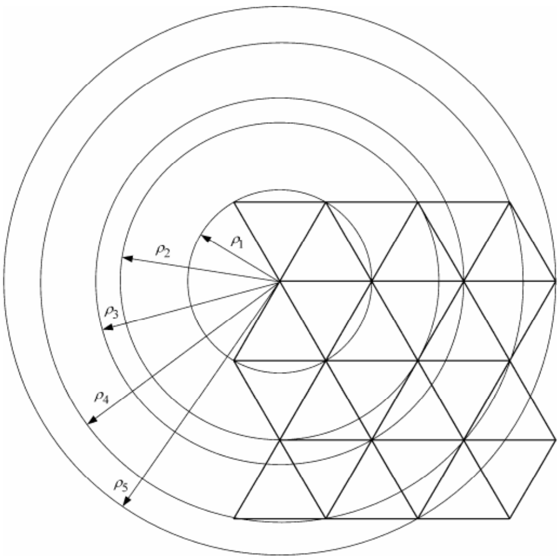

Fig. 4. Distances of the vertexes of simplex from the extreme

If the number of complex states is limited to 13, the equation system is made [1]:

$$\begin{cases} p_\beta = \sum_{\alpha=1}^{\psi'} p_\alpha p_{\alpha\beta}, \beta = 1, ..., \psi' - 1, \\ \sum_{\alpha=1}^{\psi'} p_\alpha = 1, \end{cases} \quad (5)$$

from which marginal probabilities $p_1, ..., p_{13}$ are found. Here ψ – the number of the equations in the system, ψ' – the number of the complex states of the multilink Markov chain, $\psi' < \psi$.

The marginal probabilities of the simple states a, b, c,..., f are found by adding the probabilities of complex states, having equal last simple states.

Theoretically calculated values of the probabilities are as follows: $p_a = 0,9112$, $p_b = 0,064$, $p_c = 0,00115$, $p_d = 0,000032$, $p_e = 0$, $p_f = 0$.

These probabilities allow to obtain marginal distribution density of the simplex wandering in target environment (Fig. 5):

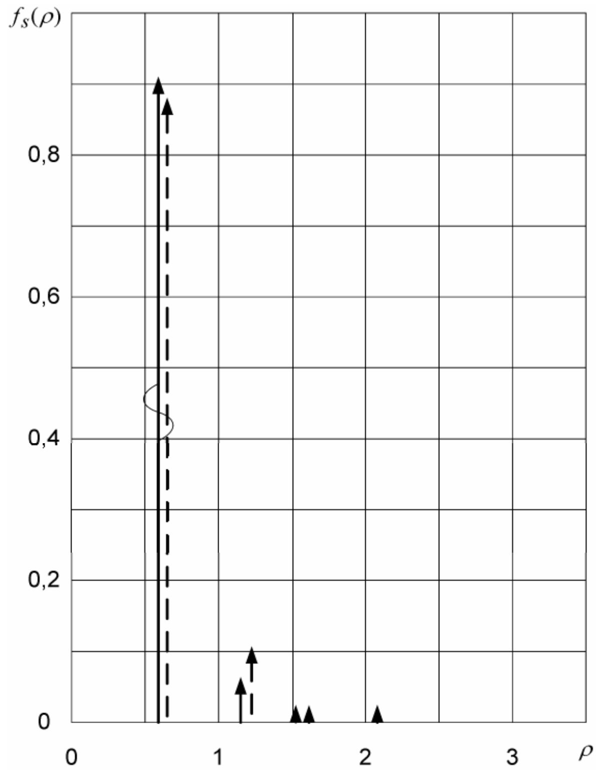

Fig. 5. Marginal distribution density

$$f_s(q) = p_a \delta\left(q - \frac{\rho_a}{\lambda}\right) + \\ + p_b \delta\left(q - \frac{\rho_b}{\lambda}\right) + ... + p_f \delta\left(q - \frac{\rho_f}{\lambda}\right), \quad (6)$$

where $\delta(\cdot)$ – delta function; λ – length of the search step, ρ_i – distance from the extreme; q – current relative value of the distance from the extreme.

This characteristic enables to assess the precision of the final stage, for example to find the probability of event, that the simplex centre in the process of wandering will not leave the given area. In Fig. 5 the distribution density is $f_s(q)$, when $k = 2$ is shown in solid line.

Theoretical calculations were verified by simulation [4]. The software was designed in which the free reflection algorithm of simplex search was used. After carrying 10000 steps of search, under the influence of normally distributed disturbances, when $\sigma = 1$ the following values of probabilities of simple states were found: $p_a = 0,8813$, $p_b = 0,1076$, $p_c = 0,0111$, $p_d = 0$, $p_e = 0$, $p_f = 0$. These probabilities in Fig. 5 are shown in a dashed line. It is obvious, that the results of theoretical calculation and simulation differ insignificantly.

Conclusions

Marginal distribution density of simplex wandering in the environment of the target was calculated. The results of the calculation show that the biggest is the probability of the simplex when near the extreme. If due to strong disturbances, there is sometimes executed an incorrect step, the probability that simplex will return towards the extreme is very big. It is almost impossible, that two or more incorrect steps in the row will be executed, while the transitional probabilities show that the probability of the correct step of the simplex exceeds in several times the probability of the incorrect step. This can be explained by the fact that when receding from the extreme, the useful signal increases and disturbances have less impact on the direction of the search.

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Search optimization of real objects is always carried under conditions of disturbances, which emerge because of imprecise measurement, influence of unknown factors on object functioning indicators, object characteristic change and other reasons. Random disturbances create step direction errors, false view of extreme position and so decrease the effectiveness of search. Because of that, when creating optimization algorithms, applied for solving stochastic problems, it is necessary to perform the search systems research working under disturbances conditions. In this work the marginal distribution density of free reflection simplex search algorithm was found when working under the condition of normally distributed disturbances, with square function target. Simplex state probabilities were calculated using Monte Carlo method. Il. 5, bibl. 5 (in English; summaries in English, Russian and Lithuanian).

Д. Шульскис. Предельная плотность распределения алгоритма симплексного поиска свободного отражения при квадратной функции цели // Электроника и электротехника. – Каунас: Технология, 2008. – № 1(81) . – С. 89–92.

Поисковая оптимизация реальных объектов всегда проводится в условиях помех, которые появляются в последствии неточного измерения, восдействия незнакомых факторов на показатели функционирования объекта, изменения свойств объекта и других причин. Случайные помехи создают отклонения от направления шага, обманчивый вид положения цели и так уменьшают эффективность поиска. Поэтому при создании алгоритмов оптимизации применяемых для решения стохастических задач, обязательно выполнить исследования поисковой системы, работающей в условиях помех. В этой работе была поставлена цель найти предельную плотность распределения алгоритма симплексного поиска свободного отражения при воздействии нормально распределенных помех, когда функция цели квадратная. Для вычисления вероятностей состояния симплекса использован метод Монте Карло. Ил. 5, библ. 5 (на английском языке; рефераты на английском, русском и литовском яз.).

D. Šulskis. Laisvo atspindžio simpleksinės paieškos algoritmo ribinis pasiskirstymo tankis esant kvadratinei tikslo funkcijai // Elektronika ir elektrotechnika. – Kaunas : Technologija, 2008. – Nr. 1(81). – P. 89–92.

Paieškinė realių objektų optimizacija visuomet vyksta trukdžių sąlygomis. Trukdžių atsiranda dėl netikslaus matavimo, nežinomų veiksnių įtakos objekto funkcionavimo rodikliams, objekto savybių pakitimo ir dėl kitų priežasčių. Atsitiktiniai trukdžiai sudaro žingsnio krypties paklaidas, apgaulingą ekstremumo padėties vaizdą ir taip sumažina paieškos efektyvumą. Todėl kuriant optimizacijos algoritmus, pritaikytus stochastiniams uždaviniams spręsti, būtina atlikti paieškinės sistemos, veikiančios trukdžių sąlygomis, tyrimus. Siekta surasti laisvo atspindžio simpleksinės paieškos algoritmo ribinį pasiskirstymo tankį, veikiant normaliai pasiskirsčiusiems trukdžiams, kai tikslo funkcija yra kvadratinė. Simplekso būsenų tikimybėms skaičiuoti panaudotas Monte Karlo metodas. Il. 5, bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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