

Algorithm of Combined Simplex Search with State Recognition

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Introduction

The optimal working mode can vary if the parameters of object fluctuate in time, so it should be continually tuned.

The optimization object can be described by the following equation:

$$y = Q(x, z[t]) + \varepsilon, \quad (1)$$

where y – observational aim parameter value, Q – real aim parameter value, x – k -dimensional controllable variable, z – e -dimensional observational not controllable variable, ε – random noise with normal distribution, mean value 0 and dispersion σ^2 , t – discrete measurement time values.

Function $Q(x, z[t])$ has a maximum at the $x^*[t]$ for each value of $z(t)$. Task of a searching system is to approach the drifting aim and follow it finding a control value $x[t]$ as close as possible to $x^*[t]$. Such kind of task emerges in directed antenna control systems for communications with moving objects, robotics, etc.

An aim of this research is to improve algorithms of simplex search for non-stationary object optimization using effective combined, adaptive methods of simplex search with state recognition.

Optimization using combined simplex search

A direction of each step during the search process is evaluated using measured results of objective function y . There are different values of $z[t]$ at the each measurement moment, so the direction of a next step is found with some error. This error can be eliminated or reduced using combined (active – passive) simplex search with evaluation of observational random variables [1, 2].

The combined simplex search is performed by the rules of a searching algorithm:

The objective function is approximated by separable equation at every step of search:

$$\hat{Q}(x, y, z) = C(x) + H(z) = \sum_{i=1}^k c_i f_i(x) + \sum_{i=1}^e h_i \varphi_i(z), \quad (2)$$

where c_i, h_i – coefficients, $f_i(x)$ and $\varphi_i(z)$ – given functions.

Find the adjusted values of objective function at the simplex apexes:

$$E_j = y_j - \sum_{i=1}^e h_i \varphi_i(z_{ji}), \quad j = 1, \dots, k+1. \quad (3)$$

Make a step of search by reflection of apex evaluating E_j .

If the search is far from the extreme, the linear model (Eq. 2) is accurate enough for the direction to the aim selection. Then

$$H(z) = \sum_{i=1}^e h_i z_i. \quad (4)$$

Values of h_i in (Eq. 4) can be found using iterative equation [3]:

$$h_i[n] = h_i[n-1] + \frac{\Delta y_j[n] - \sum_{i=1}^e h_i[n-1] \Delta z_{ji}[n]}{\gamma + \sum_{i=1}^e \Delta z_{ji}^2[n]} \Delta z_{ji}[n], \quad i = 1, \dots, e, \quad (5)$$

where n – step number; $\Delta y_j[n], \Delta z_{ji}[n]$ – alteration of objective function y and parameter z_i values at the repeated measurement in the obtained apex; γ – smoothing coefficient.

Values of h_i in (4) can be found without repeated measurements using equation:

$$h_i[n] = h_i[n-1] + \frac{y[n] - \hat{y}[n]}{\gamma + \sum_{i=1}^e z_i^2[n]} z_i[n], \quad i = 1, \dots, e, \quad (6)$$

where $y[n]$ – measured value of objective function, $\hat{y}[n]$ – forecasted value of objective function, estimated using model

$$\hat{y}[n] = \sum_{i=1}^e h_i[n-1] z_i[n]. \quad (7)$$

Coefficient evaluation method using repeated measurements (Eq. 5) is more accurate close to the extreme, where objective function is nonlinear, another one (Eq. 6, 7) is more effective if $\Delta z(t)$ changes.

Search with state recognition

The main principles of simplex search with state recognition algorithm construction are described in [4]. The state is defined as a fragment of search, adequate to the complex state of a multilinked Markov chain. Rules of the algorithm with state recognition depend to the state of searching system. These rules define the redistribution of transitional probabilities, transient cancelling of transition between some states and creation of unambiguous transitions adequate to cyclic processes. Such adaptive searching system with variable structure is able to adjust its parameters in subject to the different states during the search process. This feature can significantly increase the efficiency of search.

Miscellaneous attributes can be used for the state recognition in every step of search: difference of objective function value in reflected and obtained apexes, order of apex reflection and diverse rates, calculated using information about character of simplex motion (linear, circular). These attributes can evaluate the success of few steps of search.

The main tasks to be solved for synthesis of simplex search algorithms with state recognition are:

1. Synthesis of multilinked Markov chain taking account of given prehistory, selected criterion and observing some restrictions. The result of synthesis – the obtained Markov chain defines main strategy for every stage of search (climbing, accuracy, following, etc.).
2. States of Markov chain are divided in groups, depending to a situation of search (scrolling, turning, etc.) and different in information about order of apex reflection in past steps. Rules for recognition of each group should be created.
3. Optimization of transitional probabilities for each situation of search according to selected criterion. The result of this optimization is redistribution of transitional probabilities, transient cancelling of transition between some states and creation of unambiguous transitions. Hereby variations with different parameters and structure of Markov chain are obtained, according to a situation of search or a system state, and an algorithm of search can be designed.

The maximum seeking algorithm of combined simplex search with state recognition was created according to mentioned rules [5]:

1. Make a simplex around the centre x_{0i} . The coordinates of apexes are $x_{ji} = x_{0i} + \eta_{ji}L_0$. Measure values of objective function in every apex $y_j, j = 1, \dots, k+1$ and $z_j, j = 1, \dots, k+1$. Select the apex v_s of primary simplex with minimal value y_j , that is $y_s = \inf y_j$.

2. Reflect the apex v_s and calculate coordinates

$$x_{si} = \frac{2}{k} \sum_{j=1}^{k+1} x_{ji} - \frac{2+k}{k} x_{si}, \quad i = 1, \dots, k.$$

Find $y_s^H, y_j := y_s^H (j = s); z_s^H, z_j := z_s^H (j = s);$
 $m_j := m_j + 1, m_s := 0, \quad f := s, x_{ji} := x_{si}^H (j = s);$
 $n := n + 1.$

3. Select an apex v_s with minimal value y_j , i. e. $y_s = \inf_{j \neq f} y_j, j = 1, \dots, k+1$, except newly obtained apex v_f^H .
4. Reflect an apex number s , i. e. obtain coordinates of apex $x_{si}^H, m_j := m_j + 1, j = 1, \dots, k+1;$
 $m_s := 0; n := n + 1.$
5. Find the number of oldest apex r in the last simplex (number n): $m_r = \max \{m_j\}, j = 1, \dots, k+1.$
6. Find the number of next oldest apex p (except r):
 $m_p = \max_{j \neq r} \{m_j\}, j = 1, \dots, k+1.$
7. Calculate values of $h_i(n), i = 1, \dots, e$, using data of simplexes number $(n-1)$ and $(n-2)$ by Eq. 5 or Eq. 6.
8. Calculate adjusted values of objective function:
for simplex $(n-2) \quad E_p = y_p - \sum_{i=1}^e h_i z_i;$
for simplex $(n-1) \quad E_p^H = y_p^H - \sum_{i=1}^e h_i z_i.$
9. Find values of $y_s^H, y_j := y_s^H (j = s);$
 $z_s^H, z_j := z_s^H (j = s); x_{ji} := x_{si}^H (j = s).$
10. Analyze:
a) if $E_p^H - E_p \geq 0$, then $s := r;$
b) if $E_p^H - E_p < 0$, then $s := p;$
return to item 4.

Results

This algorithm of combined simplex search with state recognition was tested on the object with measurable drift of aim and affected by a high level random noise. The computer program for testing was created. The random noise with normal distribution, mean value equal 0 and dispersion $\sigma^2 = 1$ was taken from a table with 2500 items [6]. There were made 1000 steps of search in each experiment. The simplex search algorithm with forbidden backward step was used for comparison in same conditions. Number of observational not controllable parameters was taken equal to the number of controllable parameters. Model of the object was the Eq. 8:

$$Q(x, z) = \sum_{i=1}^k |x_i - v_i z_i|. \quad (8)$$

Results of these experiments are presented in Fig. 1–6 as distribution charts of a distance from the simplex center

to the aim during the search. There is two-dimensional drift of aim with the same speed on axis presented, but it doesn't matter – only resultant drift is important.

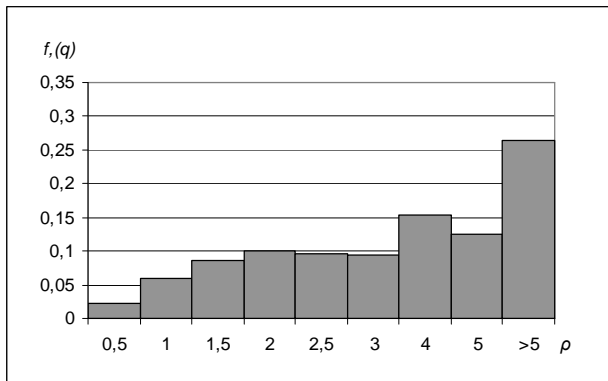


Fig. 1. Distribution chart of a distance from the simplex center to the aim using simplex search with the forbidden backward step algorithm; $k = 2, L = 2, v_0 = v_1 = 0.15, A/\sigma = 1$

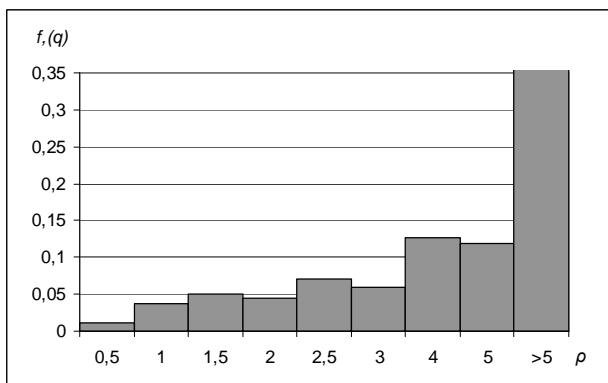


Fig. 2. Distribution chart of a distance from the simplex center to the aim using combined simplex search with a state recognition algorithm; $k = 2, L = 2, v_0 = v_1 = 0.15, A/\sigma = 1$

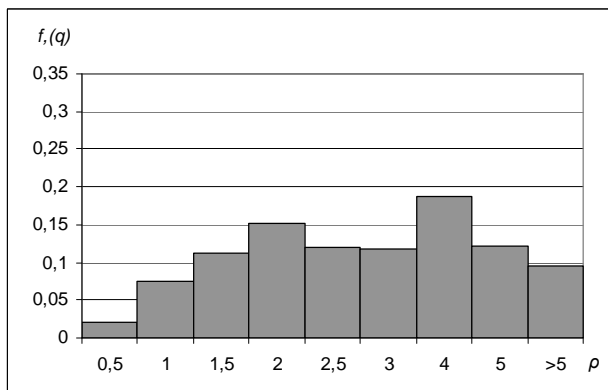


Fig. 3. Distribution chart of a distance from the simplex center to the aim using simplex search with the forbidden backward step algorithm; $k = 2, L = 2, v_0 = v_1 = 0.2, A/\sigma = 1$

There were made more than 200 tests with different sets of parameters. Tested $k = 2, 3, 4, 5$, speed of aim drift until $v_i \leq 0.25L$ (L – size of simplex edge) and noise level $0.5 \leq A/\sigma \leq 5$. Obtained results confirmed advantage of designed algorithm, especially when noise level is very high ($0.5 \leq A/\sigma \leq 2$).

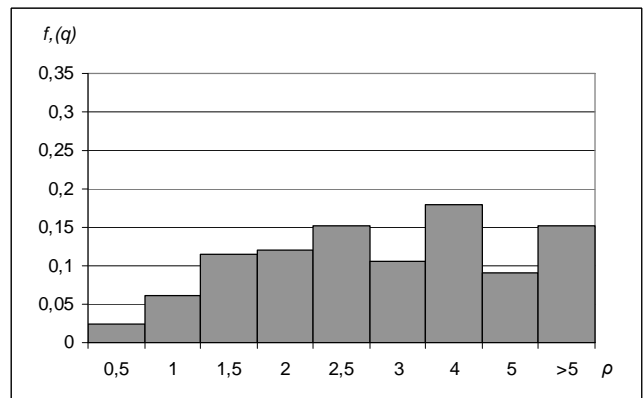


Fig. 4. Distribution chart of a distance from the simplex center to the aim using combined simplex search with a state recognition algorithm; $k = 2, L = 2, v_0 = v_1 = 0.2, A/\sigma = 1$

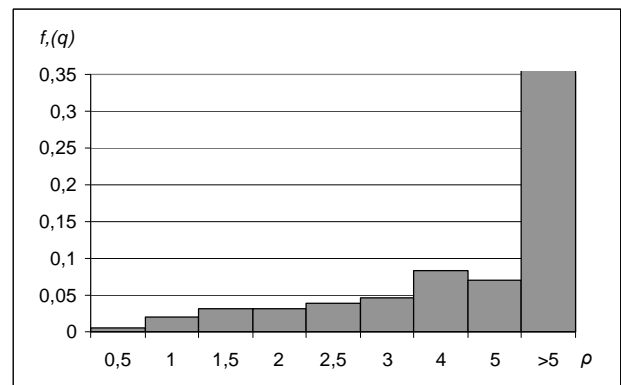


Fig. 5. Distribution chart of a distance from the simplex center to the aim using simplex search with the forbidden backward step algorithm; $k = 2, L = 2, v_0 = v_1 = 0.25, A/\sigma = 1.5$

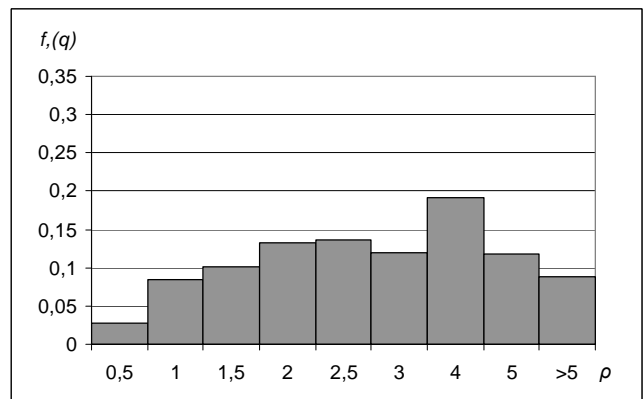


Fig. 6. Distribution chart of a distance from the simplex center to the aim using combined simplex search with a state recognition algorithm; $k = 2, L = 2, v_0 = v_1 = 0.25, A/\sigma = 1.5$

The average distance from simplex centre to the aim r_{ave} was calculated, and the combined search with the state recognition algorithm showed much lower values, for example, for $k = 2, L = 2, v_0 = v_1 = 0.35, A/\sigma = 1$ for a combined search with state recognition $r_{ave} = 8.437$, for a simplex search with the forbidden backward step $r_{ave} = 152.757$, for $k = 3, L = 3.16, v_0 = v_1 = v_2 = 0.2, A/\sigma = 1$ they are respectively $r_{ave} = 6.679$ and $r_{ave} = 57.242$.

Conclusions

1. The combined simplex search with state recognition is more effective than combined simplex search with forbidden backward step in tested conditions.

2. The designed algorithm of combined simplex search with state recognition has no limit of observational non-controllable variables.

References

1. Дамбраускас А. П. Симплексный поиск при дрейфе цели // Изв. вузов. Электромеханика. –1979. – No 2. – С. 173–179.

2. Круг Г. К., Масальский Г. Б. Симплексный инвариантный метод экспериментальной оптимизации // Вопросы кибернетики. – Москва: Академия наук СССР. – 1981. – С. 3–32.
3. Райбман Н. С., Чадеев В. М. Построение моделей процессов производства. – Москва: Энергия, 1975. – 376 с.
4. Dambrauskas A. Simpleksinės paieškos metodai. – Vilnius: Technika, 1995. – P. 192–202.
5. Dambrauskas A., Udris D. Simpleksinės paieškos tyrimas tikslinio etape // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2004. – Nr. 5(54). – P. 47–51.
6. Большев Л. Н., Смирнов Н. В. Таблицы математической статистики. – Москва: Наука, 1983. – 416 с.

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Searching optimization is usual for objects with unknown or unsolvable mathematical model, and result depends on the efficiency of algorithm. Simplex search has shown good results when optimization object and measurement results are affected by high level noise. Improvement and investigation of simplex search algorithms is an aim of this research. Characteristics of the object can vary in time during optimization process, and the search has to follow the extreme. If the variation is a result of some measurable but non-controllable parameters, the search can be improved using the information about this variation in past steps and calculating the prognosis, or performing repeated measurements of these parameters and aim value. The influence of multidimensional measurable parameters is estimated in presented algorithm of combined simplex search with state recognition, and characteristics of the optimization process are investigated. Il. 6, bibl. 6 (in English; summaries in Lithuanian, English, Russian).

A. Дамбраускас, Д. Удрис. Алгоритм комбинированного симплексного поиска с распознаванием состояния // Электроника и электротехника. – Каунас: Технология, 2008. – № 1(81). – С. 11–14.

Когда модель объекта неизвестна или труднорешаема и систему воздействуют сильные случайные помехи, могут быть применены алгоритмы симплексного поиска. Если условия поиска еще усложнены многокоординатным дрейфом цели, для успешного слежения нужны алгоритмы, способные учитывать воздействие дрейфа. Целью этого исследования стало создание и проверка комбинированного алгоритма симплексного поиска с распознаванием состояния, способного учитывать многокоординатный дрейф цели, то есть несколько неуправляемых но измеримых параметров, влияющих на значение целевой функции. Влияние дрейфа учитывается при помощи повторных измерений значения целевой функции и дрейфа, или прогнозирования этих значений. Прогнозирование опирается на предположение, что на следующем шаге поиска эти параметры изменяется так же, как и на предыдущем, то есть поиск ведется в этапе восхождения. Исследование подтвердило преимущество данного алгоритма перед алгоритмом симплексного поиска с запретом возвратного шага. Ил. 6, библи. 6 (на английском языке; рефераты на литовском, английском и русском).

A. Dambrauskas, D. Udris. Kombinuotos simpleksinės paieškos algoritmas, galintis atpažinti būsenas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2008. – Nr. 1(81). – P. 11–14.

Kai objekto modelis nežinomas ar sunkiai išsprendžiamas bei veikia stiprūs atsitiktiniai trukdžiai, ekstremumo paieškai gali būti taikomi simpleksinės paieškos algoritmai. Jeigu paiešką apsunkina tikslo dreifas, sąlygotas keleto nevaldomų, bet išmatuojamų parametrų kaitos, reikalingi algoritmai, galintys įvertinti šią įtaką. Šio tyrimo tikslas – sudaryti ir iširti galintį atpažinti būsenas kombinuotos simpleksinės paieškos algoritmą, skirtą paieškai esant daugiakoordinaciui tikslo dreifui. Tikslo dreifas gali būti įvertinamas atliekant papildomus minėtų parametrų bei tikslo funkcijos verčių matavimus arba prognozuojant šias vertes. Prognozė remiasi prielaida, kad kitame paieškos žingsnyje šios vertės keisis taip pat, kaip ir buvusiame. Tyrimai patvirtino sudarytojo algoritmo pranašumą, palyginti su uždraustos grįžties simpleksinės paieškos algoritmu, kai objektą veikia keli nevaldomi parametrai, sąlygojantys tikslo dreifą, bei stiprūs atsitiktiniai trukdžiai ($0,5 \leq A/\sigma \leq 2$). Il. 6, bibl. 6 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).