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Synthesis of Regular Digital Filters with Specified Time Characteristics

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Introduction

The designing of digital recursive filters that satisfy specified requirements to several characteristics and at the same time are simple in realization in comparison to well-known, is always relevant for the specialists in the processing of digital signals. In designing of digital recursive filters the approximations of Butterworth, Chebyshev, Bessel, izoextremal and some others are traditionally used. The traditional biquad realization is shown on Fig. 1.

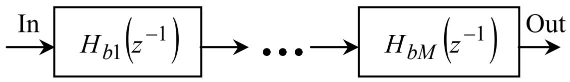

Fig. 1. Biquad realization

Here for the filters of even orders $M = N/2$ and

$$H_{bk}\left(z^{-1}\right)=\frac{b_{k0}+b_{k1}z^{-1}+b_{k2}z^{-2}}{1+a_{k1}z^{-1}+a_{k2}z^{-2}}, \text{ if } k=1...M, \qquad (1)$$

where N is a filter order.

For the filters of odd orders $M = (N+1)/2$

$$H_{bk}\left(z^{-1}\right)=\frac{b_{k0}+b_{k1}z^{-1}+b_{k2}z^{-2}}{1+a_{k1}z^{-1}+a_{k2}z^{-2}}, \qquad (2)$$

if $k = 1...(M-1)$ and

$$H_{bM}\left(z^{-1}\right)=\frac{b_{M0}+b_{M1}z^{-1}}{1+a_{M1}z^{-1}}. \qquad (3)$$

Obviously, we need to keep at once $5M$ values of coefficients for the filters of even orders and $5M-2$ coefficients for the filters of odd orders, that can be unacceptable when N is high. In a number of cases it is better to have unified subunits in the structure of such cascade combination for the hardware or program realization.

In this article the new conception of digital recursive filters designing with special biquad realization is considered. In general case it looks like in the Fig. 2.

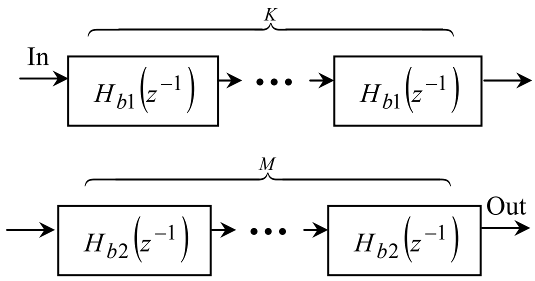

Fig. 2. The realization with unified units (regular filter)

In this case

$$H_{b1}\left(z^{-1}\right)=\frac{b_{10}+b_{11}z^{-1}+b_{12}z^{-2}}{1+a_{11}z^{-1}+a_{12}z^{-2}}, \qquad (4)$$

$$H_{b2}\left(z^{-1}\right)=\frac{b_{20}+b_{21}z^{-1}+b_{22}z^{-2}}{1+a_{21}z^{-1}+a_{22}z^{-2}}, \qquad (5)$$

where $K, M > 0$; $N = M + K$. As it is seen from the Fig. 2, the total number of coefficients in ROM is always not more than ten when any order of filter is used, but hardware or program realization is much easier. As it is shown below, the number of coefficients saved in ROM can be reduced to eight in design of the regular filters.

Optimization method of synthesis digital low-pass filter (DLPF)

Assigning the requirements for time and frequency characteristics, the following relations will be used [5]:

$$H\left(\tilde{\Omega}\right)=\begin{cases}1, & \tilde{\Omega}=0,\\ \xi\left(\tilde{\Omega}\right), & \tilde{\Omega}\in(0,\ \tilde{\Omega}_1],\\ h_{\min}, & \tilde{\Omega}\in(\tilde{\Omega}_1,\ 1],\end{cases} \qquad (6)$$

$$\Theta\left(\tilde{\Omega}\right)=\chi\left(\tilde{\Omega}\right),\quad \tilde{\Omega}\in[0,\ \tilde{\Omega}_1], \qquad (7)$$

where $H(\tilde{\Omega})$, $\Theta(\tilde{\Omega})$ accordingly are amplitude-frequency response (AFR) and phase-frequency response (PFR) of the synthesized filter; $\xi(\tilde{\Omega})$, $\chi(\tilde{\Omega})$ – some unknown frequency relations; $\tilde{\Omega}_1$ – normalized digital control frequency; $h_{\min}$ – level of AFR in stopband.

Physical nature of conditions (6) and (7) consist of following; AFR of DLPF type circuit is equals 1 if it has zero frequency. In frequency band $(0,\ \tilde{\Omega}_1]$, which can conditionally be defined as bandpass, AFR и PFR are undefined: their frequency dependences are defined by the assignment of the circuit. In frequency band $(\tilde{\Omega}_1,\ 1]$, which can conditionally be defined as stopband, value of AFR is not higher than $h_{\min}$. PFR cannot be defined same way in stopband.

Step response (SR) of DLPF type circuit meets the conditions of the following equality

$$\lim_{n\to\infty} g(n) = 1 . \tag{8}$$

For quantity valuation of characteristics of the transitional process, we will enter following parameters (Fig. 3) δ – ejection or level of ripples, τ – rise time or time of transition.

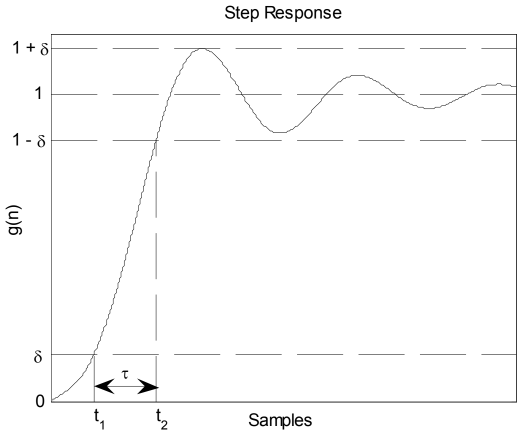

Fig. 3. Step response of linear filtering circuit

For the suggested system the iteration method of local optimization with MATLAB standard function packet *fmincon* [3, 6] was chosen. It's aim is finding the optimal coefficients of transfer functions H_{b1}, H_{b2}.

The goal function is composed depending on the requirements to AFR and SR is determined by four parameters, where $a(\tilde{\Omega})$ – attenuation (loss) characteristic:

- $a(0)$ – value of attenuation in zero frequency;
- $a_{\min}$ – minimum allowable level of AFR in the stopband;
- $\tilde{\Omega}_1$ – normalized digital control frequency on the $a_{\min}$;
- $\delta_{\max}$ – maximum allowable ripple level of SR.

The calculation of the goal function F is made as following:

$$F = \alpha F_1 + \beta F_2 ; \tag{9}$$

$$F_1 = a(0) + \sum_{\tilde{\Omega}=\tilde{\Omega}_1}^{1} \left[a_{\min} - a(\tilde{\Omega})\right], \text{ if } a(\tilde{\Omega}) < a_{\min} ; \tag{10}$$

$$F_2 = \begin{cases} \sum_{n=n_2}^{\infty} \left[g(n) - (1+\delta_{\max})\right], & \text{if } g(n) > 1+\delta_{\max}, \\ \sum_{n=n_2}^{\infty} \left[(1-\delta_{\max}) - g(n)\right], & \text{if } g(n) < 1-\delta_{\max}; \end{cases} \tag{11}$$

where α and β – weighting coefficients (often $\beta = (1-\alpha)$).

During the optimization process the coefficients a and b of transfer functions H_{b1} and H_{b2} are varied, that determines the value of goal function. The optimisation is finished when further decreasing of goal function is impossible.

The detail of the algorithm can be explained by the block scheme (Fig. 4), here F – calculated goal function, F_p – the goal function, that we found in the end of the previous optimisation, x – array of optimised coefficients H_{b1} and H_{b2}.

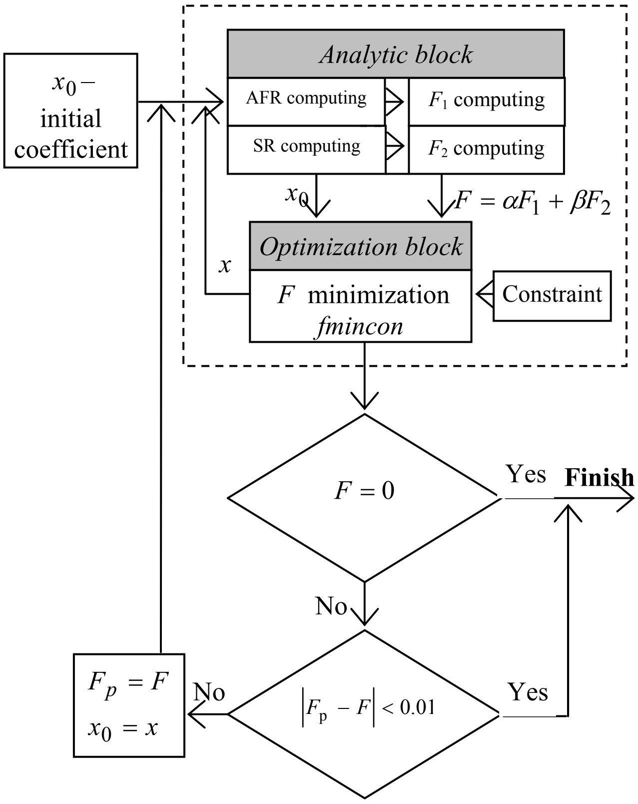

Fig. 4. The block scheme of the algorithm

Example of DLPF design with specified requirements to step response

The synthesis of digital filter with transient functions that has multiple roots in their denominator was shown by authors in [4]. The fact that the presence of multiple roots in denominator guaranties the transient characteristics with extremely low ripples (up to 0.5%) is proved. The system on the Fig. 2 exactly has the multiple roots thus it is logical to suppose that it might have the transient characteristics with exceptionally low ripples.

The following parameters were set during optimization search of coefficients:

- $a(0) = 0$,
- $a_{\min} = 50\text{dB}$,
- $\widetilde{\Omega}_1 = 0.35$,
- $\delta_{\max} = 0.01 = 1\%$.

The filters of 10th and 14th orders are synthesized as a result of a row of iterations. For good layout compare characteristics of synthesized filters with the same order Chebyshev type I digital filters with maximum allowable level of AFR in the passband $a_{\max} = 0.25\text{dB}$ and normalized cutoff frequency equal 0.3.

The loss characteristics of Chebyshev and regular filters at $N = 10$ are shown in Fig. 5.

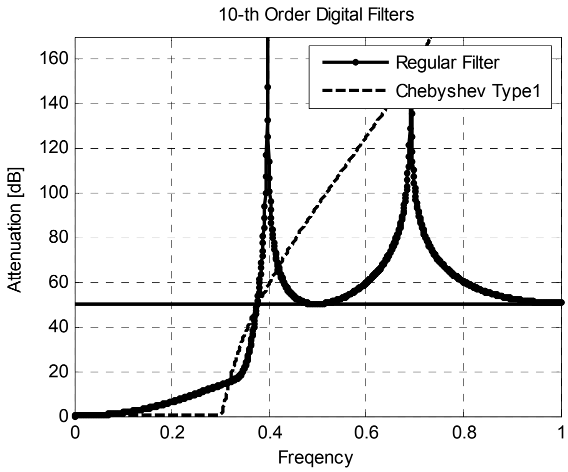

Fig. 5. Loss characteristics of 10th order compared filters

From the Fig.5 it is shown that the curve in bandpass has the form [4] not typical for the traditional filtration. This look of the AFR is typical for Jess-Schussler type filters [2,4,5] (which realization structure is not regular). The attenuation over the control frequency no less, than $a_{\min} = 50dB$, thus suggested regular filters are high selective.

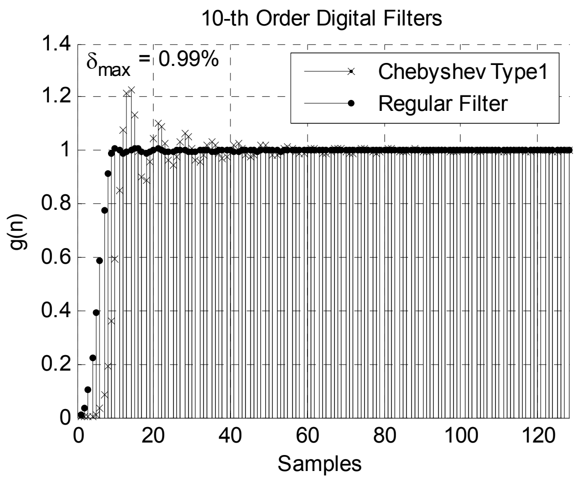

Fig. 6. Step responses of 10th order compared filters

The step response is shown in the Fig. 6. The level of its ripples doesn't exceed one percent, but rise time much less.

The coefficients obtained in the process of synthesis are as following:

```
B1 = 1.1710    1.3270    1.1710
B2 = 0.5511   -0.3496    0.5511
A1 = 1.0000   -0.8232    0.8586
A2 = 1.0000   -0.5670    0.1261
```

Here $K = 2$ and $M = 3$ (see Fig. 2). It is significant that in this case it is enough to keep in ROM the values of eight coefficients of multipliers, as $b_{10} = b_{12}$ and $b_{20} = b_{22}$.

Characteristic of time of the group delay (GD) in compared filters pass band is very interesting.

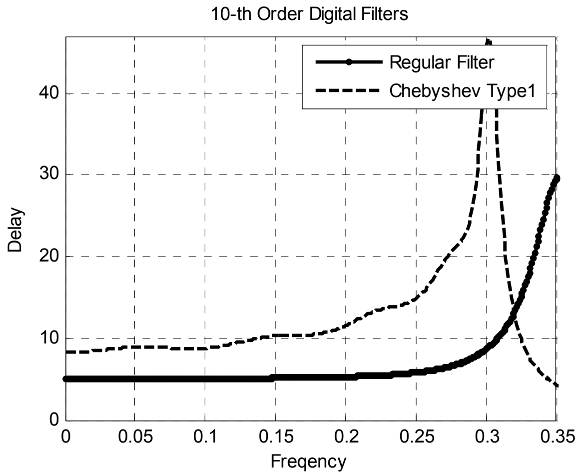

Fig. 7. Group delay of 10th order compared filters

It's great, that GD of regular filter turn out practically constant in passband. In (5) is shown, that minimal GD ejection depends on linearity of prototype PFR, because of that, we intuitively can suppose that, GD of synthesized filter in fact will appear constant in transmission zone.

We will show the characteristics of 14th order regular filter and Chebyshev type I filter in conclusion.

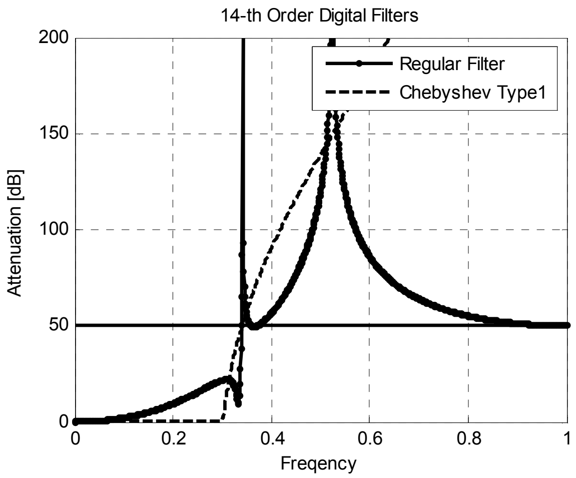

Fig. 8. Loss characteristics of 14th order compared filters

We can see, that pulsations of SR are decreased till 0,7 %, and GD is constant in passband. AFR highly competitive with Chebyshev filter by selectivity after control frequency.

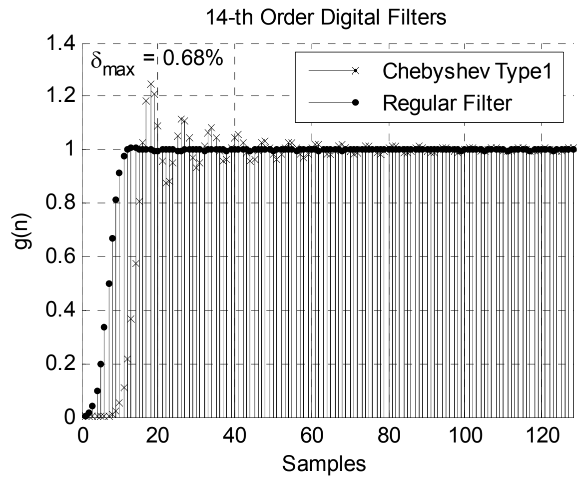

Fig. 9. Step responses of 14^{th} order compared filters.

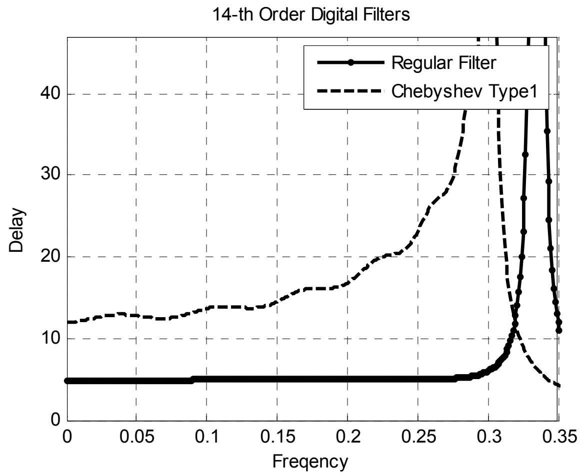

Fig. 10. Group delay of 14^{th} order compared filters.

The coefficients obtained in the process of synthesis at $K = 3$ and $M = 4$ are as following:

```
B1 = 1.3155   -1.2529    1.3155
B2 = 1.4536    0.2166    1.4536
A1 = 1.0000   -0.9829    0.9807
A2 = 1.0000   -0.6493    0.1705
```

Conclusions

The new conception of digital filters is shown (fig.2). Using such regular system it is possible to achieve the high selective filters and support minimal or specified ripples of step response (fig.5-10). Extra great quality of regular filters is that group delay has low level and practically constant in passband.

And all that, with realization simplicity and with small requirement of working memory (ROM), cause the number of coefficients, which should be stored in memory, in this occasion not more than 8.

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S. Sharkovsky. Synthesis of Regular Digital Filters with Specified Time Characteristics // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 6(78). – P. 25–28.

Offered a new conception of digital regular design, which time and frequency characteristics are satisfied the specification, but realization is more simple, compare with standard procedures. Chose basic cascade connection of unified biquades. Usually no more then eight constants should be stored in memory. The program realizations as well as hardware realization of regular filters are more simple then traditional. Biquades coefficients are found by local optimization. In publication is shown DLPF 10^{th} and 14^{th} order design examples, which characteristics are comparing with characteristics of Chebyshev filters with same order and same requirements to control frequency. Ill. 10, bibl. 6 (in English, summaries in English, Russian and Lithuanian).

С. Шарковский. Синтез регулярных цифровых фильтров с заданными временными характеристиками // Электроника и электротехника. – Каунас: Технология, 2007. – № 6(78). – С. 25–28.

Предложена новая концепция синтеза цифровых регулярных фильтров, частотные и временные характеристики которых удовлетворяют заданным, а реализация более простая по сравнению со стандартными процедурами. Выбрано базовое каскадное соединение одинаковых биквадов. В памяти при этом нужно хранить не более восьми констант. И программная, и аппаратная реализация регулярных фильтров существенно проще по отношению к традиционным. Коэффициенты биквадов находятся локальной оптимизацией. Приведен пример синтеза ЦФНЧ $10^{го}$ и $14^{го}$ порядков, характеристики которого сравниваются с характеристиками фильтра Чебышева того же порядка и с теми же требованиями к контрольной частоте. Ил. 10, библ. 6 (на английском языке; рефераты на английском, русском и литовском яз.).

S. Sharkovsky. Skaitmeninių reguliuojamų filtrų su nustatytomis laikinėmis charakteristikomis tyrimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 6(78). – P. 25–28.

Pasiūlyta nauja skaitmeninių reguliuojamų filtrų koncepcija, įgalinanti nustatytas dažnines ir laikines charakteristikas realizuoti paprasčiau, palyginti su įprastinėmis procedūromis. Parinktas bazinis pakopinis vienodų pakopų surinkimas. Šiuo atvejų atmintyje saugoma ne daugiau kaip 8 konstantos. Reguliuojamų skaitmeninių filtrų programinė ir aparatinė realizacija yra gerokai paprastesnė, palyginti su įprastine. Pakopų koeficientai parinkti vietinės optimizacijos būdu. Pateiktas DLPF 10 ir 14 eilės filtro pavyzdys. Čia charakteristikos panašios į tos pačios eilės Čebyševo charakteristikas. Il. 10, bibl. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).