ELECTRONICS AND ELECTRICAL ENGINEERING

ISSN 1392 – 1215 2007. No. 8(80)

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Algorithm for Detecting Deterministic Chaos in Pseudoperiodic Time Series

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Introduction

A long-standing fundamental issue in nonlinear time series analysis is to determine whether a complex time series is regular, deterministically chaotic, or random. An accurate identification of the dynamics underlying a complex time series, is of crucial importance in understanding the corresponding physical process, and in turn affects the subsequent model development. A steady stream of efforts has been made, and a number of effective methods have been proposed (also in the latest years [1]-[3]) to tackle this difficult problem. The vast majority of these methods are based on attractor reconstruction from time series and such characteristics as largest Lyapunov exponent, K2 entropy, and correlation dimension calculation [4], [5]. However, the existence of noise, which may mask or mimic the deterministic structure of the time series, can lead to spurious results [6]. Finally, most of these approaches depend heavily on a good reconstruction of the phase-space geometry of the dynamical system. Since there is no unique way to choose the embedding dimension and the time lag, the accuracy of such methods is hard to guarantee [6], [7]. For a certain class of time series this task is simpler. Zhang *et al* [6] have proposed a different method to detect deterministic structure from a pseudoperiodic time series. By using the correlation coefficient as a measure of the distance between cycles, they were exempted from the phase-space reconstruction and construct a hierarchy of pseudocycle series. Appropriate statistics are then applied to reveal the temporal and spatial correlation encoded in this hierarchy of the pseudocycle series, which allows for a reliable detection of determinism and chaos in the original time series. The algorithm developed here is also based on the concept of the using correlation coefficient as a measure of the distance between cycles, but the execution of the idea in this paper is different. We present the straightforward and noisy resistant algorithm to detecting chaos in pseudoperiodic time series by using the correlation coefficient as a measure of the distance between cycles, but without building a hierarchy of pseudocycle series. Proposed algorithm is similar to Rosenstein algorithm [8] for largest Lyapunov exponent calculating, but without reconstructing of the attractor dynamics. Same as in work [6], throughout the paper, we use the *x* component of the well-known Rossler system and an experimental laser dataset for illustration, both of which are chaotic and contain obvious periodic component. The laser dataset is the record of the output power of the NH_3 laser available in Santa Fe Competition (Data Set A).

Description of the algorithm

1) Given a pseudoperiodic time series $\{X_i\} = \{x_1, x_2, \cdots, x_N\}$ of length N, one can define *m*-lengths sequence cycles $C^{(m)}(i) = \{x_i, x_{i+1}, \cdots, x_{i+m-1}\}$. These cycles represent *m* consecutive *x* values, commencing with the ith point and *m* is defined approximately as one period T_p of pseudoperiodic time series. For a given $C^{(m)}(i)$: $i = 1, 2, \cdots, N/2 - m + 1 - T_p - T_d$ (T_d is explained below) the correlation coefficient ρ_{ij} as the distance between each pair of cycles *C(i)* and *C(j)* for $|j - i| \geq T_p$ is calculated. The correlation coefficient characterizes the similarity between cycle *C(i)* and *C(j)*. The larger the correlation coefficient, the higher the level of similarity. Considering the continuity and smoothness of the vector fields of deterministic systems, two cycles with a larger ρ_{ij} will also be close in the phase space, i. e., for the relation between cycles describing, the correlation coefficient can be used equivalently as the phase-space distance [6].

2) The search of most similar cycles is executed using a sliding overlapping window of constant length $T = N/2$ for all *i*. For a given *i* the values of lag *j* changes from $1{+}T_p$ to $N/2{+}T_p$. The constraint $|j - i| \geq T_p$ is necessary to exclude temporally correlated points.

3) The algorithm locates most similar ij^{th} pair of cycles (with maximum correlation coefficient ρ_{mi}) of each point *i*. Like Rosenstein algorithm [8], the averaged divergence between two nearby cycles $\rho_m(k)$ at time steps *k* ($k = 1, 2, \cdots, T_d$) is calculated

$$\rho_m(k) = \frac{1}{\Delta t}\left\langle \ln \rho_{mi}(k) \right\rangle , \qquad (1)$$

where $\langle \cdots \rangle$ denotes the average over all values of i, Δt – the sampling period of the time series. This process of averaging is the key to calculating divergence between two nearby cycles in presence of noise

4) For chaotic systems, the distance between two nearby cycles will increase exponentially over time due to the very nature of sensitivity to initial conditions. Therefore, the correlation between two nearby cycles, which decreases smoothly and monotonously with the distance between cycles [6], is also expected to drop exponentially with the step k. The semilogarithmic plot $\ln(\rho_m(k)) \sim k$ (or versus time $t = k \cdot \Delta t$) thus appears to be a line nearby straight, whose slope is actually related to the largest Lyapunov exponent. The larger the $\left|\Delta \ln \rho_m(k)/\Delta k\right|$, the higher the level of chaos. So we can use $\left|\Delta \ln \rho_m(k)/\Delta k\right|$ as an indicator of chaos. Since $\rho_m(k)$ is close to 1, $\ln(\rho_m(k)) \approx \rho_m(k) - 1$ and we can estimate slope as $\left|\Delta \rho_m(k)/\Delta k\right|$. The curve saturates at longer times and T_d is defined normally only for the slope region.

Results

We consider the influence of different types of noise on the measure we have defined. In the case of additive noise, i.e., measurement or instrumentation noise, all the pair-wise correlation coefficient $\rho_{mi}(k)$ will decrease. However, since the additive noise has no preference in influencing different cycles in the time series, $\rho_{mi}(k)$ will decrease roughly to the same extent, and their averaged divergence remains nearly unchanged. Fig. 1 shows a plot of $\langle \rho_{mi}(k) \rangle$ versus k (in each figure "<Divergence>" and "Iteration" are used to denote $\langle \rho_{mi}(k) \rangle$ and k, respectively) for the x component of the Rossler system with additive white Gaussian noise and colored noise (1/f noise) of different levels. The Rossler system is given by

$$\begin{cases} \dfrac{dx}{dt} = -(y+z), \\ \dfrac{dy}{dt} = x + a \cdot y, \\ \dfrac{dz}{dt} = b + z(x-c); \end{cases} \qquad (2)$$

with parameters a = 0.2, b = 0.2 and c = 5.7.

Fig. 2 shows plot for experimental laser data set with additive white Gaussian noise and colored noise of different levels. In Fig.1, 2, we can see that algorithm can successfully detect chaos in the presence of additive white Gaussian and colored noise – the slope of lines indicates chaos for signal-noise ratio (SNR) up to 10 dB. For a periodic sinusoidal signal with noise there are no such relations – the curves of $\langle \rho_{mi}(k) \rangle$ versus k remain flats for SNR up to 10 dB for white Gaussian noise and for colored noise (Fig.3).

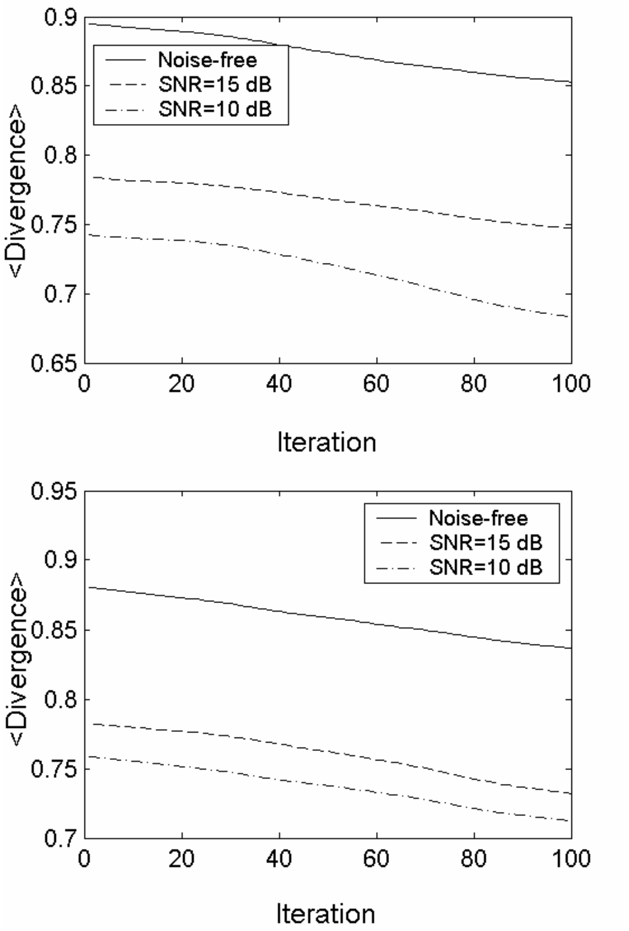

Fig. 1. Plot of <Divergence> versus iteration for the x component of the Rossler system (series length 1600) with additive Gaussian noise (upper panel) and additive colored noise (lower panel) of different levels

In the case of additive white Gaussian noise the correlation coefficients between different pairs of cycles are random and $\langle \rho_{mi}(k) \rangle$ will assume statistically the same value for different k, while for the colored noise, though cycles of the noisy periodic signal might appear "correlated" due to the intrinsic correlation of the colored noise, we cannot find the scaling region in the plot of $\langle \rho_{mi}(k) \rangle \sim k$, since $\langle \rho_{mi}(k) \rangle$ are roughly the same for k shorter than the decorrelation time of the noise [6].

Discussion and conclusions

Through this approach, we can also discriminate between a low-dimensional chaotic signal and a periodic signal with noise. For low-dimensional pseudoperiodic chaotic signal with slow divergence between nearest neighbors it is possible to find the similarity between relative length cycles without a risk that first parts of two cycles are close and the late parts could be far away from each other due to the influence of the positive Lyapunov exponents. Multiple averaging of distance measuring (correlation coefficient) between two nearby relative long cycles allows reducing the influence of random highdimensional noise. For the chaotic time series, as we have seen, correlation coefficient $\langle \rho_{mi}(k) \rangle$ will decrease with k, and a scaling region is present in the plot of $\langle \ln \rho_{mi}(k) \rangle \sim k$ or $\langle \rho_{mi}(k) \rangle \sim k$. While for periodic signals with noise, there are no such relations. The algorithm performed reasonably well for SNR up to 10 dB.

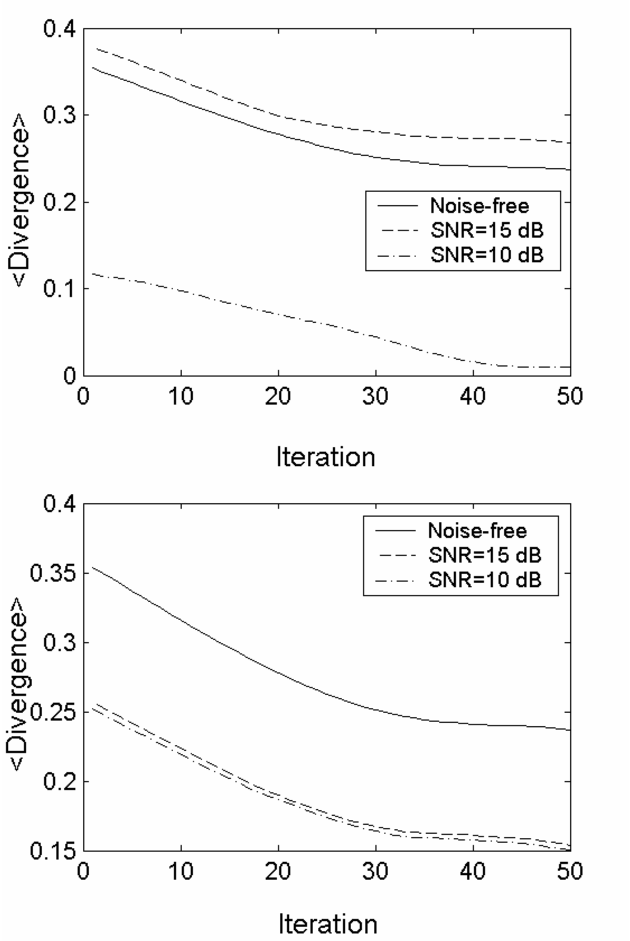

Fig. 2. Plot of <Divergence> versus time for the experimental laser data set (series length 1000) with additive Gaussian noise (upper panel) and additive colored noise (lower panel) of different levels

The traditional universal algorithms for calculating largest Lyapunov exponents [8], [9] cannot reliable estimate the largest Lyapunov exponents at noise level about SNR = 10 dB.

In summary, we have proposed a noise resistant algorithm to detect deterministic structure and chaos for time series data exhibiting strong pseudoperiodic behavior. The intrinsic correlation of the data set is studied on the scale of single cycles by using a similarity measure, thus phase-space reconstruction can be avoided. Differently from method proposed by Zhang *et al* [6], where the pseudoperiodic time series are segmented into consecutive (no overlapping) cycles according to the local minimum (or maximum), we use overlapping cycles similarly to phase-space vectors of reconstructed dynamics. This allows obtaining the results with relative small data sets. For example, in paper [6] 1596 cycles for the Rossler system and 1224 cycles for laser data set are used to produce a wider scaling region for visual inspection. In our work the series length for Rossler system is 1600 and for laser data set – 1000 points, i.e. comparable with series length for calculating largest Lyapunov exponents according to the Rosenstein method [8].

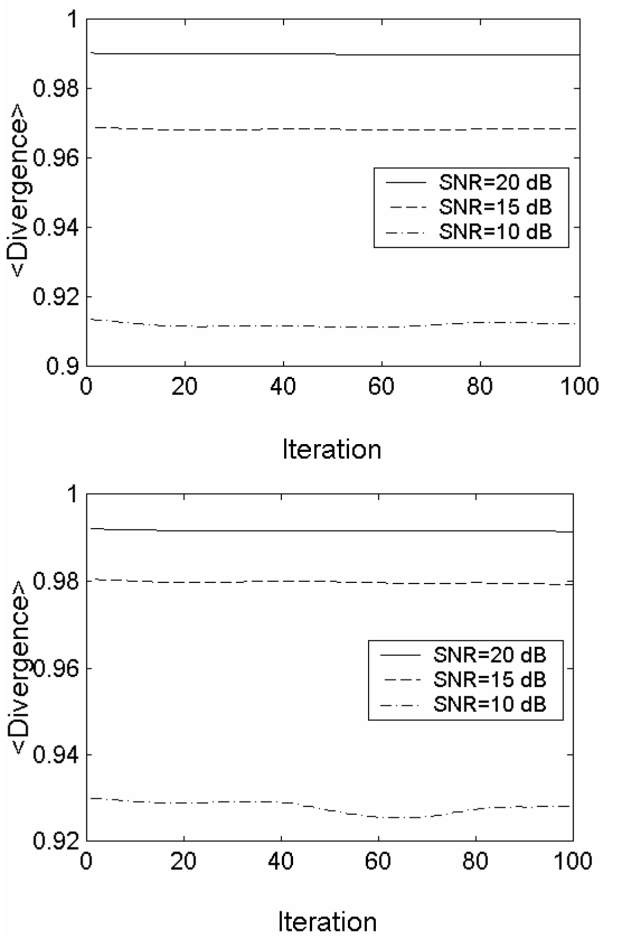

Fig. 3. Plot of <Divergence> versus time for a periodic sinusoidal signal (series length 1600) with additive Gaussian noise (upper panel) and additive colored noise (lower panel) of different levels

In case of the high level of noise, algorithm can be successfully applied together with nonlinear phase-space reconstruction and the principal components analysis (PCA) – a well known technique, which can be used for pseudoperiodic signal prefiltering [10]. Starting from a time series $\{X_i\} = \{x_1, x_2, \cdots, x_N\}$, a matrix Y is constructed by using the nonlinear embedding phase-space technique [4], [5] as follows:

$$Y(i, j) = x(j + (i - 1)\tau), \qquad (3)$$

where $j = 1, \cdots, N - m\tau$, $i = 1, \cdots, m$, and m and τ are the reconstruction dimension and time delay, respectively. By applying PCA [5], [11] for noise reduction and choosing only some eigenvectors with the largest eigenvalues of covariance matrix, the projected vectors are calculated with reduced dimension $l < m$. Finally, chaos is detected according to the introduced algorithm. If $l > 1$, two-dimensional correlation coefficient between two matrices of l-dimensional vectors is used as a measure of the distance between cycles instead of ordinary correlation coefficient. By using PCA as a primary noise reduction method, we have reliably detected chaos for noisy Rossler signal (with white Gaussian noise) with a proposed algorithm for SNR up to 6 dB (we chose $m = 10$, $\tau = 1$ and $l = 2$).

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Submitted for publication 2007 07 15

K. Pukenas, K. Muckus. Algorithm for Detecting Deterministic Chaos in Pseudoperiodic Time Series // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 8(80). – P. 53–56.

A new straightforward algorithm is proposed to detect deterministic structure from a pseudoperiodic time series without embedding. The correlation coefficient as a measure of the distance between certain cycles is used and averaged diverging slope between closest cycles, as an indicator of chaos, is calculated according well known Rosenstein method for calculating largest Lyapunov exponents. We demonstrate that this method can reliably identify chaos in the presence of noise of different sources at a signal-noise ratio up to 10 dB for both artificial data and experimental time series. Ill 3, bibl. 11 (in English; summaries in English, Russian and Lithuanian).

К. Пукенас, К. Муцкус. Алгоритм обнаружения детерминистического хаоса в псевдопериодических временных рядах // Электроника и электротехника. – Каунас: Технология, 2007. – № 8(80). – С. 53–56.

Предлагается новый простой алгоритм для обнаружения детерминистического начала в псевдопериодических временных рядах без реконструкции фазового пространства. В качестве меры дистанции между определенными циклами временного ряда применен коэффициент корреляции, на основании которого рассчитывается усредненная наклонная дивергенции между ближайшими (с наименьшей дистанцией) циклами, являющаяся индикатором детерминистического хаоса. Усредненная зависимость расхождения между ближайшими циклами от времени рассчитывается по методу Розенштейна для определения максимальной экспоненты Ляпунова. Показывается, что алгоритм способен надежно обнаруживать хаотическую природу смоделированных и экспериментальных псевдопериодических временных рядов при наличии шумов различного происхождения и при отношении сигнал-шум выше 10 дБ. Ил. 3, библ. 11 (на английском языке; рефераты на английском, русском и литовском яз.).

K. Pukėnas, K. Muckus. Algoritmas deterministinio chaoso detekcijai pseudoperiodinėse laiko eilutėse // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 8(80). – P. 53–56.

Pateikiamas naujas paprastas deterministinio chaoso detekcijos pseudoperiodinėse laiko eilutėse algoritmas be fazinės erdvės rekonstrukcijos. Naudojant koreliacijos koeficientą kaip distancijos tarp tam tikrų laiko eilutės ciklų matą, skaičiuojama suvidurkinta divergencijos tarp artimiausių ciklų (su mažiausia distancija) priklausomybės nuo laiko kreivė, kurios nuožulnumas yra deterministinio chaoso indikatorius. Suvidurkintai divergencijai tarp artimiausių ciklų skaičiuoti panaudotas Rozenšteino metodas didžiausiajai Liapunovo eksponentei apskaičiuoti. Straipsnyje parodoma, kad algoritmas gali patikimai aptikti chaotinę sumodeliuotų ir eksperimentinių pseudoperiodinių laiko eilučių prigimtį veikiant įvairiems triukšmams, kai signalo ir triukšmo santykis didesnis kaip 10 dB. Il. 3, bibl. 11 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

DOI: 10.5755/j02.eie.11006