

## Simplified Calculation of Linear Induction Drives Characteristics

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### Introduction

In the industry automation systems widely are used small power and relatively small speed linear induction motors (LIM) and on it base designed controlled linear electric drives linear electric drive [1–9]. The mechanical characteristics of these LIM and controlled linear electric drive are close to linear characteristics. The influence of slip to the LIM inductor current is small also the dynamic force and current amplitudes are low. Theoretical performance and mechanical characteristics of motors and drives derived from the LIM electromagnetic expressions are complicated [10]. Problem gets more difficult because of LIM is asymmetrical three phase electrical energy receiver with reverse current component, which creates breaking force component. Because of that, theoretical mechanical characteristic of a low speed LIM crosses the speed axis lower synchronous speed point, when the low speed LIM supplied by the symmetrical three phase source mechanical characteristic with influence of end effect crosses speed axis over mentioned point. When designing the LIM and linear electric drive it is useful to use simplified LIM and linear electric drive mechanical and performance characteristic expressions, derived with respect to mentioned linear induction motor and on it base designed electric drive properties.

The examples of inductors and secondary elements used in the design of LIM, based on the calculation method, explained in this paper, are shown in Fig. 1.

### Simplified expression of linear induction motor primary current

In the paper [3] on the principles of LIM series equivalent circuit the mechanical and performance characteristic expressions for multipolar drive when the width of the secondary element is equal to inductor width are obtained. LIM inductor current under this working conditions are calculated by:

$$I_1 = U_1 \sqrt{\frac{1 + \varepsilon_0^2 s^2}{A_1(s)}}; \quad (1)$$

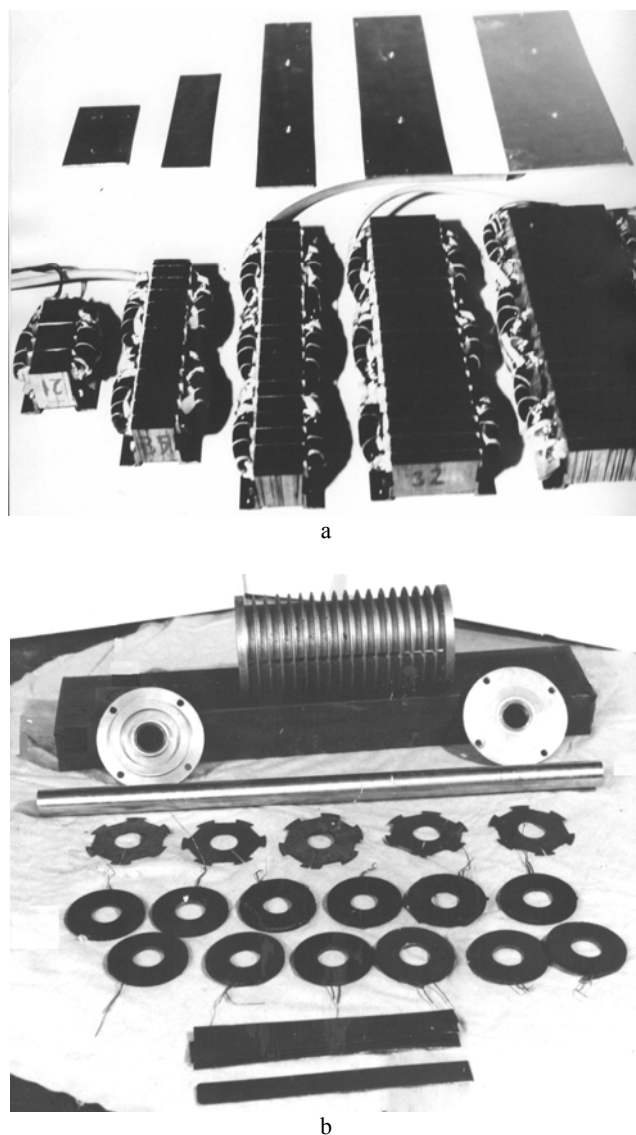


Fig. 1. Examples of linear induction motor elements designed in the VGTU Automation department: a – flat construction LIM; b – cylindrical construction LIM (CLIM)

where  $U_1$  – LIM inductor phase voltage;  $\varepsilon_0 s = \varepsilon$  – Reynold's magnetic number;

$$A_1(s) = (R_1^2 + X_{11}^2 - 2a_2 X_{11} X_m + a_1^2 X_m^2) \varepsilon_0^2 s^2 + 2a_1 R_1 X_m \varepsilon_0 s + (R_1^2 + X_{11}^2) \cong R_1^2 + X_{11}^2 \quad (2)$$

$$X_{11} = X_m + X_1; \quad (3)$$

where  $R_1$  – resistance of LIM inductor phase winding;  $X_1$  – leakage reactance of LIM inductor phase winding;  $X_m$  – magnetizing reactance;  $a_1$  and  $a_2$  – approximation coefficients, associated with LIM series equivalent circuit inserted resistance and reactance, which reflect influence of secondary element to inductor, with magnetic Reynolds number, magnetizing reactance and slip.

Expression (1) in [3] derived from the respect to LIM natural end effect and inductor three phase current system asymmetry. However when calculating mechanical characteristics, mentioned factors causes different sign errors, so in the first multifunction linear electric drive design stage we can not to take into account these parameters.

By examine (1) root numerator binomial and denominator polynomial numerical values, when the slip  $s$  changes, it is possible to simplify expression of current components. Results for experimental motor are shown in Table 1.

Parameters of experimental linear induction motor:  $R_1 = 2,82 \Omega$ ;  $X_1 = 5,96 \Omega$ ;  $X_m = 8,24 \Omega$ ;  $\varepsilon_0 = 0,284$ ;  $\alpha_1 = 0,618$ ;  $\alpha_2 = 0,427$ .

**Table 1.** Results for experimental linear induction motor

Notation	Expression	Calculation error		
		s=0	s=0,5	s=1
$B$	$1 + \varepsilon_0^2 s^2$	1	1,02	1,081
$\Delta B\%$	$\frac{B(s) - B(0)}{B(0)} \cdot 100\%$	0%	2%	8,1%
$A_{11}$	$(R_1^2 + X_{11}^2 - 2a_2 X_{11} X_{p1} + a_1^2 X_{p1}^2) \varepsilon_0^2 s^2$	0	2,78	11,14
$A_{12}$	$2a_1 R_1 X_{p1} \varepsilon_0 s$	0	4,08	8,16
$A_{13}$	$R_1^2 + X_{11}^2$	210	210	210
$A_1(s)$	$A_{11} + A_{12} + A_{13}$	209,6	216,46	228,9
$\Delta A_1\%$	$\frac{A_1(s) - A_1(0)}{A_1(0)} \cdot 100\%$	0%	-1,24%	-1,02%
$C$	$\frac{1 + \varepsilon_0^2 s^2}{A_1(s)}$	$4,77 \cdot 10^{-3}$	$4,71 \cdot 10^{-3}$	$4,72 \cdot 10^{-3}$

Notation	Expression	Calculation error		
		s=0	s=0,5	s=1
$\Delta C\%$	$\frac{C(s) - C(0)}{C(0)} \cdot 100\%$	0%	-1,24%	-1,02%
$D$	$\sqrt{\frac{1 + \varepsilon_0^2 s^2}{A_1(s)}}$	0,0691	0,0686	0,0687
$I_1$	$U_1 \sqrt{\frac{1 + \varepsilon_0^2 s^2}{A_1(s)}}$ (when $U_1 = 220 \text{ V}$ )	15,1	15,1	15,1
$I_1$	$U_1 \sqrt{\frac{1 + \varepsilon_0^2 s^2}{A_1(s)}}$ (when $U_1 = 127 \text{ V}$ )	8,78	8,711	8,72
$\Delta I_1\%$	$\frac{I_1(s) - I_1(0)}{I_1(0)} \cdot 100\%$	0%	-0,79%	-0,68%

From the Table 1 we can see that in the paper [1] derived LIM inductor current expression can be strongly simplified, when calculation errors are not exceeding some percentage:

$$I_1 = U_1 \sqrt{\frac{1 + \varepsilon_0^2 s^2}{R_1^2 + X_{11}^2}} \quad (4)$$

Simplified binomial  $1 + \varepsilon_0^2 s^2$  and polynomial (2) expressions let us get simplified expressions of LIM performance characteristics.

### Linear induction motor performance characteristics

Considering expression (4) we get these formulas for supply network apparent and active power calculation:

$$S_1 = m_1 U_1^2 \sqrt{\frac{1 + \varepsilon_0^2 s^2}{A_1(s)}} = m_1 U_1^2 \sqrt{\frac{1 + \varepsilon_0^2 s^2}{R_1^2 + X_{11}^2}}; \quad (5)$$

$$P_1 = m_1 U_1^2 \frac{R_1 \varepsilon_0^2 s^2 + a_1 X_m \varepsilon_0 s + R_1}{R_1^2 + X_{11}^2}; \quad (6)$$

where  $m$  – number of LIM inductor phases.

Power factor and efficiency of linear inductor motor are calculated as:

$$\cos \varphi_1 = \frac{R_1 \varepsilon_0^2 s^2 + a_1 X_m \varepsilon_0 s + R_1}{\sqrt{(1 + \varepsilon_0^2 s^2)(R_1^2 + X_{11}^2)}}; \quad (7)$$

$$\eta = \frac{a_1 X_m \varepsilon_0 (1-s)s}{R_1 \varepsilon_0^2 s^2 + a_1 X_m \varepsilon_0 s + R_1}. \quad (8)$$

### Expressions of linear induction motor mechanical characteristics

Force, developed by linear induction motor, is calculated by the formula

$$F = \frac{m_1 I_1^2 R_2'}{2\mathcal{F}_1}. \quad (9)$$

Expression (9) can be expressed by the Kloss formula

$$F = \frac{2F_k(1+a_0s_k)}{\frac{s}{s_k} + \frac{s_k}{s} + 2a_0s_k}; \quad (10)$$

where  $F_k$  and  $s_k$  – LIM breakdown force and slip;

$$F_k = \pm \frac{m_1 U_1^2 X_m a_1}{2\mathcal{F}_1 [(R_1^2 + X_{11}^2) \pm a_1 R_1 X_m]}; \quad (11)$$

$$s_k = \pm \frac{1}{\varepsilon_0} \sqrt{\frac{R_1^2 + X_{11}^2}{R_1^2 + X_{11}^2 - 2a_2 X_{11} X_m + a_1^2 X_m^2}}; \quad (12)$$

$a_0$  – coefficient, calculated as

$$a_0 = \frac{a_1 R_1 X_m \varepsilon_0}{R_1^2 + X_{11}^2}. \quad (13)$$

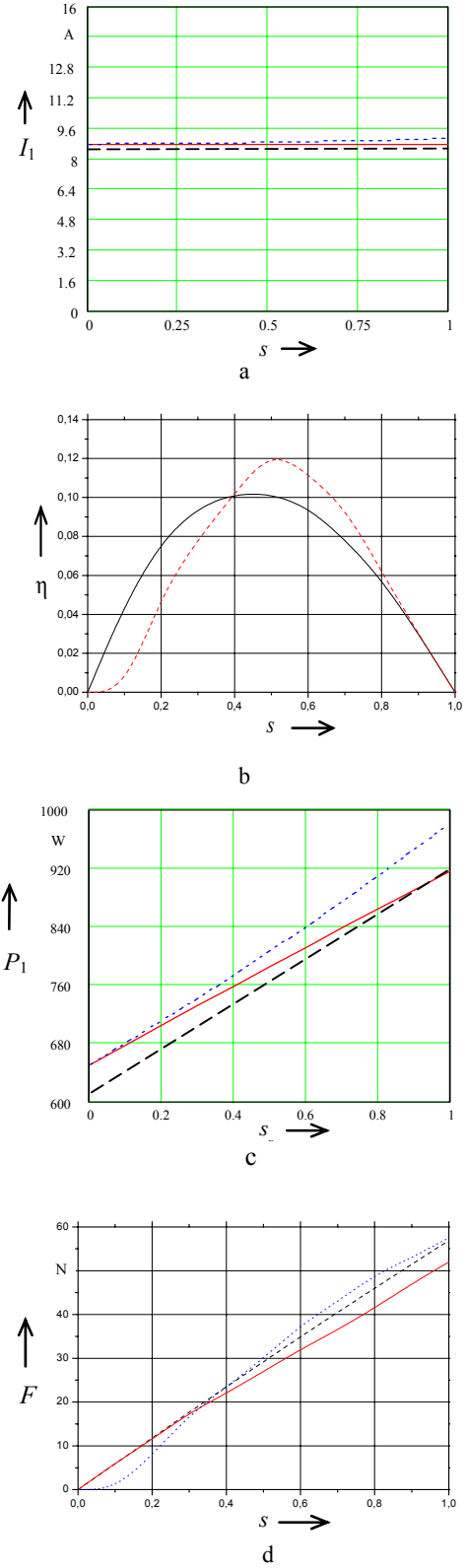
Given expressions valid and for characteristic computation of cylindrical type of LIM (CLIM). In this case  $a_1 = a_2 = 1$ . Then

$$s_k = \pm \frac{1}{\varepsilon_0} \sqrt{\frac{R_1^2 + X_{11}^2}{R_1^2 + X_{11}^2}}. \quad (14)$$

We can propose that the LIM breakdown slip is less than CLIM breakdown slip because of expression (12) root value is less than the root value of expression (14).

### The comparison of theoretical and experimental results

In a Fig. 2 the static characteristics of theoretical and experimental results of LIM with narrow secondary element are shown. The LIM starting currents and forces values are shown by dots in this figure are calculated by using formulas of this paper. From the Fig. 2 we can say that these parameters have to be calculated by simplified method to increase the multiple-choice designing of linear electric drives.



**Fig. 2.** Performance (a, b c) and mechanical (d) characteristics of LIM with narrow secondary element: — calculations by expressions in [3]; - - - experimental results; · · · calculation by simplified expressions

### Conclusions

1. Small power and low speed linear induction motor electrical characteristics let us to simplify

complicated theoretical LIM mechanical and performance expressions. This also enables to simplify designing of multiple-choice systems with linear electric drives.

2. Theoretical characteristic of a low speed LIM crosses the speed axis over synchronous speed point because of end effect and goes under because of asymmetry of currents. However in the first stage of design it is useful not to take into account LIM features.

3. Plain LIM breakdown slip is less than slip of cylindrical LIM (CLIM).

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### **Z. Savickienė, A. J. Poška. Simplified Calculation of Linear Induction Drives Characteristics // Electronics and Electrical Engineering. – Kaunas: Technologija, 2007. – No. 5(77). – P. 15–18.**

The paper presents new and simplified expressions for performance and mechanical characteristics of the linear induction motors (LIM). It enables to increase the speed of designing of different multifunction mechanisms of linear induction motors. On the base of the given expressions, results compared to experimentally and theoretically derived calculations with advanced origin conditions. The simplified computation methodology is based on facts, that theoretical mechanical characteristic of a low speed LIM crosses the speed axis over synchronous speed point because of influence of end effect and goes below the speed point because of asymmetry of currents. However in the first stage of design it is useful not to take into account LIM features which influences the different sign errors. III, 2, bibl. 10 (in English; summaries in English, Russian and Lithuanian).

### **З. Савицкене, А. Ю. Пошка. Упрощенный расчет характеристик линейного асинхронного электропривода // Электроника и электротехника. – Каунас: Технология, 2007. – № 5(77). – С. 15–18.**

Приведены новые упрощенные выражения для расчета рабочих и механических характеристик низкоскоростных линейных асинхронных двигателей (ЛАД) малой мощности. Они позволяют ускорить процесс многовариантного проектирования линейного электропривода для различных механизмов. Результаты расчетов, проведенных по полученным выражениям сравнены с экспериментальными данными и результатами расчетов по исходным теоретическим выражениям. Упрощенная методика расчета характеристик разработана с учетом того, что теоретические механические характеристики низкоскоростных линейных асинхронных двигателей из-за влияния продольного краевого (концевого) эффекта проходят выше точки синхронной скорости, а из-за асимметрии фазных токов, – выше ее, т. е. не учет указанных явлений приводит к разнозначным погрешностям. Ил. 2, библи. 10 (на английском языке; рефераты на английском, русском и литовском яз.).

### **Z. Savickienė, A. J. Poška. Supaprastintas tiesiaiegių asinchroninių elektros pavarų charakteristikų skaičiavimas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2007. – Nr. 5(77). – P. 15–18.**

Pateiktos naujos supaprastintos mažos galios ir greičių tiesiaiegių asinchroninių elektros variklių (TAV) darbo ir mechaninių charakteristikų išraiškos. Jos įgalina paspartinti daugiavariantį įvairių mechanizmų tiesiaiegių elektros pavarų projektavimą. Pagal šias išraiškas atliktų skaičiavimų rezultatai palyginami su eksperimentiniais ir teoriniais, gautais skaičiuojant pagal sudėtingesnes pradinės išraiškas. Supaprastinta charakteristikų skaičiavimo metodika grindžiama faktais, kad mažų greičių TAV teorinės mechaninės charakteristikos dėl galų efekto greičių ašį kerta virš sinchroninio greičio taško, o dėl srovių asimetrijos – žemiau jo. Todėl pradinėje projektavimo stadijoje tikslinga neatsižvelgti į nurodytas TAV savybes, lemiančias skirtingo ženklo paklaidas. Il. 2, bibl. 10 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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