

## **The Generalized Model of the Linear Induction Motor**

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### **Introduction**

Linear asynchronous machine (LAM), as an object of further scientific investigation, has drawn a wide interest of the world scientists at the beginning of the second half of the last century. At that time there appeared fundamental research works of the first of this science scholars such as British scientist E. R. Laithwaite, Japanese S. Yamamura, Y. Isikava, H. Ito and others.

A decade later Lithuanian scientists initiated and participated in the investigation of LAM phenomenon directed by V. Česonis and A. Smilgevičius [1]. The author of the article was among the first to participate in the preliminary studies as well and succeeded in enriching the theory of these machines by analyzing the nonsymmetrical LAM mathematical model by the theoretical approach to the phenomenon and managed to generalize LAM theory more comprehensively than other researchers. The second author of this article participated in LAM control theory elaboration; the results of his investigation were presented in his doctoral thesis.

The research on LAM, operating at the mode of the linear induction motors (LIM), has been continued since that time at Vilnius Gediminas Technical University, Department of Automation [2, 5, 7]. However, I would like to regret the fact that these researches are being carried out without mentioning the sources of LAM theory and without the analysis of the fundamental theory of LAM as the classical one. Moreover, in the thesis of the researchers and publications issued there a tendency to diverge into the research of episodic type of local character of the phenomenon is very often found there, and there is missing interrelation with fundamental LAM theory.

The above mentioned situation encouraged the authors of these article to reveal that objectively as far as they succeeded and to evaluate the present situation in LAM scientific research as well as try to systematize LAM researches made, submit the generalized mathematical model of LAM operating under the LIM mode, to describe on its basis the peculiarities of LAM as the electric machine.

It has to be pointed out that application of such a principle based on the method of systematization makes it possible to carry out a research of wide LIM scientific

spectrum on the basis of one generalized LIM theory basis. To observe such a principle might serve in systematizing of the variety of LIM research works that have already been carried out as well as are being carried in Lithuania and relate them with the classical theory of LIM, develop the so called „Lithuanian school of scientific research on LIM“.

### **Summarized LIM mathematical model and its peculiarities**

There exist many various types of LAM alternatives [6]. The majority of scientific research done has been related to the linear asynchronous machines. The most part of them has been aimed to investigate LAM, operating under the motor mode. The main results of such investigations are presented in [3, 4, 5] sources as well as in [3] or in the variant [4] which is the Russian language version, it could be justly called as fundamental theory of LIM. There have been known attempts to systematize LIM theory aspects into one solid summation, the latter would allow in the recourses of similar preliminary assumptions to carry out comparative analysis of LIM possible variants and to evaluate separate peculiarities of LIM. One of the ways to achieve such a goal has been a search for a generalized LIM model.

As a matter of fact, under the wide variety of LIM it is rather a complicated task to compile a generalized mathematical model. The attempts to create it have been made at the beginning of the formation of the theory of machines. One has to state that such efforts have been practically successful and as proof of that here are given the references [3, 4].

The authors of the article consider a generalized mathematical model of LIM to be one under the basis of which it could be possible to investigate presumably more alternatives of LIM and the obtained characteristics to compare as interconnected ones.

The authors of the article contemplate that such a well known and reflecting variety of the investigated models of LIM till present time might serve the alternative made of two flat type inductors with active surfaces, with the secondary element (SE) located in between which consists of ferromagnetic band covered on both sides with

the non-ferrous conductive material. SE and its place of location in the space between the inductors may be symmetrical and non-symmetrical. That is why a mathematical model corresponding special type of such a LIM could be relatively called symmetrical and non-symmetrical.

A symmetrical mathematical LIM is considered to be such a model where SE in the form of ferromagnetic band with covered on both sides equal layers of conductive ferromagnetic material and is placed symmetrically between the two inductors and air gaps. Such a LIM symmetrical model has been analyzed widely enough in the works of S. Yamamura and other scientists; the information on them has been presented in scientific sources [3, 4].

Non-symmetrical LIM mathematical model is considered to be the model where the SE is made of ferromagnetic band placed symmetrically in the air gap between the inductors and covered on both sides of unequal thickness conductive material layers of positioned within unequal distances through the air gaps from the inductors. Such non-symmetric LIM model is rather properly analyzed in the scientific publication given in [5].

The difference between symmetrical and nonsymmetrical models is that nonsymmetrical model in comparison with the symmetrical one is more generalized and makes it possible to investigate a wider spectrum of LIM types.

Due to the fact that both the symmetrical and nonsymmetrical models of LIM include a number of possible LIM alternatives, they could justly be called generalized models.

One has to signify that because both symmetrical and nonsymmetrical LIM generalized models theoretically have been analyzed under the same preliminary conditions, so the symmetrical model is a separate case of nonsymmetrical model, i.e., the research done on nonsymmetrical model in work [5] is the continuation of the works [3, 4].

It is necessary to pay attention to one more indicated essential peculiarity of LIM. In both cases LIM is analyzed in the static form and the final results of investigations are the process of determining static characteristics. As in this case LIM characteristics describe its operation under the settled mode (SE velocity  $v = 0$ ), the application of these results is used to investigate the dynamics of electric drives with LIM which is a very limited one (except graph analytic methods). At present the usage of structural characteristics for investigation of dynamics from the point of view of scientific research is an old fashioned one and is not satisfactory correct.

The main feature of the generalized LIM model is the fact that on its basis it is possible not only to carry out the comparative analysis of many realistic LIM types, but carry out the search according to the desirable criteria of the most optimal LIM version.

### General LIM characteristics

As a rule nearly in all known cases LIM characteristics are obtained by solving the equation system

in the space between the LIM inductors in the existing electromagnetic field characterizing the system.

This solution may be carried out by selecting one-dimensional, two-dimensional or three-dimensional model. The fuller and relatively complete research may be called the solutions received with the help of three-dimensional model. The solutions are reflected in the sources [3, 4] comprehensively enough. That is why at present when the results of the solution of three-dimensional LIM model are known, to continue the research and make the generalized conclusions in accordance with the simplest LIM models (one dimensional or two dimensional) only together with a more generalized model because to behave otherwise scientifically is not correct.

It is unallowable when the essential phenomena, distinguishing LIM from rotor type induction motors, are not evaluated or not accented in the research. The main specific phenomena are the following:

1) Final (entrance and exit) effects the significance of which is stressed and described in the sources [3, 4]; the influence by which the characteristics of high speed and low speed LIM are differently influenced.

2) The phenomena that are executed in the solid SE of LIM (the effects of edges) [3, 4], which particularly distinguish LIM from the classical theory of induction machines. If one is not guided by the theory of final and edge effects and their compensation theory, the results of theoretical research of electric drives with LIM do not exceed classical theory of rotor asynchronous electric machines, which may be considered as sufficiently formulated enough, from the point of view of boundaries and scientific researches they are worthless.

### The research model of LIM

The idealized LIM mathematical model [5] is considered the main one and it is presented in Fig. 1.

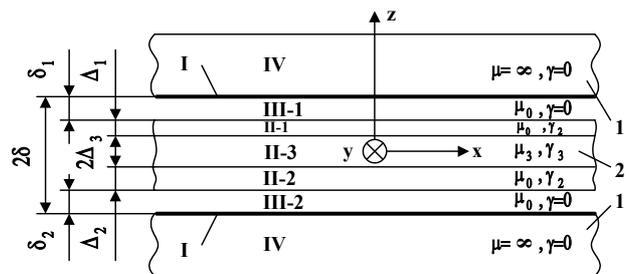


Fig. 1. Generalized LIM mathematical model

This is previously described and generalized nonsymmetrical LIM model. It is comprised of two inductors marked in the Fig. 1 and SE, indicated in Fig. 2. In the air space between the inductors 1 (the distance between them is  $2\delta$ ), there is symmetrically located ferromagnetic band of  $2\Delta_3$  thickness, covered on both sides by the conductive layers of  $\Delta_1$  and  $\Delta_2$  thickness of non ferromagnetic material and located apart from active surfaces of the inductors by corresponding distances (air gaps)  $\delta_1$  and  $\delta_2$ .

In general theory of three-phase electric machines there are often the assumption made concerning real

preliminary windings (of inductors), located into the grooves of magnetic core changed by the equivalent layer of the current. After having taken this assumption the density of the equivalent layer of the current which flow in the real windings (creating the same main spatial harmonics of magneto motive force) may be expressed by the formula [4]:

$$J_1 = \frac{3\sqrt{2}w_1 k_{ap}}{p\tau} I_1; \quad (1)$$

where  $w_1$  – is the number of windings;  $k_{ap}$  – is the coefficient of the winding;  $p$  – is the number of pole pairs;  $\tau$  – length of pole;  $I_1$  – is electric current.

In such type of LIM model it is possible to distinguish four zones in accordance with the magnetic permeability and electric conductance:

I – is the layer of the current characterized by  $J_1$  density. II – is SE which is compiled of II-3 ferromagnetic material layer, the density, magnetic permeability and electric conductance of which are correspondingly  $2\Delta_3$ ,  $\mu_3$ ,  $\gamma_3$ ; II-1 and II-2 are two layers of nonmagnetic material the thickness of which is  $\Delta_1$  and  $\Delta_2$ ; the magnetic permeability and electric conductance are  $-\mu_0$ ,  $\gamma_2$ ; III – III-1 and III-2 are air gaps the length of which is  $\delta_1$  and  $\delta_2$ , but magnetic permeability and electric conductance are  $\mu_0$ ,  $\gamma=0$ ; IV – are magnetic cores, the magnetic permeability and electric conductance of which are  $-\mu=\infty$ ,  $\gamma=0$ .

As it has been proved in [5], by using the method of vector potential, by solving Maxwell equations and from the equations of magnetic field turning to the theory of electric chains (after having used in the theory of electric machine the applied representative scheme of LIM), the force of LIM is developed in the direction of  $x$  is expressed by the formula:

$$F_x = \frac{m_1 L_{\delta 0} \alpha K_m U_1^2}{(r_1 - \omega_1 L_{\delta 0} K_m)^2 + (x_{\sigma 1} + \omega_1 L_{\delta 0} K_r)^2}; \quad (2)$$

$$L_{\delta 0} = \mu_0 \frac{m_1 w_1^2 k_{ap}^2 2c}{p}; \quad (3)$$

$$K = K_r + iK_m; \quad (4)$$

$$K^{N-N} = \frac{a_1 sh\alpha\delta_1 + \beta b_1 ch\alpha\delta_1}{a_1 ch\alpha\delta_1 + \beta b_1 sh\alpha\delta_1}; \quad (5)$$

$$K^{N-S} = \frac{a_2 sh\alpha\delta_1 + \beta b_2 ch\alpha\delta_1}{a_2 ch\alpha\delta_1 + \beta b_2 sh\alpha\delta_1}; \quad (6)$$

$$a_1 = ch\lambda_3 \Delta_3 ch\lambda_2 \Delta_1 + \Theta sh\lambda_3 \Delta_3 sh\lambda_2 \Delta_1; \quad (7)$$

$$b_1 = ch\lambda_3 \Delta_3 sh\lambda_2 \Delta_1 + \Theta sh\lambda_3 \Delta_3 ch\lambda_2 \Delta_1; \quad (8)$$

$$a_2 = sh\lambda_3 \Delta_3 ch\lambda_2 \Delta_1 + \Theta ch\lambda_3 \Delta_3 sh\lambda_2 \Delta_1; \quad (9)$$

$$b_2 = sh\lambda_3 \Delta_3 sh\lambda_2 \Delta_1 + \Theta ch\lambda_3 \Delta_3 ch\lambda_2 \Delta_1; \quad (10)$$

$$\alpha = \frac{\pi}{\tau}; \quad \beta = \frac{\alpha}{\lambda_2}; \quad \Theta = \frac{\lambda_2 \mu_3}{\lambda_3 \mu_0}; \quad \omega_1 = 2\pi f_1; \quad (11)$$

$$\lambda_2 = \alpha \sqrt{1+i\varepsilon_2}; \quad \lambda_3 = \alpha \sqrt{1+i\varepsilon_3}; \quad (12)$$

where  $m_1$ ,  $f_1$ ,  $\omega_1 - U_1$  is the number of phases of power supply voltage, frequency and angular frequency;  $w_1$ ,  $k_{ap}$  – is the number of windings of phase coil and the coefficient of coil;  $2c$  – is the width of inductors;  $\mu_0 = 4\pi 10^{-7} H/m$  – is the magnetic permeability of void;  $r_1$ ,  $x_{\sigma 1}$  – are the active and dispersion resistances of inductor winding;  $\varepsilon_2$ ,  $\varepsilon_3$  – are the Reynold's magnetic numbers of nonmagnetic and magnetic SE layers;  $L_{\delta 0}$  – is the value having the value of inductance,  $K$ ,  $K^{N-N}$ ,  $K^{N-S}$  – are the complex coefficients, the inductance poles of which are homonymous (N – N) and (N – S) are opposite;

In accordance with the formulas (2-12) it is possible not only to investigate possible LIM type characteristics and the peculiarities of their management by means of classical machine theory methods, but also to optimize LIM parameters by means of comparative method of analysis in accordance with separate criteria.

To calculate LIM characteristics following (2) there have been used preliminary data presented in Table 1.

**Table 1.** Preliminary data

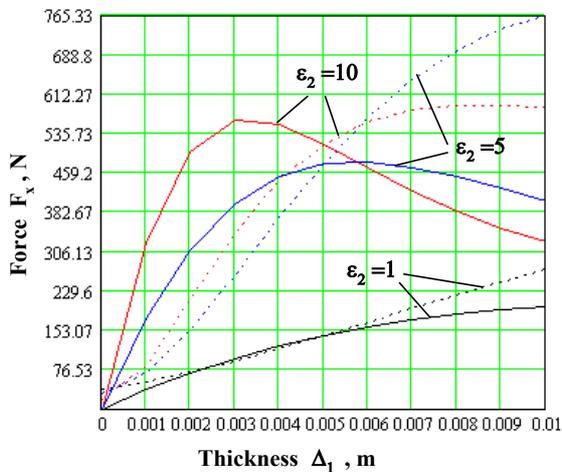
No.	Title of the parameters	Marking	Value	Mathunits
1.	Number of phases	$m_1$	3	-
2.	Number of coils	$w_1$	960	-
3.	Coefficient of windings	$k_{ap}$	1	-
4.	Width of inductor	$2c$	0,1	m
5.	Number of pair of poles	$p$	4	-
6.	Active resistance	$r_1$	2,82	$\Omega$
7.	Inductive resistance	$x_{\sigma 1}$	5,96	$\Omega$
8.	Length of pole	$\tau$	0,06	m
9.	Reynolds's number	$\varepsilon_2$	1,5,10	-
10.	Reynolds's number	$\varepsilon_3$	1	-
13.	Magnetic permeability	$\mu_3$	10000	-
14.	Air gap length	$\delta_1$	5	mm
15.	SE thickness	$\Delta_1$	0...10	mm
16.	SE thickness	$\Delta_3$	5	mm
17.	Voltage	$U_1$	220	V
18.	Network frequency	$f_1$	50	Hz

To calculate LIM characteristics in accordance with the indicated theory there are used a software package *Mathcad 2001 Professional*.

The calculated characteristics are submitted in Fig. 2, their coincidence with the experimentally verified ones is within the boundaries of error of a certain percent.

The authors of this article are sure that when following the recommended position presented in this article and the principal of LIM theory systematization of electric drives with LIM for the research there are opening new very wide perspectives for Lithuanian scientists

engaged in the investigation of LIM systems as well there are possibilities with the help of their research works done in close relation to the fundamentals of LIM theory to make an input and contribution into the scientific world's fund joining the researchers who try to expand LIM theory.



**Fig. 2.** The graphs of calculated characteristics: dotted lines – (N – S) poles; continuous – (N – N) poles

## Conclusions

1. Scientific research of electric drives and other systems with LIM investigation is advisable to be expanded in connection to the fundamental theory of LIM.
2. On the basis of generalized LIM mathematical model the developed theory shall comprise wide possibilities of the systems with LIM for the solution of optimization problems.

**L. Radzevičius, E. Matkevičius. The Generalized Model of the Linear Induction Motor // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 7(71). – P. 5–8.**

Scientific article presents the theory systematizing by means of systematic principal the linear asynchronous machine (LAM), operating under the mode of linear induction motor (LIM) and it has been proved that one of the most promising trends in developing the research on electric drives with LIM is to use comparative analysis methods and by solving the optimization problems of these drives in accordance with the desirable criteria and the most rational is the research base of the generalized LIM model compiled under the background of LIM fundamental theory. The article points out that the scientific achievements based on applying modern mathematical methods could successfully be used for solving engineering problems connected with LIM application. III.2, bibl. 7 (in English; summaries in English, Russian and Lithuanian).

**Л. Радзевичюс, Э. Маткявичюс. Обобщенная модель линейного асинхронного двигателя // Электроника и электротехника. – Каунас: Технология, 2006. – № 7(71). – С. 5–8.**

В настоящей статье на основании системного принципа представляется обобщенная теория линейной асинхронной машины (ЛАМ), работающей в режиме линейного асинхронного двигателя (ЛАД) и доказанно, что одной из перспективных направлений в области электроприводов с ЛАД на основании методов сравнительного анализа при решении задач этих приводов по желаемым критериям самой рациональной является обобщенная модель ЛАД, созданная на основании фундаментальной теории ЛАМ. Акцентируется, что эта научная база с применением современных математических методов успешно может быть применена в инженерной практике для решения с ЛАД связанных инженерных задач. Ил. 2 библи. 7 (на английском языке; рефераты на английском, русском и литовском яз.).

**L. Radzevičius, E. Matkevičius. Tiesiaeigio asinchroninio variklio apibendrintasis modelis // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 7(71). – P. 5–8.**

Pateikta sisteminiu principu apibendrinta tiesinių asinchroninių mašinų (TAM), veikiančių tiesiaeigio asinchroninio variklio (TAV) režimu, teorija ir įrodyta, kad viena iš perspektyvesnių krypčių plėtojant elektros pavarų su TAV tyrimus lyginamosios analizės metodais bei sprendžiant šių pavarų optimizavimo pagal norimus kriterijus uždavinius yra apibendrintojo TAV modelio pagrindu sukurtos TAM fundamentaliosios teorijos mokslinė bazė. Parodoma, kad ši mokslinė bazė, taikant šiuolaikinius matematinius metodus, sėkmingai gali būti panaudota inžineriniams uždaviniams, susijusiems su TAV naudojimu, spęsti. Il.2, bibl. 7 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

3. Based on the generalized LIM mathematical model there has been developed the theory of machines which might open wide possibilities in applying comparative analysis method, considering separate criteria to be able to optimize LIM parameters and characteristics as well as make a wide scientific field for systems with LIM to investigate.
4. The article recommends and derives LIM theoretical conclusions followed by the mathematical expressions representing them in order to be able to implement that into engineering practice.

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Presented for publication 2006 02 23