

## Analysis of Geometrical Accuracy of Long Grating Scale Calibration Comparator

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### Introduction

One of the main components of a line position calibration error in linear scale calibrators are geometrical errors of the comparator, i.e. the errors associated with the carriage's movement deviations – its slight angular fluctuations about the main movement axis and the axes perpendicular thereto, and the linear displacements along those perpendicular axes. The most important are the carriage's angular fluctuations about the abovementioned perpendicular axes due to nonobservance of *Abbe* principle.

Geometrical errors are reduced by means of increase in the production accuracy, or they are compensated. The increase in the production accuracy is associated with growing expenses, and most importantly, is limited which fact renders the accuracy insufficient for precision scale calibration.

Geometrical errors are further reduced by means of computational and active methods of error compensation.

In the first case the movement errors are measured and approximated by parametric functions, measurement error-to-movement errors dependence models are created and the errors are compensated in accordance with those models on the real time scale. The value of residual errors depends on the stability of movement errors and the accuracy of approximation of their systemic components by parametric functions [1].

In the case of active error compensation the compensational values of the angular fluctuations of the carriage are determined in accordance with the difference of the displacement of two various points of the carriage on the compensation plane. Active angular fluctuation compensation errors are associated with the impact produced by the changes in the environment parameters on the laser beam wavelength. Long measuring systems with a laser beam spanning over a few meters in the air are especially sensitive to such changes in the environment parameters. The type of the long linear scale calibration system under discussion is exactly the same.

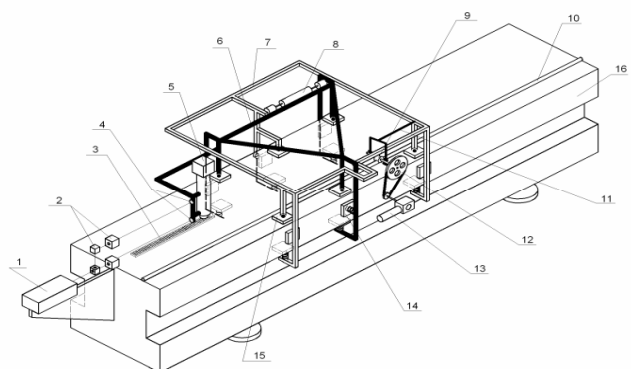
Errors of these methods are analyzed in this paper.

They will be compared from the point of view of accuracy and efficiency.

### The Object of the Research

The research has been conducted by using a maximum 3500 mm long linear grating scale calibration comparator [2, 3]. Fig. 1 shows the principal schematic diagram of the comparator. The diagram shows all main units and systems of the comparator designed for precise realization and measurement of movement and detection of the scale line position.

A length comparator has been constructed wherein a microscope with an ultra-fast global shutter photosensitive cell matrix moves along the measuring glass scale on a granite base. Heterodyne laser interferometers with He-Ne lasers are used for precise detection of the line scale displacement.



**Fig. 1.** The comparator's scheme with the main units and systems. 1- laser, 2- system of compensation *Abbe* errors in the vertical plane, 3- scale to be calibrated, 4- retroreflectors, 5- microscope with CMOS camera, 6- precise part of carriage, 7- powered part of carriage, 8- joint of carriage's parts, 9- friction wheel, 10- beam of friction gear, 11- joint of friction gear with carriage's parts, 12- belt gear, 13- gear-motor, 14- springy aerostatic bearing, 15- rigidly mounted aerostatic bearing, 16- granite basis with the guide-ways

The comparator's basis comprises a 4 meter long massive fine-structured granite traverse placed on four pneumatic supports on the horizontal plane for damping of high frequency vibrations and a carriage moving along the traverse. The maximum level of ground base vibrations does not exceed 0.7  $\mu\text{m}$  at 25 Hz and 0.1  $\mu\text{m}$  at the rest range [5]. After feeding the air into the aerostatic bearings of the carriage it slides on the air bearings along the basis on six high-precision guide-ways. The clearance between air bearings and the guide-ways amounts to 6  $\mu\text{m}$ . Air consumption is very low. The carriage is designed so that it moves on rigidly mounted aerostatic bearings that are tight adjusted by the springy aerostatic bearings mounted on the opposite side of the carriage's guide-ways. Thus, the desirable small clearance of the bearings is regulated and the rigidity thereof is increased.

The carriage is pulled along the granite guide-ways by means of a program-controlled friction gear. The carriage and gear system design eliminates the influence of the drive on the precision of linear movement. To this end, the carriage consists of the two parts, i.e. a power part and a precise part (see Fig. 1). A drive is connected to the first part, whereas all measuring systems designed for the measurement of the carriage's movements and for the detection of the calibrated scale lines are connected to the precise part.

Microscope with a CMOS camera is mounted on the carriage, and is adjustable by means of the focus setup system. Digital measuring microscope enables precise positioning of the length calibration systems, estimation of the line edge quality and precise location of lines.

Compensation of *Abbe* errors on the vertical plane is implemented in the additional optical interferometer system with the error calculation software. *Abbe* error on the horizontal plane is minor and is not subject to calculation. Temperature of scale and air along the path of the laser beam is measured in ten fixed positions. Results are obtained by the data logger and sent to the main computer. Temperature extension of a scale is compensated in real time.

The measured length is normalized to the designed value at 20°C by means of measuring scale temperature and using the coefficient of thermal expansion. Environmental corrections for the refractive index of the air are made by means of calculations using *Edlen's* equation. The whole calibration process and all system operations are controlled by the main PC operating under the specific operation algorithm inclusive of error compensation.

## Methodology

It is assumed that the analyzed error  $\Delta_Q$  consists of the systematic component definable by function  $F_Q$  that approximates this error, and random error  $E_Q$ .

$$\Delta_Q = F_Q(\xi, \theta) + E_Q; \quad (1)$$

here  $\xi=(\xi_1, \xi_2, \xi_k)$  – arguments (coordinates, temperature, etc.), registered during each measurement;  $\theta=(\theta_1, \theta_2, \dots, \theta_n)$  – parameters of function  $F_Q$ .

The following is collected for the approximation of error  $\Delta_Q$ : basic arguments  $\xi_j$  the number whereof is minimized; approximation function  $F_Q$  that is simple for calculation and the parameters whereof are memorized in advance; values of parameters  $\theta$  enabling to minimize the standard deviation (STD) of the error values from the approximation function

$$S_{pF} = \left\{ \frac{1}{N} \sum_{j=1}^n \left| \Delta_Q(\xi(j)) - F_Q(\xi(j), \hat{\theta}) \right|^2 \right\}^{\frac{1}{2}}. \quad (2)$$

The maximum deviation of errors from the approximation function is also calculated

$$M_{pF} = \max_{1 \leq j \leq N} \left| \Delta_Q(\xi(j)) - F_Q(\xi(j), \hat{\theta}) \right|. \quad (3)$$

The accuracy of error approximation by parametric functions is estimated by deviations  $S_{pF}$ ,  $M_{pF}$  and, by comparing the deviations of the approximation function values from the mean errors  $S_{pV}$ ,  $M_{pV}$  at the STD and maximum deviations of the error values from the average  $S_{pV}$ ,  $M_{pV}$  respectively, which are calculated according to analogical formulas (Eq. 1, Eq. 2).

The character of errors and reasons of their occurrence are determined, the possibilities to evaluate and eliminate them are achieved during the analysis of spectral density. The evaluation is performed by period-diagram  $I(\nu)$  of errors [4]:

$$I(\nu) = \frac{1}{2\pi M} \left| \sum_{k=1}^M (\bar{\delta}_k - \mu) e^{-ik2\pi\nu} \right|^2, \quad 0 \leq \nu \leq \frac{1}{2}, \quad (4)$$

here  $\mu = \frac{1}{M} \sum_{k=1}^M \bar{\delta}_k$ ,  $k=1, 2, \dots, M$  – is an average of the displacements,  $\bar{\delta}_k$  – are results of experiment,  $\nu$  – is a frequency.

The analysis of the main errors of the guide-way form has shown that its general harmonic components are low frequency components. The summary value of deviations of the guide-way form within the whole length of the 4 m base does not exceed 4  $\mu\text{m}$  on the vertical plane and 3  $\mu\text{m}$  on the horizontal plane.

The period diagram gives the clear expressed maximum on the period-diagram at frequency  $\nu_j=0.008$ , which corresponds to the dominated component of the guide-way form error. The STD of the approximation function values from the values of the form error is  $S_{pF}=0.06 \mu\text{m}$ .

After evaluating the abovementioned analyses the following algebraic polynomial splines have been adopted for approximation of the angular fluctuations on the vertical and horizontal planes

$$F(x) = \sum_{i=0}^m a_i x^i + \sum_{j=1}^v \sum_{k=1}^d b_{jk} (x - u_j)^{m+1-k}, \quad (5)$$

here  $m$  is a spline row,  $m = 1, 2, \dots; v$  – number of units  $u_1, u_2, \dots, u_v$ ;  $d$  – spline defectiveness,  $d = 1, 2, \dots, m+1$ ;

$$X_+ = \begin{cases} x, & x > 0 \\ 0, & x \leq 0 \end{cases}, \theta = (a_0, \dots, a_n; u_1, \dots, u_v; b_{11}, \dots, b_{vd}), n_F = m + 1 + (d + 1)V. \quad (6)$$

The tests were conducted in a thermostable laboratory. During the tests the following ambient conditions were maintained in the laboratory: temperature at  $+20 \pm 0.15^\circ\text{C}$ , moisture  $40 \pm 0.10\%$ , the air pressure changed within the range of  $1000 \pm 10$  hPa. To avoid formation of intensive air flows and turbulences in the room, the air was fed into and evacuated from the room in many places.

To obtain the data for the computational approximation, the angular fluctuations of the carriage were measured with a *Hewlett Packard* HP 5526A model laser interferometer at constant velocity of the carriage and steady scale calibration speed of 4 mm/s.

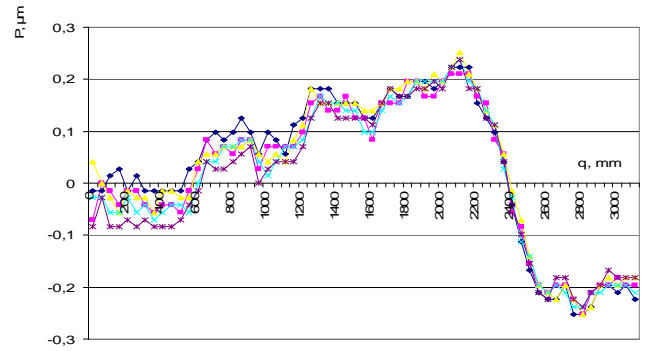
The angular fluctuation of the carriage is measured several times while the carriage is moving in different directions. The values of the corrected data of the scale line position measurement are found by multiplying the angular fluctuation values by the distance between the carriage's displacement measurement line and the plane of the measured scale lines. The average of various values of its displacement is found from the corrected data of several measurements. This line of average values is approximated by the spline (Eq. 5).

In the case of active compensation, while calibrating scale lines with a *Zygo* LOT ZMI 2000 model heterodyne laser interferometer, the difference of the displacements of the carriage's two points in the direction of measurement on the vertical plane is measured. The corrected value of the line position reading error is calculated by multiplying the abovementioned difference of displacements by the distance between the abovementioned points and the distance between the carriage's displacement measuring line and the plane of the measured scale lines. In the course of measurements performed with both laser interferometers the ambient air temperature, air humidity and air pressure are registered, and the impact of the above factors on the wavelength of the laser beam is corrected in real time.

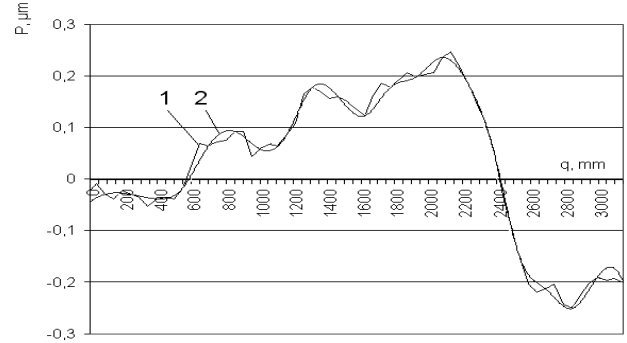
## Related work

Graphs of measurement corrections of the carriage's angular fluctuations about the horizontal transverse axis plotted against the displacement value are presented in Fig. 2. Line 1 of these average values and graph 2 of the spline approximating this line are shown in Fig. 3.

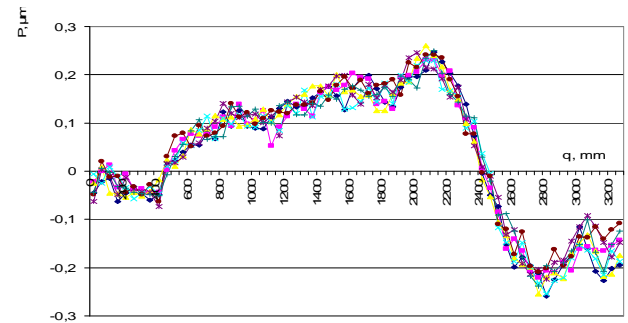
After calculating it was found that the maximum value of the compensated systematic corrections defined by the approximation spline (Eq. 5) is  $0.475 \mu\text{m}$ , and the STD of the uncompensated corrections occurring due to a random component of the carriage's angular fluctuations, from the approximation function is  $S_{pF} = 0.022 \mu\text{m}$  and the maximum deviation from the approximation function is  $M_{pF} = 0.076 \mu\text{m}$ .



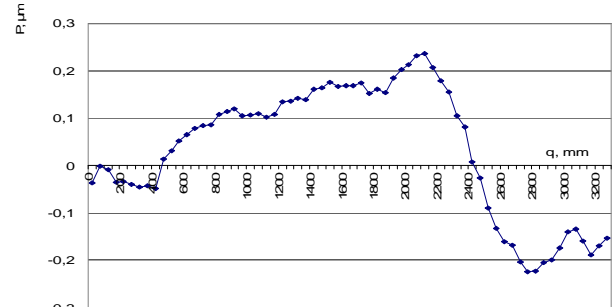
**Fig. 2.** Graphs of the corrections of angular fluctuations about the horizontal axis



**Fig. 3.** Line 1 of average values of the corrections of the angular fluctuations about the horizontal axis and the line's approximation 2



a



b

**Fig. 4.** Graphs of the correction measurements, evaluated by the active compensation method of the carriage's angular fluctuations about the horizontal axis (a) and the mean values thereof (b)

The accuracy of approximation is evaluated in respect of the mean values by the STD of the approximation function values. It is  $S_{FV} = 0.028 \mu\text{m}$ . It illustrates a sufficiently high accuracy of approximation. In the case of

ideal compensation, i.e. by calculating the compensation values in accordance with the average corrections of special measurements, the maximum value of compensated corrections is  $0.488 \mu\text{m}$ , and the STD of uncompensated corrections is  $S_{pV} = 0.018 \mu\text{m}$ .

Measurement corrections of the carriage's angular fluctuations about the horizontal transversal axis (a) and the line of their mean values (b), evaluated by the active compensation method, are presented in Fig. 4.

In this case the amount of the compensated error should be evaluated by the sum of the absolute magnitudes of positive and negative mean values. It is equal to  $0.498 \mu\text{m}$ . The amount of the uncompensated corrections is determined by the mean square deviation of their values in respect of the averages,  $S_{pV} = 0.0196 \mu\text{m}$ .

The compensated systematic component of corrections of the carriage's fluctuations about the vertical axis for the research comparator is  $0.085 \mu\text{m}$ . The mean square deviation of the uncompensated corrections from the average values reaches  $0.0067 \mu\text{m}$ .

## Conclusions

1. Errors of laser displacement measuring systems manifest themselves in large path measuring systems, which errors cause considerable random errors of compensation of the moving units, minor angular fluctuations and the measurement errors resulting from nonobservance of *Abbe* principle.

2. Two methods that have been submitted for the analysis of large path precision stable measuring systems – the computational method under which the angular

fluctuation errors are measured and approximated by parametrical functions, and are calculated and arranged according to them in real time, and the active method wherein angular fluctuations are directly measured and the errors resulting from them are also compensated in real time, provide comparable accuracy.

3. With respect to less stable systems whereof random error component is relatively larger in comparison with the one under investigation, the active compensation method provides a higher accuracy of compensation of angular fluctuation errors of the moving units.

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**A. Barakauskas, A. Kasparaitis, A. Šukys, P. Kojelavičius.** Analysis of Geometrical Accuracy of Long Grating Scale Calibration Comparator // *Electronics and Electrical Engineering*. – Kaunas: Technologija, 2008. – No. 1(81). – P. 3–6.

Line detection errors in linear scale comparators occur in the cases where *Abbe* principle is ignored. These errors consist of random and systematic components and can be minimized by increasing the accuracy of component design and production, as well as by compensating geometrical deviations. Design methods intended to increase the accuracy of the precision movement of comparator parts are described here. Experimental error investigation results are presented. Computational and active methods of geometrical errors compensation are established and described. Investigation results are arranged and efficiency of the examined methods is compared. Ill. 4, bibl. 5 (in English; summaries in English, Russian and Lithuanian).

**А. Баракаускас, А. Каспарайтис, А. Шукис, П. Коялавичюс.** Анализ геометрической точности компаратора растровых линеек большой длины // *Электроника и электротехника*. – Каунас: Технология, 2008. – № 1(81). – С. 3–6.

Погрешности определения положения линии в компараторах линейных шкал проявляются при несоблюдении принципа *Аббе*. Эти погрешности охватывают случайные и систематические составляющие и могут быть уменьшены путем увеличения проектной и технологической точности, а также путем компенсации геометрических погрешностей. Изложены проектные методы для увеличения точности прецизионного движения узлов компаратора. Приведены результаты экспериментального исследования. Составлены и приведены компьютерный и активный метод компенсации геометрических погрешностей. Изложены результаты исследования и сопоставлена эффективность испытанных методов. Ил. 4, библи. 5 (на английском языке; рефераты на английском, русском и литовском яз.).

**A. Barakauskas, A. Kasparaitis, A. Šukys, P. Kojelavičius.** Ilgujų rastrinių skalių kalibravimo komparatoriaus geometrinio tikslumo analizė // *Elektronika ir elektrotechnika*. – Kaunas: Technologija, 2008. – Nr. 1(81). – P. 3–6.

Linijinių skalių komparatoriuose linijos padėties nustatymo paklaidų atsiranda, kai neatsižvelgiama į *Abbe*s principą. Šias paklaidas sudaro atsitiktinės ir sistemingosios dedamosios. Jos gali būti sumažintos didinant elementų projektinį ir technologinį tikslumą, taip pat kompensuojant geometrines paklaidas. Aprašyti projektiniai komparatoriaus elementų precizinio judesio tikslumo padidinimo metodai. Pateikti paklaidų eksperimentinio tyrimo rezultatai. Sudaryti ir aprašyti geometrinių paklaidų skaičiuojamosios ir aktyviosios kompensacijos metodai. Išdėstyti tyrimo rezultatai ir palygintas bandytų metodų efektyvumas. Il. 4, bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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