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Signal Reconstruction from Multiple Level Crossings Using Asymmetric Constructing Functions

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Introduction

Signal reconstruction is one of the main tasks in signal processing, where a discrete signal has to be transformed into an analog form. Commonly used analog-to-digital converters sample signals uniformly. The sampling rate is determined by maximal frequency in signal spectrum. A level-crossing sampling approach differs from traditional sampling scheme. For level-crossing sampling quantization levels are regularly disposed along the amplitude range of the signal [1]. A sample is captured only when the analog input signal crosses one of these levels. In general, samples are not uniformly spaced out in time and the sampling density depends on the signal's local properties [2]. This allows minimizing activity, power consumption and hardware complexity of the circuit that performs the digitizing. The paper describes the method proposed for signal reconstruction from its level-crossing samples.

Asymmetric constructing functions

To reconstruct the uniformly sampled signal a corresponding reconstruction filter is used. In time domain reconstruction becomes convolution of samples with impulse response of the filter:

$$\hat{s}(t) = \sum_{n=0}^{N-1} s(t_n) \cdot h(t - t_n), \quad (1)$$

where $\hat{s}(t)$ is reconstructed signal, $s(t_n)$ are the original signal samples and $h(t)$ is the impulse response that can be considered as signal constructing function. In case the sampling rate equals the Nyquist rate, an ideal filter is used and constructing function is the well-known sinc-function, which has fairly poor decay properties. In contrast, the case of oversampling gives some freedom in the choice of $h(t)$. If the use of cubic cardinal spline $\eta^3(t)$ is considered, then

$$h(t) = \eta^3(t) = \frac{6\alpha}{\alpha^2 - 1} \cdot \sum_{k=-\infty}^{\infty} \alpha^{|k|} \cdot \beta^3(t-k) \quad (2)$$

decays comparatively fast (Fig. 1.), where $\alpha = \sqrt{3} - 2$ and $\beta^3(t)$ is the cubic B-spline [3].

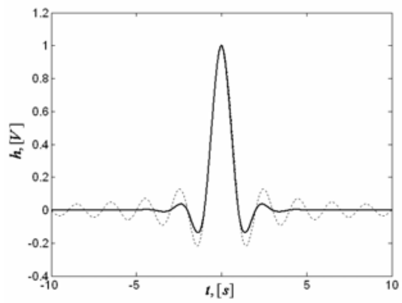

Fig. 1. Sinc-function (dotted line) and the cubic cardinal spline

To reconstruct the nonuniformly sampled non-stationary signal, the use of asymmetric constructing functions with fast decay time is considered. This allows improving the locality of signal reconstruction.

The asymmetric cubic cardinal spline $\kappa^3{}_n(t)$ is determined by expression:

$$\kappa_n^3(t) = \sum_{m=-5}^{6} \left[\eta^3 \left(\frac{t - t_{n-m} - \Delta t_{n-m} \cdot m}{\Delta t_{n-m}} \right) \cdot \Delta u_{n-m} \right], \quad (3)$$

where n is the index of level-crossing sample, $\Delta t_n = t_{n+1} - t_n$ and $\Delta u_n = u(t - t_n) - u(t - t_{n+1})$ is the difference of two unit step functions. An example of asymmetric cubic cardinal spline is shown in Fig. 2.

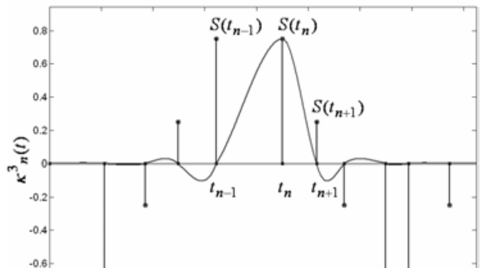

Fig. 2. Asymmetric third-order cardinal spline

Note the constructing function $\kappa^3{}_n(t)$ differs for each individual sample. In case of uniform sampling $\kappa^3{}_n(t) = \eta^3((t - t_n) \cdot f_s)$ for all n, where f_s is signal sampling frequency.

Signal reconstruction

Signal reconstruction formula is similar to (1):

$$\hat{s}(t) = \sum_{n=0}^{N-1} s(t_n) \cdot \kappa_n^3(t). \tag{4}$$

In this case the calculation of $\kappa_n^3(t)$ is quite simple in comparison with iterative estimation of local atoms in [5].

If two successive level-crossing samples are at the same level, i.e. $s(t_n) = s(t_{n+1})$, than $\hat{s}(t)$ between these samples will often be less in magnitude than the original signal (Fig. 4.). To prevent it, the following correction method is presented.

Correction of the reconstructed signal

To increase the magnitude of the reconstructed signal between two successive samples $s(t_n) = s(t_{n+1})$, the use of the quadratic Bezier spline [6] is considered.

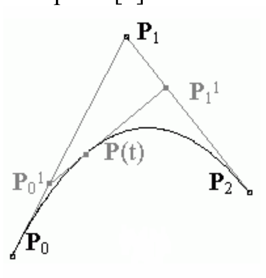

Fig. 3. Quadratic Bezier spline

If three points P_0, P_1 and P_2 are given (Fig. 3.), then

$$P(t) = (1-t)^2 P_0 + 2(1-t)tP_1 + t^2 P_2,\ t \in [0,1]. \tag{5}$$

If each point is determined by its rectangular coordinates (x, y) and $P_{0y} = P_{2y}$, than the maximal value of the ordinate of $P(t)$ is

$$P_y^{(\max)} = P_{0y} + \frac{1}{2} \cdot \frac{\Delta x}{k_1^{-1} - k_2^{-1}}, \tag{6}$$

where $\Delta x = P_{2x} - P_{0x}$ and k_1, k_2 are the slopes of the lines P_0P_1 and P_1P_2.

To correct the reconstructed signal, formula (6) can be used to determine the coefficient C:

$$C = \frac{P_y^{(\max)} - P_{0y}}{\hat{S}_n^{(\max)} - s(t_n)} = \frac{1}{2} \cdot \frac{1}{\hat{S}_n^{(\max)} - s(t_n)} \cdot \frac{t_{n+1} - t_n}{k_1^{-1} - k_2^{-1}} \tag{7}$$

and

$$\hat{s}_{corr}(t) = C \cdot (\hat{s}(t) - s(t_n)) + s(t_n), \tag{8}$$

where $\hat{S}_n^{(\max)}$ is the maximal value of the reconstructed signal between two successive samples $s(t_n) = s(t_{n+1})$ and $\hat{s}_{corr}(t)$ is the reconstructed signal after correction.

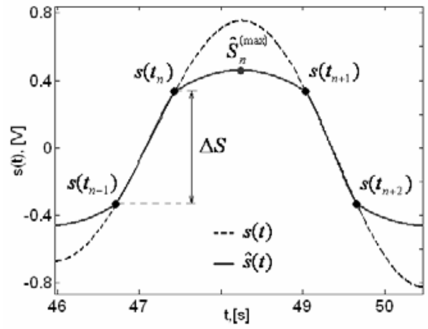

Fig. 4. Original $s(t)$ and reconstructed $\hat{s}(t)$ signals

To determine the slopes k_1 and k_2, two methods are considered. The first one can be used, if $s(t_{n-1}) \neq s(t_n) = s(t_{n+1}) \neq s(t_{n+2})$ and the slopes are

$$k_1 = \frac{\Delta S}{\Delta t_{n-1}} \text{ and } k_2 = -\frac{\Delta S}{\Delta t_{n+1}}, \tag{9}$$

where ΔS is the difference between two successive quantization levels (Fig. 4.). The precision of the estimated slopes is obviously limited by ΔS, which shouldn't be too small. To improve it, the second method is considered. Each time the signal crosses one of the defined levels S_i an additional level $S_i \pm dS$ is used to determine the derivative of the signal (Fig. 5.). In this case the slopes are

$$k_1 = \frac{dS}{dt_n} \text{ and } k_2 = -\frac{dS}{dt_{n+1}}. \tag{10}$$

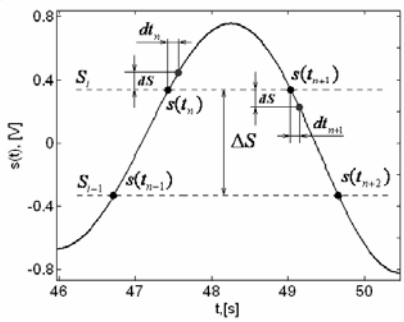

Fig. 5. Derivative estimation of the signal

The precision of derivatives is determined by dS, which can be small enough, and the condition $s(t_{n-1}) \neq s(t_n) = s(t_{n+1}) \neq s(t_{n+2})$ may not hold. If the estimated coefficient C in (7) exceeds allowable value due to big difference Δt_n, than original signal between t_n and t_{n+1} must have local extremes (Fig. 6.).

To limit the corrected signal in allowable amplitude range, the following technique is proposed. Firstly the values $1 > d_1 > d_2 > d_3 > \ldots$ are chosen such that $d_j = \frac{|a_j - \overline{S}_{i-1}|}{S_i - \overline{S}_{i-1}}$, where a_j indicates the level at which corrected signal will be reflected. Secondly the maximal value of the corrected signal before limitation $\hat{s}_{corr\,\max}$ must be equal $\overline{S}_{i-1} = \frac{S_i + S_{i-1}}{2}$ after the limitation (Fig. 6.). To ensure this, the following expression must hold true:

$$d_1 + d_2 + d_3 + ... = \frac{\hat{s}_{corr\,\max} - \overline{S}_{i-1}}{S_i - S_{i-1}} . \qquad (11)$$

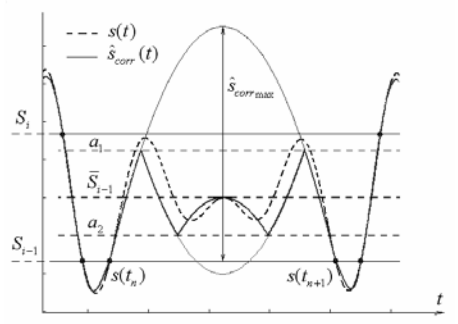

Fig. 6. The limitation of the corrected reconstructed signal

When initial d_j values are chosen, the equation (11) mostly will not be satisfied, so the smallest $j_{\min}$ is found such that

$$\begin{cases} \dfrac{\hat{s}_{corr\,\max} - \overline{S}_{i-1}}{S_i - S_{i-1}} \le \sum_{j=1}^{j_{\min}} d_j , \quad if \; j_{\min} = 1, \\ \sum_{j=1}^{j_{\min}-1} d_j < \dfrac{\hat{s}_{corr\,\max} - \overline{S}_{i-1}}{S_i - S_{i-1}} \le \sum_{j=1}^{j_{\min}} d_j , \quad if \; j_{\min} > 1. \end{cases} \qquad (12)$$

After the estimation of $j_{\min}$, new values of d_j are calculated:

$$d'_j = \frac{\hat{s}_{corr\,\max} - \overline{S}_{i-1}}{\left(S_i - S_{i-1}\right)\sum_{j=1}^{j_{\min}} d_j} \cdot d_j . \qquad (13)$$

For new d'_j equation (11) holds true and new a'_j values are found:

$$a'_j = \left(-1\right)^{j+1} d'_j \left(S_i - \overline{S}_{i-1}\right) + \overline{S}_{i-1} . \qquad (14)$$

Index $j_{\min}$ indicates a number of times the corrected signal is reflected in the process of limitation at levels a'_j.

Simulation results

The performance of described signal reconstruction technique has been investigated by computer simulations on two test signals. The first one is a chirp with constant amplitude, while the second is a chirp with time varying amplitude. The result of reconstruction of the first signal is illustrated in Fig. 7. Note that in this case $s(t_n) \neq s(t_{n+1})$ for all n and no correction of reconstructed signal is needed. In contrast, the difference between original signal and recovered one in Fig. 8. is quite obvious. The result gets better after correction and limitation of the reconstructed signal. This can be seen in Fig. 9. If more quantization levels for level-crossing sampling are used, the reconstruction result of the second test signal is illustrated in Fig. 10.

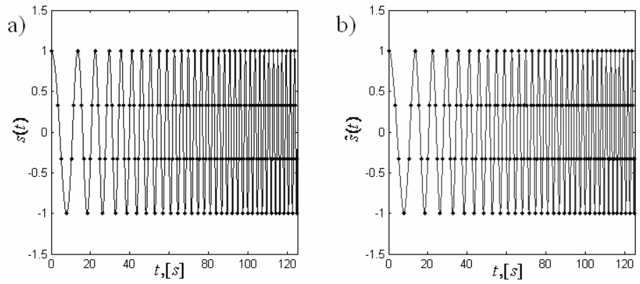

Fig. 7. Reconstruction of the first test signal: a) original signal; b) reconstructed signal

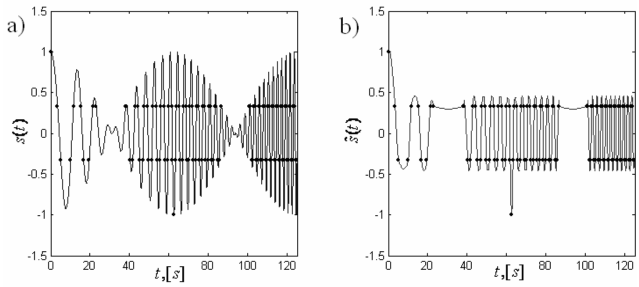

Fig. 8. Reconstruction of the second test signal: a) original signal; b) reconstructed signal

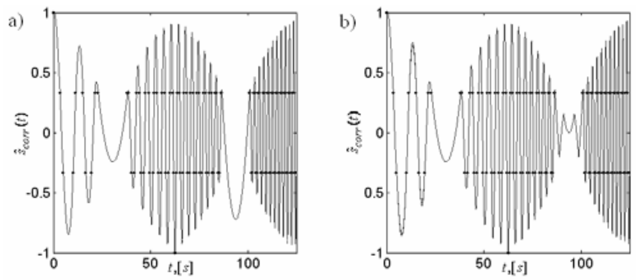

Fig. 9. Reconstructed second test signal after a) correction and b) correction and limitation

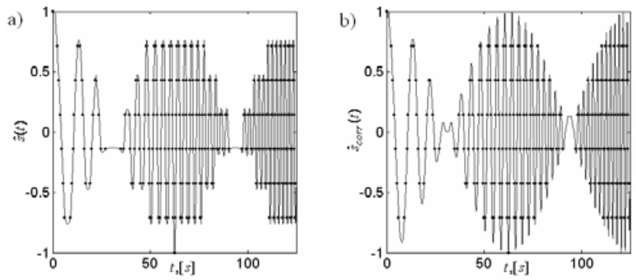

Fig. 10. Reconstructed second test signal (sampled by 8 levels-crossings) a) before correction and b) after correction and limitation

To estimate the quality of reconstruction, effective values of error signal $\varepsilon(t) = \hat{s}(t) - s(t)$ and original $s(t)$ are compared. The results are summarized in Table 1. The reconstruction error for the first test signal is $\varepsilon_{eff} / s_{eff} = 5\%$.

Conclusions

As can be seen from the results of simulations, the reconstruction of the signal improves as the number of level-crossing samples increases. The reconstruction error can be considerably reduced after the correction and limitation of the reconstructed signal.

Table 1. Estimated reconstruction errors for the second test signal

$\frac{\varepsilon_{eff}}{s_{eff}}$, [%] / Number of levels (samples)	After reconstruction	After correction	After correction and limitation
4 (109)	51	38	20
8 (288)	16	7	5

The advantages of the proposed signal reconstruction method:

- rather fast with higher precision as the number of samples increases;
- the value of the reconstructed signal is determined only by twelve samples localized around time moment at which signal value is being calculated (this gives a chance to build an almost real time signal reconstruction systems).

The reconstruction result gets worse if three or more successive samples are at the same level. The reconstructed signal between two successive samples is always smooth (except when the limitation is done) so that rapid changes in original signal are not taken into account. This effect can be reduced by increasing the number of levels when performing signal sampling.

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R. Shavelis. Signal Reconstruction from Multiple Level Crossings Using Asymmetric Constructing Functions // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 5(77). – P. 57–60.

Signal reconstruction is one of the main tasks in signal processing, where a discrete signal has to be transformed into an analog form. Commonly used analog-to-digital converters sample signals uniformly. The sampling rate is determined by maximal frequency in signal spectrum. A level-crossing sampling approach differs from traditional sampling scheme and results in nonuniformly spaced samples with sampling density depending on the signal's local properties. The method is proposed for signal reconstruction from its level-crossing samples, using asymmetric cubic cardinal splines. Achieved results are demonstrated by simulations. Ill. 10, bibl. 6 (in English; summaries in English, Russian and Lithuanian).

Р. Шавелис. Восстановление дискретизованных по уровням сигналов с использованием асимметричных конструирующих функций // Электроника и электротехника. – Каунас: Технология, 2007. – № 5(77). – С. 57–60.

Восстановление сигналов представляет собой одну из основных задач обработки сигналов, где дискретизованный сигнал должен быть преобразован в аналоговую форму. Широко используемые аналого-цифровые преобразователи дискретизуют сигналы равномерно по времени. При этом частота дискретизации сигнала определяется максимальной частотой в его спектре. Дискретизация сигналов по уровням отличается от упомянутой и приводит к неравномерному распределению отсчетов во времени, зависящему от специфических свойств сигнала. Предлагается метод восстановления дискретизованных по уровням сигналов с использованием ассимметричных кубических фундаментальных сплайнов. Полученные результаты продемонстрированы результатами моделирования. Ил. 10, библ. 6 (на английском языке; рефераты на английском, русском и литовском яз.).

R. Shavelis. Diskretizuotų pagal lygius signalų atkūrimas taikant asimetrines konstruojančiąsias funkcijas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 5(77). – P. 57–60.

Signalų atkūrimas yra viena pagrindinių jų apdorojimo užduočių, kai diskretizuotas signalas turi būti keičiamas į analoginį. Plačiai taikomi analoginiai kodiniai keitikliai signalus tolygiai diskretizuoja laike. Signalo diskretizavimo dažnis nustatomas pagal maksimalų jo spektro dažnį. Diskretizacija pagal lygius skiriasi nuo minėtosios, nes gaunamas netolygus laike atskaitų išdėstymas, priklausantis nuo signalo savybių. Siūlomas metodas diskretizuotiems pagal lygius signalams atkurti taikant asimetrinius fundamentaliuosius splainus. Pateikiami gauti modeliavimo rezultatai. Il. 10, bibl. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).