ELECTRONICS AND ELECTRICAL ENGINEERING
ISSN 1392 – 1215 2007. No. 7(79)
ELEKTRONIKA IR ELEKTROTECHNIKA

T170 ELECTRONICS
ELEKTRONIKA

Analytical Model of System Enabled to Serve *n* Types of Messages

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Introduction

In the qualitative analysis of nodes of networks providing Value Added Services (VAS), nodes are presented as a system of queues and processors responsible for service [1, 2]. Different types of messages arrive all having different mean service time.

In the qualitative analysis of such complicated system analytical [3] or simulation models are used [1, 2].

Some analytical methods are based on the steady state solution of corresponding Markov chain, but if the system has a lot of feedback connections, queues can store a large number of messages then the number of states and consequently the number of balance equations needed to solve becomes to large to be solvable [4].

Queuing theory models are often used as a base for VAS system models. In the [5, 6] papers networks which provide VAS are modeled. Interaction between SSP (Service Switching Point) and SCP (Service Control Point) is studied. Models in paper [5] use M/M/n/K/K queuing system with assumed simplification that only one message can be sent from SSP to SCP for service, another message is sent only after the service of the first message is completed. In reality SSP point can send messages independently of whether the last message was serviced in SCP or not.

O. Haase, K. Murakami, T. Porta are suggesting a model for next generation network which provides voice services [7]. Separate network elements are studied as an independent node with infinite capacity queues. Different types of messages arrive with different intensities. Mean service time of i'th type message equals $\overline{T_i}$ (fig. 1). Interarrival times and service times are distributed exponentially.

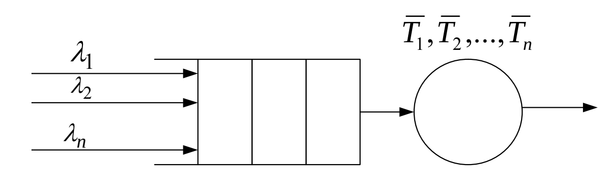

Fig. 1. System of service of *n* different types of messages, each with different service times

here λ_n – n'th type message arrival intensity, $\overline{T_n}$ – n'th type message mean service time.

In the [7] paper analytical model of M/M/1 system is used. It lets us determine mean time in system of all different types of messages.

$$\overline{D}_{A',i} = \frac{\overline{T_i}}{\left(1 - \sum_{i=1}^{n} \rho_i\right)}, \quad (1)$$

here $\rho_i = \lambda_i \cdot T_i$ – load of processor which serves i'th type messages, n – number of distinct message types.

Total time in system is calculated by summing mean message time spent in all network elements (nodes and links).

Analytical model of the node

In this paper analytical model is presented, that gives more precise estimate of message delay in the node, than the one presented in [7].

Node which is serving different types of messages with different known intensities is studied. Service distribution of the combined flow of all message types is not exponential. Consequently the system has to be studied not as a M/M/1 but instead as a M/G/1 system.

Mathematical expressions used for computation of the mean i'th type message time in a node with infinite capacity queue are presented.

Mean time spent in queue is the same for all messages. It equals

$$\overline{W} = \frac{\sum_{i=1}^{n} \lambda_i \cdot \overline{T}^2}{\left(1 - \sum_{i=1}^{n} (\lambda_i \cdot T_i)\right)}, \quad (2)$$

here $\overline{T}^2 = \sum_{i=1}^{n} \left(\frac{\lambda_i \cdot T_i^2}{\sum_{i=1}^{n} \lambda_i} \right)$ is the second moment of message service time.

i‘th type message time in system can be found by adding mean time spent in queue and mean service time of i’th type message

$$D_i = W + T_i \,. \tag{3}$$

Mean time in system of all messages

$$D_{vid} = \frac{\sum_{i=1}^{n} \lambda_i \cdot D_i}{\sum_{i=1}^{n} \lambda_i} \,. \tag{4}$$

Validation of result obtained in analytical model

We will compare the results obtained by using analytical model and the results obtained with simulation model.

Simulation model was suggested in paper [9]. Its correct resemblance of the system being modeled was tested by comparing obtained results with the ones obtained using Markov chain method than the queue capacity is small.

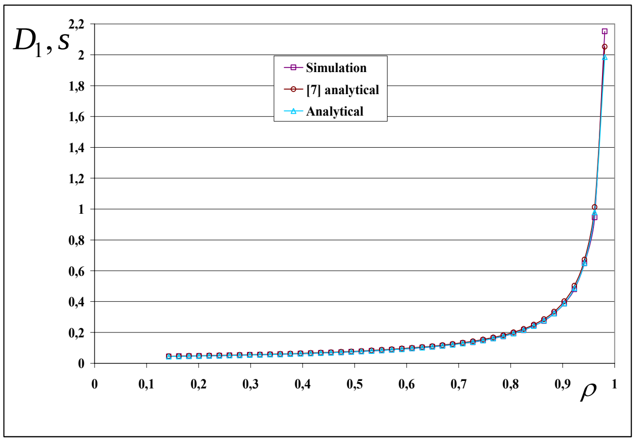

Fig. 2. Dependence of the first type message time in system, on processor load

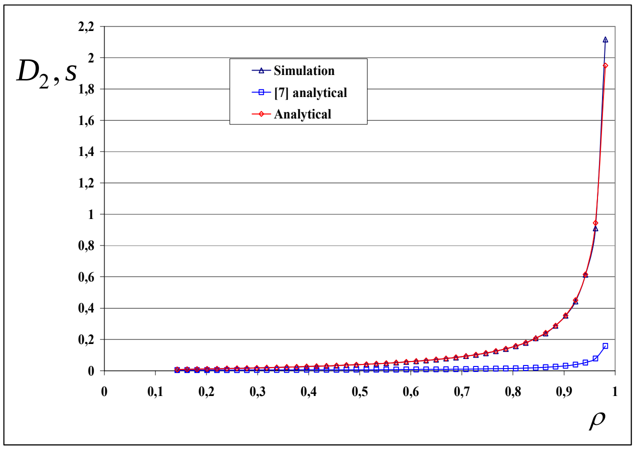

Fig. 3. Dependence of the second type message time in system on processor load

Analytical model can also be compared with the one using Markov chain methods, but because analytical model uses infinite capacity queue, it can be done only than the probability of message being lost is infinitely small.

Dependence of mean message time in system on processor load in the results obtained in simulation, analytical models and the model presented in paper [7] is shown in Fig. 2–5.

Arrival intensities of the second and the third type messages and the service times of all type messages remains constant ($\lambda_2 = 7$ msg/s, $\lambda_3 = 3$ msg/s, $T_1 = 0{,}039$ s, $T_2 = 0{,}003$ s, $T_3 = 0{,}034$ s). Only arrival intensity of the first type messages changes ($\lambda_1 \in [0{,}5;22]$ msg/s).

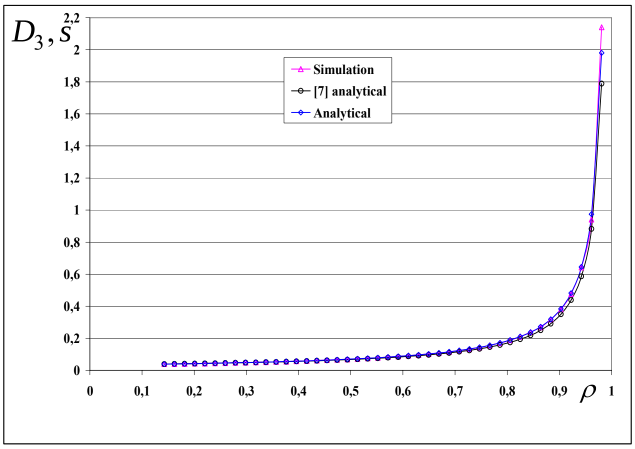

Fig. 4. Dependence of the third type message time in system on processor load

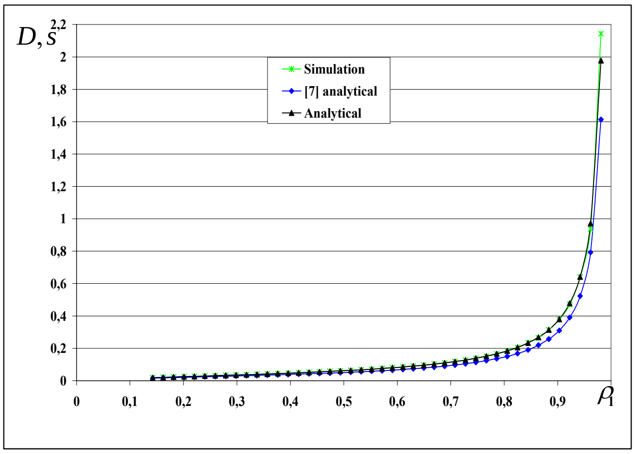

Fig. 5. Dependence of mean time in system spent by all messages on processor load

For comparison of results obtained in simulation, analytical models and the model presented in paper [7] we will use sample Kolmogorov-Smirnov test.

Sample $X = \{x_1, x_2, \ldots, x_n\}$ has an empirical distribution function $F(x)$ defined as [9]:

$$F(x) = \frac{k}{n},\; x_i^* < x < x_{i+1}^* \,, \tag{5}$$

where $X^* = \{x_1^*, x_2^*, ..., x_n^*\}$ is the rank order of sample X, k - number of observations less then x. Hypothesis H_0 „results obtained with simulation model and analytical methods are identical“. So theirs distribution function have to be close to each other. Maximal distance between each sample‘s empirical distribution function is found [9]:

$$D = \max |F_1(x) - F_2(x)| . \qquad (6)$$

Based on Kolmogorov-Smirnov test it can be stated that:

- if $R \in K$, then hypothesis H_0 contradicts observation (is incorrect);
- if $R \notin K$, then hypothesis H_0 is correct;

here K is critical region.

For result comparison sample of 112 elements of each characteristic was used, significance level was selected as $\alpha = 0{,}05$. Critical region $K = [0{,}1141195; +\infty]$ is obtained.

Results of the hypothesis H_0 “results obtained with simulation model and analytical model are identical” are shown in Table 1.

Table 1. Results of hypothesis testing

Characteristic	D	H_0
Mean service time of all messages	0.00909091	true
Mean service time of the first type messages	0.00909091	true
Mean service time of the second type messages	0.00909091	true
Mean service time of the third type messages	0.0181818	true

Kolmogorov-Smirnov test confirms hypothesis that all corresponding results are from the same distributions.

Results of the hypothesis H_0 “results obtained with simulation model and analytical model from paper [7] are identical” are shown in table 2.

Table 2. Results of hypothesis testing

Characteristic	D	H_0
Mean service time of all messages	0.0818182	true
Mean service time of the first type messages	0.0454545	true
Mean service time of the second type messages	0.627273	false
Mean service time of the third type messages	0.0272727	true

Kolmogorov-Smirnov test shows that only part of the tested results obtained with simulation model and analytical model [7] are from the same distributions.

Using (7) we can find relative errors results of both analytical models in respect to results obtained with simulation.

$$\Delta_{D_{A,i}} = \frac{|D_{A,i} - D_{I\_beg,i}|}{D_{I\_beg,i}} \cdot 100\% , \qquad (7)$$

here $D_{I\_beg,i}$ is mean time in system of i’th type message obtained with simulation model than the queue capacity is infinite.

$$\Delta_{D_{A',i}} = \frac{|D_{A',i} - D_{I\_beg,i}|}{D_{I\_beg,i}} \cdot 100\% . \qquad (8)$$

Results are presented in Fig. 6. While load on the processor which is servicing messages is less than 0.9 results obtained with analytical model differ from corresponding simulation results less than 1%. If the load on the processor is more than 0.9 difference rises but is still less than 8%.

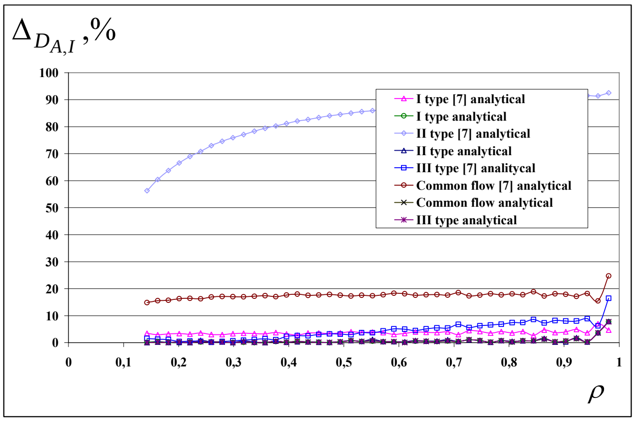

Fig. 6. Relative error of mean service time of analytical model used in this paper and the one used in paper [7] dependence on processor load

Using analytical model presented in [7] mean service time in system of the first and the third messages types differ from corresponding means obtained using simulation less than 5% if the load on processor is less then 0.6. If the load is less than 0.96, relative error does not increase above 10%. However mean time of second type messages which are serviced several times faster than the other ones, even if the load on the processor is low differs more than 50% from simulation results and then the load increases to 0.8 relative error is more than 90%.

Conclusions

One server queuing system, where different types of messages arrive, each type with interarrival times and service times distributed exponentially but with different distribution means and if the service time of some type messages differs considerably from the mean service time of all messages, has to be modeled using models for the M/G/1 system.

Correctness of raised hypotheses about the equality of corresponding results and small relative difference between the corresponding results of analytical and simulations models lets us conclude that proposed analytical model is adequate for use in models of the system being studied. Relative difference between the proposed model and simulation model is less then 4% then processor load is less than 0.95.

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Submitted for publication 2007 05 09

R. Gedmantas, A. Jarutis, J. Jarutis. Analytical Model of System Enabled to Serve *n* Types of Messages // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 7(79). – P. 71–74.

In the qualitative analysis of value-added complementary services it is necessary to take into account different algorithms for service realization. Consequently different types of messages are created each with different service time distributions. Analytical or simulation models are used for analysis of such systems. Nodes providing Value-Added Services VAS are presented as a system of queues and processors responsible for service in the analysis of theirs efficiency, qualitative parameters of VAS services. Different types of messages arrive to the queue each with different service times. Analytical model of such system is presented in this paper. It is based on the M/G/1 system. Results of the model are compared with the results obtained in simulation by using two-sample Kolmogorov-Smirnov test. Ill. 6, bibl. 8 (in English; summaries in English, Russian and Lithuanian).

Р. Гедмантас, А. Ярутис, И. Ярутис. Аналитическия модель системы, обслуживающей *n* типов заявок // Электроника и электротехника. – Каунас: Технология, 2007. – № 7(79). – С. 71–74.

При анализе обслуживания дополнительных услуг повышенной ценности следует учесть разные алгоритмы предоставления услуг. В результате создаются потоки заявок разного типа, обслуживаемые разными временами. Для их исследования используются аналитические или имитационные методы. Дополнительные услуги повышенной ценности предоставляющие сетевые узлы при анализе их работоспособности, анализе качественных показателей предоставления услуг предоставляются как система, состоящая из буферов и их обслуживающих процессоров. В работе предложена аналитическая модель такой системы, основанная на системе M/G/1. Результаты аналитической модели сравнены с результатами имитационной модели на основе критерия Колмогорова-Смирнова. Ил. 6, библ. 8 (на английском языке; рефераты на английском, русском и литовском яз.).

R. Gedmantas, A. Jarutis, J. Jarutis. Sistemos, aptarnaujančios *n* skirtingų tipų paraiškų, analitinis modelis // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 7(79). – P. 71–74.

Atliekant pridėtinės vertės papildomų paslaugų teikimo analizę, būtina įvertinti skirtingus jų realizavimo algoritmus. Sukuriami skirtingo tipo, skirtingos aptarnavimo trukmės paraiškų srautai. Tokių tinklų analizei naudojami analitiniai ar imitaciniai modeliai. Pridėtinės vertės paslaugas PVP teikiančių tinklų mazgai, atliekant jų darbingumo, paslaugų teikimo kokybinių rodiklių analizę, pateikiami kaip sistema, susidedanti iš buferių ir juos aptarnaujančių procesorių. Į buferius patenka skirtingų tipų pranešimai, kurių vidutinė aptarnavimo trukmė taip pat yra skirtinga. Darbe pasiūlytas tokios sistemos analitinis modelis, pagrįstas M/G/1 modeliu. Analitiniu modeliu gauti rezultatai palyginti su rezultatais, gautais naudojant imitacinį modelį, tam panaudojant Kolmogorovo-Smirnovo kriterijų. Il. 6, bibl. 8 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).