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# Modeling Information Complex Processing with MMS-Kalman Filter

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## Introduction

First modeling results of vehicle place determination with Minimum Mean Square (MMS) algorithm and Kalman filter were described in [1]. In department we are testing adaptive algorithms for place [2], but in some cases, when parameters of the object motion are constant, MMS-Kalman algorithm gives precision results of place determination. Process of filtering is divided on tow steps. In fist step place rough determination take place, but in second step filtering with parameter determination is used. For place location most frequently are used tow kinds of sensors: satellite Global Positioning Systems (GPS) and Inertial Navigation Systems (INS). In first stage of place location is used information of GPS and Minimum Mean Square (MMS) algorithm. In second stage are used GPS and INS information with complex information processing. In case if navigation system have not INS sensors in second step only GPS information can be used. For this purpose we estimate MMS-MMS algorithm also.

## Estimation of MMS-MMS algorithm for mobile object place determination

MMS-MMS filter algorithm is used for estimation of dynamic object place location with satellite Global Positioning System. For rough place determination is used information of GPS and Minimum Mean Square (MMS) algorithm. Look how it works.

If $Y$ is vector of measurements with dimension $N$, which is linearly depending from estimated parameters $X$ with dimension $n$

$$Y = HX + \eta\,, \qquad (1)$$

where $\eta$ – measurement error vector.

We will search estimation of parameters $X$ when square error $\varepsilon^2$ is minimised:

$$\varepsilon^2 = (Y - HX)^T (Y - HX). \qquad (2)$$

For solving this task we will use differencing for $X$ and result make equal to zero:

$$\frac{\partial \varepsilon^2}{\partial X} = -2H^T(Y - HX)\Big|_{X=\hat{X}} = 0. \qquad (3)$$

Solving of equation (3) gives:

$$\hat{X} = \left(H^T H\right)^{-1} H^T Y. \qquad (4)$$

When pseudo distances $\tilde{P}_{izm\_i}\,; i = \overline{1, N}$ from $N$ satellites are measured, but coordinates x, y, z of place and clock shift D’ are determinate $X = |x \;\; y \;\; z \;\; D'|^T$, all measurements are unite in one vector

$$\tilde{P}_{izm} = P_{izm}(X) + \eta\,. \qquad (5)$$

Matrix $H$ for our case is equal:

$$H = \left[\frac{\partial P_{izm}^T\left(\hat{X}_0\right)}{\partial X}\right]^T. \qquad (6)$$

Using (4) for place location with GPS and MMS algorithm we have:

$$\hat{X} = \left(H^T H\right)^{-1} H^T Y =$$
$$\left(H^T H\right)^{-1} H^T \left(\tilde{P}_{izm} - P_{izm}\left(\hat{X}_0\right) + H\hat{X}_0\right) =$$
$$= \hat{X}_0 + \left(H^T H\right)^{-1} H^T \left(\tilde{P}_{izm} - P_{izm}\left(\hat{X}_0\right)\right), \qquad (7)$$

where $\hat{X}_0$ – matrix of knowing coordinates before estimation.

Results of modelling place rough determination with MATLAB, when knowing coordinate $x_0$ are x – 300 km, time shift is 5 km and random measurement errors are independent with $\sigma_D$ = 1 m, are shown in Fig. 1.

On the Fig. 1 we can see, that after 3-4 steps of iteration place determination error are smaller than same meters. More detailed it may be seen from Fig. 2, where MMS algorithm is used for place determination, when knowing coordinates are precise. On the Fig. 2 we can see also, that using this MMS algorithm for place determination does not decrease fluctuation errors.

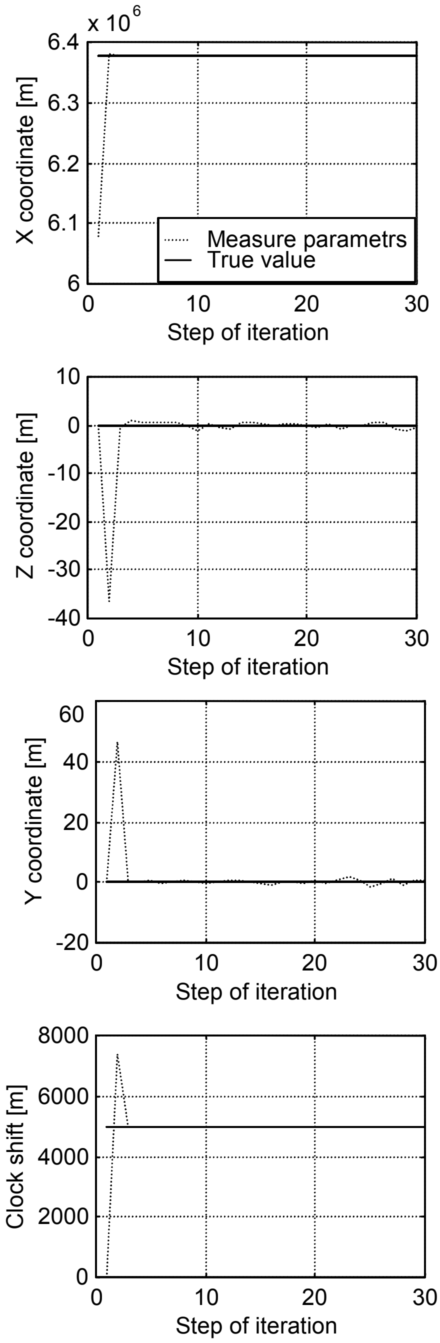


**Fig. 1.** Place rough determination with MMS algorithm

For decreasing these errors and determination of movement parameters MMS algorithm also may be used [3]. More rapidly MMS algorithms determinate dynamic object velocity Vxe and first point coordinate xeo using i=1…n points:

$$xeo := \frac{\left(\sum_i i^2\right)\cdot\sum_i x_i - \sum_i i\cdot\left[\sum_i \left(i\cdot x_i\right)\right]}{d}; \qquad (8)$$

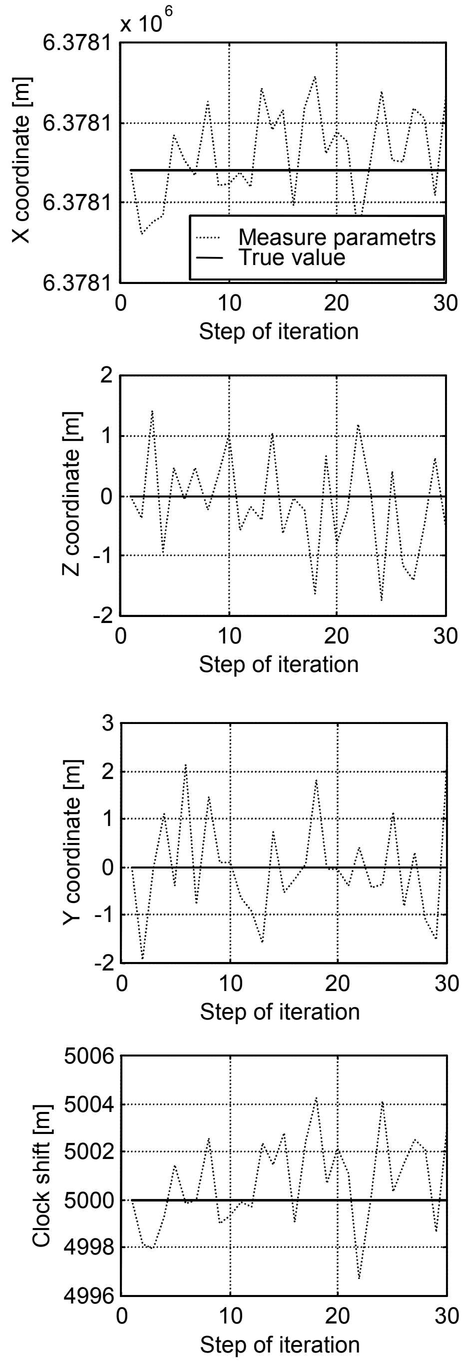


**Fig. 2.** Place precise determination with MMS algorithm

$$Vxeo := \frac{n\cdot\left[\sum_i \left(i\cdot x_i\right)\right] - \sum_i i\cdot\sum_i x_i}{d}; \qquad (9)$$

where parameter d is calculate using expression:

$$d := n\cdot\sum_i i^2 - \left(\sum_i i\right)^2. \qquad (10)$$

If dynamic object movement parameters are constant in n points, which are used for parameter determination and n is hundred or more, algorithm gives very good results. Fig. 3 shows modelling result using MMS algorithm for case when $V_x$ = 0.2 m/s, but Fig. 4 when $V_x$ = 0.02 m/s and number of points is 100. Estimated coordinate xe fluctuation errors decrease √n times: $\sigma_{xe} = \sigma_x/\sqrt{n}$.

If parameters of dynamic object is changing adaptive MMS algorithm with various n may be used. For this expected place is calculate using estimated velocity and time step. For example expected coordinate $x_{n+1}$ is: $x_{n+1} = x_n + Vxe \cdot \Delta t$. If measured coordinate differ more than some value Δx MMS filter parameter n is decrease till value 2…5, but if differ is smaller then n = n+1 till n reaches maximal value.

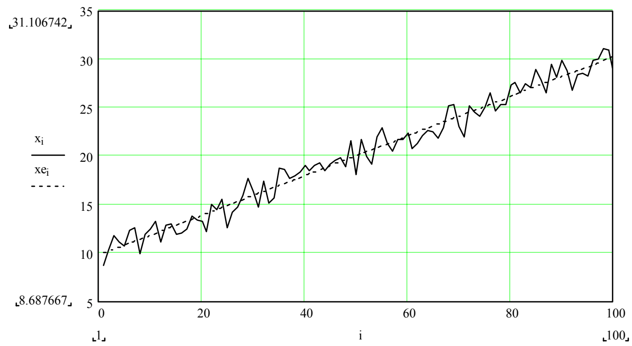


**Fig. 3.** x coordinate and its change velocity estimation with MMS algorithm ($V_x$ = 0.2 m/s)

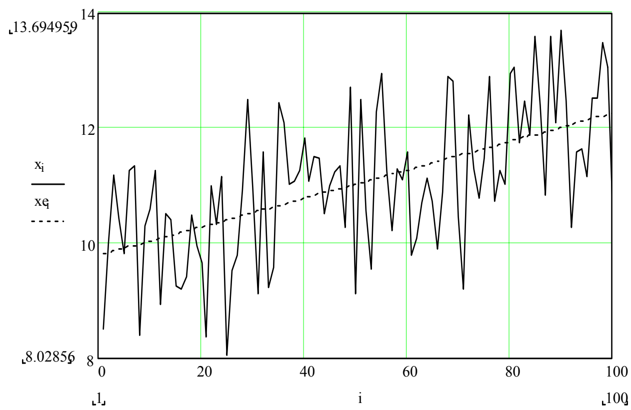


**Fig. 4.** x coordinate and its change velocity estimation with MMS algorithm ($V_x$ = 0.02 m/s)

**Estimation of MMS-Kalman filtering algorithm for mobile object place determination**

For rough place determination Kalman algorithm also may be used, but results of modelling shows, that estimation time is more than with MMS algorithm. More interesting is testing Kalman algorithm application for decreasing fluctuation errors on the second stage of coordinate processing. For decreasing of Global Positioning Systems fluctuation errors additional information from Inertial Navigation System is used. Filter algorithms are [2]:

$$\begin{cases} \boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T (\boldsymbol{H}_k \boldsymbol{P}_k^- \boldsymbol{H}_k^T + \boldsymbol{R}_k)^{-1}, \\ \hat{\boldsymbol{X}}_k = \hat{\boldsymbol{X}}_k^- + \boldsymbol{K}_k (\boldsymbol{Z}_k - \boldsymbol{H}_k \hat{\boldsymbol{X}}_k^-), \\ \boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k) \boldsymbol{P}_k^-, \\ \boldsymbol{P}_{k+1}^- = \Phi_k \boldsymbol{P}_k \Phi_k^T + \boldsymbol{Q}_k, \\ \hat{\boldsymbol{X}}_{k+1}^- = \Phi_k \hat{\boldsymbol{X}}_k. \end{cases} \quad (11)$$

where $\boldsymbol{H}_k$ – measurement transmission matrix, $\boldsymbol{Z}_k$ – vector of measured parameters, $\boldsymbol{X}_k$ – systems state vector, $\boldsymbol{K}_k$ – Kalman filter transmission matrix, $\boldsymbol{\Phi}_k$ – system state transmission matrix, $\boldsymbol{P}_k$ – measurement noise covariance matrix, $\boldsymbol{Q}_k$ – systems noise covariance matrix. Index " – " show that parameters are calculate before measurements in "k" cycle.

Complex date processing with Kalman filter was modelled only for one coordinate x. GPS date random error σ was bring 1 m, but INS velocity measuring error – 0.01m/s for random error and 1m/s for constant error. In result of complex date processing constant velocity error also is calculate. Matrixes $\Phi_k$ and $H_k$ in this case are equal:

$$\Phi_k := \begin{pmatrix} 1 & \Delta t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$H_k := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & -\Delta t & 0 \end{pmatrix};$$

where Δt is interval between measurements.

Modelling results when $V_x$ = 0.2 m/s are show in Fig. 5, but when $V_x$ = 0.02 m/s – in Fig. 6. As it is visible from Fig.5 and 6 mistakes of x coordinate location are reduce and value of estimated coordinate error is so that was get with MMS algorithm on the second step of estimation. Kalman algorithm works better when velocity is changing and it also estimates the constant error of INS.

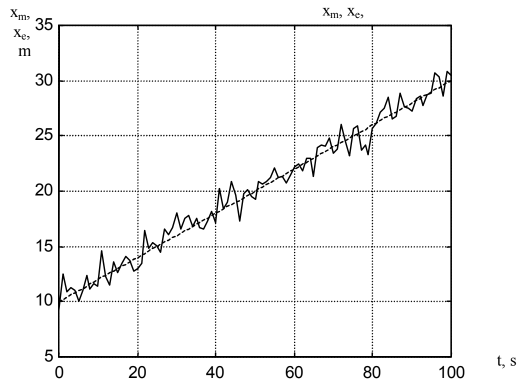


**Fig. 5.** x coordinate and its change velocity estimation with MMS algorithm ($V_x$ = 0.2 m/s): --- estimated value of x coordinate $x_e$, — measured value of x coordinate $x_m$

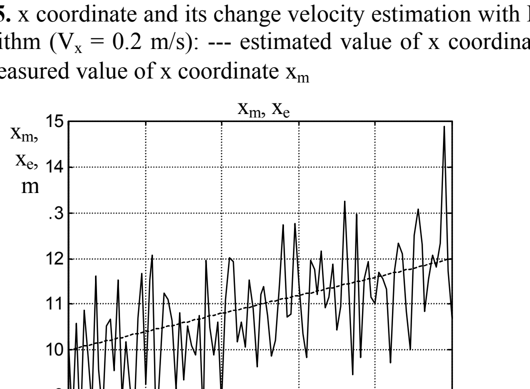


**Fig. 6.** x coordinate and its change velocity estimation with MMS algorithm ($V_x$ = 0.02 m/s): --- estimated value of x coordinate $x_e$, — measured value of x coordinate $x_m$

## Conclusion

The described methods of GPS information filtering with MMS-MMS algorithms and complex information processing of GPS and INS with MMS-Kalman algorithm shows increase in accuracy of estimation of position of mobile objects. The offered complex MMS-Kalman algorithm reduce time of coordinate estimation because MMS algorithms are very effective for quick place processing, but Kalman recursive algorithm allows to reduce fluctuation errors and estimate the dynamic parameters of mobile object. In case if navigation system have not INS sensors GPS information can be filtered with MMS-MMS algorithm.

## References


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**A. Kluga, J. Kluga. Modeling Information Complex Processing with MMS-Kalman Filter // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 6(78). – P. 17–20.**

Comparison of tow algorithms for mobile object place location with satellite systems is shown in this work. First is the minimum mean-square (MMS) recursive algorithm, which is used for place rough determination and random error filtering (MMS-MMS algorithm). Second is MMS-Kalman filtering algorithm. Estimation of algorithms was obtained by modeling mobile object place location with satellite systems in various dynamic and noise signal situations. Is show, that in case if navigation system have not INS sensors GPS information can be filtered with MMS-MMS algorithm, but if GPS and INS sensors are used MMS-Kalman algorithm can be used. Ill. 6, bibl. 3 (in English, summaries in English, Russian and Lithuanian).

**А. Клуга, Я. Клуга. Моделирование комплексной обработки информации с фильтром ММК-Калмана // Электроника и электротехника. – Каунас: Технология, 2007. – № 6(78). – С. 17–20.**

Рассмотрено сравнение двух алгоритмов для определения местоположения подвижного объекта с помощью спутниковых систем. Первый – это рекурсивный алгоритм минимальной среднеквадратической погрешности ММК, который используется для приближенного определения местоположения и для фильтрации случайных погрешностей (ММК-ММК). Второй – это алгоритм ММК-Калмана. Оценка алгоритмов получена путем моделирования место определения мобильного объекта с помощью спутниковых систем в различных условиях помех и динамики. Показано, что если навигационная система не имеет инерционного датчика, для обработки информации GPS может использоваться ММК-ММК алгоритм, а при использовании системы ИНС – алгоритм ММК-Калмана. Ил. 6, библ. 3 (на английском языке; рефераты на английском, русском и литовском яз.).

**A. Kluga, J. Kluga. Kompleksinės informacijos apdorojimo modeliavimas panaudojant Kalmano (minimalios vidutinės kvadratinės paklaidos) filtrą // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 6(78). – P. 17–20.**

Pateikiamas mobilių objektų vietos nustatymo, naudojant palydovines sistemas algoritmų palyginimas. Pirmasis algoritmas – tai rekursinis minimalios vidutinės kvadratinės paklaidos metodas, kuris yra taikomas vietai apytiksliai nustatyti ir atsitiktinėm klaidoms filtruoti. Antrasis – Kalmano filtracijos rekursinis algoritmas. Lyginta naudojant mobilaus objekto vietos nustatymo modeliavimą, esant įvairioms triukšmų ir dinamikos sąlygoms. Parodyta, kad, jei navigacijos sistemos neturi inertinės navigacinės sistemos jutiklio, informacija gali būti filtruojama rekursiniu minimalios vidutinės kvadratinės paklaidos metodu, tačiau jei tas jutiklis yra kartu su GPS gali būti naudojamas Kalmano filtracijos rekursinis algoritmas. Il. 6, bibl. 3 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).