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Edge Finite Elements for 3D Electromagnetic Field Modeling

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Introduction

Finite element method is widely used to analyze, simulate, optimize and design electromagnetic devices. Lots of modern CAD, CAM, CAE programs are based on this method. Fig. 1 summarizes the process of finite element analysis [1,2].

Although numerical application of finite element method is easy and straightforward and it produces accurate results, several problems have been identified when the ordinary nodal based finite elements are employed to compute vector electromagnetic fields. These problems are: long computation time; large amount of memory; satisfaction of the appropriate boundary conditions at material and conducting interfaces; difficulty in treating the conducting and dielectric edges and corners due to the field singularities associated with these structures and etc.

In order to solve these problems, a novel approach has been developed: the approach uses so-called vector finite elements or edge finite elements which assign degrees of freedom to the edges rather than to the nodes of the finite elements. The edge finite element is geometrically the same as its nodal counterpart. The nodal element has one shape function associated with each of the nodes, the edge element has one shape function for each of the edges.

Edge finite elements

Edge finite element method has several advantages in 3-D electromagnetic field computation compare to the nodal finite element technique. Edge finite elements satisfy continuity of only tangential or normal field components across interfaces between two adjacent finite elements. This property can be used when modeling electromagnetic field, since edge finite element approximation does not impose any additional constraints on the approximated field apart from those prescribed by the nature of the field itself.

Overall performance of the software, based on edge finite element method, is also higher and requires less amount of memory compare to the software, which implements the conventional nodal finite element method. This fact is important when analyzing complex 3-D electromagnetic field distributions, because edge finite element method allows fast and accurate analysis even on low-end personal computers.

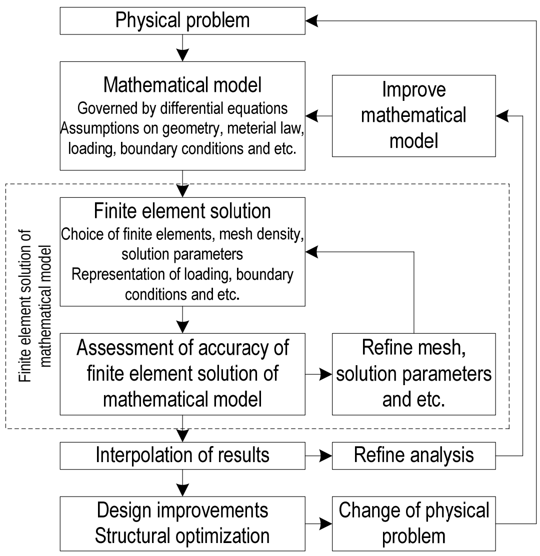

Fig. 1. Process of finite element analysis

Although edge finite element method has several advantages for finite element analysis of vector field problems, and also exhibits considerable performance level of electromagnetic field calculations, edge finite elements also have several problems due to its numerical implementation. The most important problems are source current input, uniqueness of the solution, choosing the adequate gauge and gauging procedure, slower convergence rate of the iterative solver etc.

The edge finite element is geometrically the same as its nodal equivalent. In this case, the unknowns are the component of the field that lies along the edges of the elements.

The approximation of the electromagnetic field in finite element analysis is performed using finite number of discrete values of the electromagnetic field at specific points of mesh, nodes for nodal analysis or edges for edge analysis. The calculation of the electromagnetic field parameters at any point inside the analyzed domain is performed by using simple approximation function, which is called shape function.

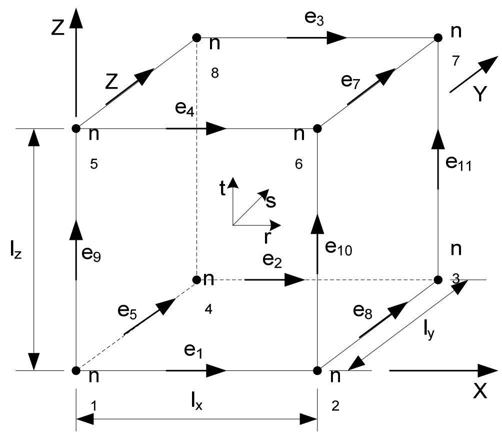

Fig. 2. Rectangular brick edge finite element

Rectangular brick element presented in Fig. 2 has eight nodes n_1, n_2, ..., n_8 and twelve edges e_1, e_2, ..., e_{12}. Two coordinate systems are formed: global coordinate system (x, y, z) and local coordinate system (r, s, t), with origin at the center of gravity of the brick element with global coordinates $(l_x/2, l_y/2, l_z/2)$, where l_x, l_y, l_z are the lengths of each brick dimension. The relations between the global and the local coordinate systems are described by the following equations:

$$x = (r+1)(l_x/2), \tag{1}$$

$$y = (s+1)(l_y/2), \tag{2}$$

$$z = (t+1)(l_z/2). \tag{3}$$

Further, the edge functions for this finite element are generated. For example, the edge shape function for edge e_1, going from node n_1 toward node n_2, has the following shape [1-3]:

$$\vec{N}_{e_1} = N_{n_1}\nabla N_{n_2} - N_{n_2}\nabla N_{n_1}, \tag{4}$$

$$N_{n_1} = ((1-r)(1-s)(1-t))/8, \tag{5}$$

$$N_{n_2} = ((1+r)(1-s)(1-t))/8; \tag{6}$$

where N_{n_1} and N_{n_2} – nodal shape functions at points n_1 and n_2.

The gradient functions of the rectangular brick element nodal shape functions can be evaluated using the following expressions:

$$\nabla N_{n_1} = \begin{bmatrix} \dfrac{\partial N_{n_1}}{\partial x} \\ \dfrac{\partial N_{n_1}}{\partial y} \\ \dfrac{\partial N_{n_1}}{\partial z} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial N_{n_1}}{\partial r}\dfrac{\partial r}{\partial x} + \dfrac{\partial N_{n_1}}{\partial s}\dfrac{\partial s}{\partial x} + \dfrac{\partial N_{n_1}}{\partial t}\dfrac{\partial t}{\partial x} \\ \dfrac{\partial N_{n_1}}{\partial r}\dfrac{\partial r}{\partial y} + \dfrac{\partial N_{n_1}}{\partial s}\dfrac{\partial s}{\partial y} + \dfrac{\partial N_{n_1}}{\partial t}\dfrac{\partial t}{\partial y} \\ \dfrac{\partial N_{n_1}}{\partial r}\dfrac{\partial r}{\partial z} + \dfrac{\partial N_{n_1}}{\partial s}\dfrac{\partial s}{\partial z} + \dfrac{\partial N_{n_1}}{\partial t}\dfrac{\partial t}{\partial z} \end{bmatrix} =$$

$$= \begin{bmatrix} \dfrac{\partial N_{n_1}}{\partial r}\dfrac{\partial r}{\partial x} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{4l_x}(1-s)(1-t) \\ 0 \\ 0 \end{bmatrix}, \tag{7}$$

$$\nabla N_{n_2} = \begin{bmatrix} \dfrac{1}{4l_x}(1-s)(1-t) \\ 0 \\ 0 \end{bmatrix}. \tag{8}$$

Since $\nabla N_{n_1} = -\nabla N_{n_2} = \nabla r$, the shape function for edge e_1 assumes the following expression:

$$\vec{N}_{e_i} = \begin{bmatrix} \vec{N}_x^{1,2,3,4} & 0 & 0 \\ 0 & \vec{N}_y^{5,6,7,8} & 0 \\ 0 & 0 & \vec{N}_z^{9,10,11,12} \end{bmatrix}. \tag{9}$$

The rectangular brick element shape functions for all other edges e_i are expressed analogously. Shape functions for edge finite elements are vector functions. The approximation of the electromagnetic field inside each finite element can be performed using the following expression:

$$\vec{E} = \sum_{i=1}^{12} \vec{N}_{e_i} E_i; \tag{10}$$

where E_i – the tangential components of the unknown electromagnetic field parameter along each edge, N_{e_i} – the matrix of the shape function.

The entire set of shape functions for rectangular brick edge element can be expressed as

$$\vec{N}_{e_i} = \begin{bmatrix} \vec{N}_x^{1,2,3,4} & 0 & 0 \\ 0 & \vec{N}_y^{5,6,7,8} & 0 \\ 0 & 0 & \vec{N}_z^{9,10,11,12} \end{bmatrix}. \tag{11}$$

It can be seen from the equation (11), that the first four edges have the same direction as the x axis, the second four edges have the same direction as the y axis and the remaining four edges have the same direction as the z axis. Further calculations are performed in the same way as in the nodal finite element method.

Edge finite element applications

There is a variety of programs, which are designed in order to create engineering environment for electromagnetic field modeling. Finite element method is most widely developed at the present time, so most of these programs implement this method as their base technique. These programs include *ANSYS/Emag* (ANSYS Inc.), *Maxwell 2D* and *Maxwell 3D* (Ansoft Corporation), *COSMOS/EMS* (Structural Research & Analysis Corporation), *Algor/Electrostatic* (Algor Inc.), *QuickField* (Tera Analysis Ltd.), *FlexPDE* (PDE Solutions Inc.) and etc. These programs differ in number of solved different electromagnetism problems, computer resource demand, preprocessor and postprocessor possibilities, integration with other finite element and automated design programs. Software package ANSYS/Emag has the most extensive modeling capabilities using edge finite element method.

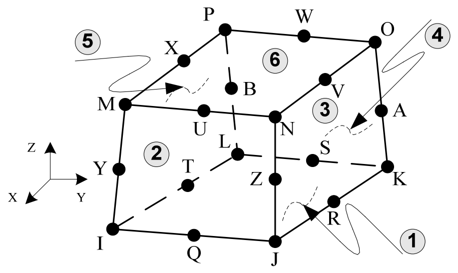

Fig. 3. 20-node brick edge finite element *SOLID117*

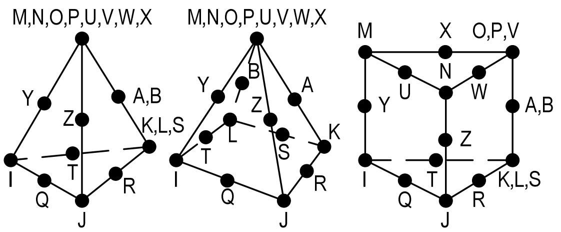

Fig. 4. Possible geometric shapes of the element *SOLID117*

Table. 1. Shape functions

Nodal shape functions	Edge shape functions
$N_I = (1-r)(1-s)(1-t)$	$E_Q = (1-s)(1-t)gradr$
$N_J = r(1-s)(1-t)$	$E_R = r(1-t)grads$
$N_K = rs(1-t)$	$E_S = -s(1-t)gradr$
$N_L = (1-r)s(1-t)$	$E_T = -(1-r)(1-t)grads$
$N_M = (1-r)(1-s)t$	$E_U = (1-s)tgradr$
$N_N = r(1-s)t$	$E_V = rtgrads$
$N_Q = rst$	$E_W = -stgradr$
$N_P = (1-r)st$	$E_X = -(1-r)tgrads$
	$E_Y = -(1-s)(1-r)gradt$
	$E_Z = s(1-r)gradt$
	$E_A = stgradt$
	$E_B = (1-s)rgradt$

Solid finite element *SOLID117* (Fig. 3-4), implemented in the ANSYS/Emag program, is typically used in the analysis of low-frequency problems based on edge finite element formulation [3,4]. The corner nodes of element *SOLID117* $I,\ldots,P$ are used to describe the geometry, orient the edges, support time integrated electric potential degrees of freedom. The side nodes $Q,\ldots,A$ are used to support the edge-flux degrees of freedom, define the positive orientation of an edge to point from the adjacent (to the edge) corner node with lower node number to the other adjacent node with higher node number. The edge-flux degrees of freedom are used in both magnetostatic and dynamic analyses; the electric potential degrees of freedom are used only for dynamic analysis.

The vector potential A and time integrated electric scalar potential V can be described as

$$A = A_Q E_Q + \ldots + A_B E_B \,, \qquad (12)$$

$$V = V_I N_I + \ldots + V_P N_P \,; \qquad (13)$$

where $A_Q,\ldots,A_B$ – edge-flux; $V_I,\ldots,V_P$ – time integrated electric scalar potential; $E_Q,\ldots,E_B$ – vector edge shape functions (Table 1); $N_I,\ldots,N_P$ – scalar nodal shape functions (Table 1).

Shape function for magnetic vector potential component A_x

$$A_x = \frac{1}{8}\big(A_{xI}(1-s)(1-t)(1-r) + A_{xJ}(1+s)(1-t)(1-r) + A_{xK}(1+s)(1+t)(1-r) + A_{xL}(1-s)(1+t)(1-r) + A_{xM}(1-s)(1-t)(1+r) + A_{xN}(1+s)(1-t)(1+r) + A_{xO}(1+s)(1+t)(1+r) + A_{xP}(1-s)(1+t)(1+r)\big). \qquad (14)$$

Form functions for magnetic vector potential components A_y and A_z are expressed analogously.

The global Cartesian coordinates *X*, *Y*, *Z* can be expressed by the master coordinates *r*, *s*, *t*.

$$X = N_I(r,s,t)X_I + \ldots + N_P(r,s,t)X_P \,, \qquad (15)$$

$$Y = N_I(r,s,t)Y_I + \ldots + N_P(r,s,t)Y_P \,, \qquad (16)$$

$$Z = N_I(r,s,t)Z_I + \ldots + N_P(r,s,t)Z_P \,; \qquad (17)$$

where X_I, Y_I, Z_I – global Cartesian coordinates of the corner nodes; $N_I,\ldots,N_P$ – first order scalar nodal shape functions.

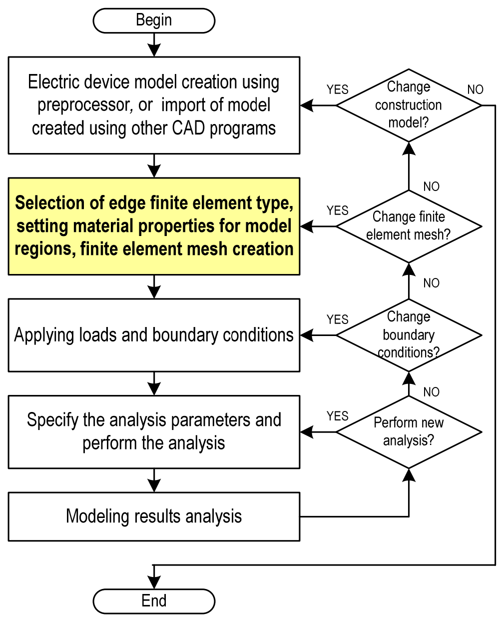

Fig. 5. Electronic device modeling algorithm

The 20-node rectangular brick finite element geometry can be changed to 10-node tetrahedron, 13-node pyramid or 15-node wedge shapes (Fig. 4). It has been determined, that tetrahedral shapes have advantage in air domains with no electric currents, whereas hexahedra shapes are recommended for regions with electric currents [4]. Pyramids are used when it is needed to maintain efficient meshing between hexahedral and tetrahedral regions. Wedges are generally applied for 2-D like geometries, when longitudinal dimensions are longer than transverse sizes. In this case the cross-section can be meshed by area meshing and wedges are generated by extrusion.

Since the process of modeling of electromagnetic fields created by various electronic devices using edge finite elements is analogous to the modeling using node finite elements, algorithms presented in [5-7] can be also used when solving electromagnetic field problems based on edge finite element formulation. The generalized algorithm electromagnetic field modeling is presented in Fig. 5.

Conclusions

When using edge finite element technique, the number of generated unknown values is larger than using nodal finite element technique for the mesh of the same spacing. When using the edge finite element approximation the higher number of unknowns is compensated by lower connectivity between edges or a sparsity of the rigidity matrix. Therefore in result, the edge finite element method requires less amount of memory and faster analysis can be achieved.

Edge finite element method can be successfully used when solving various electromagnetic field simulation problems, where the application of the nodal finite element method is complicated, e.g. when using nodal-based approximation of a vector variables in finite element computations.

There are many possible application areas for edge finite element method, including various magnetostatic, low and high frequency eddy current problems, also problems of modeling of electromagnetic field created by permanent magnets. The span of edge finite element method application areas continuously expands.

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Edge Finite Elements are analyzed, since they are gaining more and more popularity in modeling of electronic devices. Comparison of Edge Finite Elements and Nodal Finite Elements is presented, their advantages and lacks are introduced, also recommendations for their application are given. Form functions used to describe rectangular finite element in 3D space are presented. Possibilities of the various finite element analysis software packages are analyzed. Properties of the edge finite element *SOLID117*, which is implemented in the software package ANSYS/Emag and intended for modeling of low-frequency electromagnetic fields, are discussed here. Generalized electromagnetic field modeling algorithm using edge finite elements was created. Il. 5, bibl. 7 (in English; summaries in English, Russian and Lithuanian).

П. Тарвидас. Конечные *edge*-элементы для моделирования электромагнитного поля в трехмерном пространстве // Электроника и электротехника. – Каунас: Технология, 2007. – № 4(76). – С. 29–32.

Анализируются конечные *edge*-элементы, так как они получают все больше популярности в моделировании электронных устройств. Сравнены конечные *edge*-элементы и узловые конечные элементы, представлены их преимущества и недостатки, также даются рекомендации для их использования. Представлены функции формы, используемые для описания прямоугольного конечного элемента в трехмерном пространстве. Проанализированы возможности различных программ анализа методом конечных элементов. Обсуждены свойства конечного *edge*-элемента *SOLID117*, который осуществлен в пакете программ ANSYS/Emag и предназначен для того, чтобы моделировать электромагнитные поля низкой частоты в пространстве. Создан обобщенный алгоритм моделирования электромагнитных полей используя конечные *edge*-элементы. Ил. 5, библ. 7 (на английском языке; рефераты на английском, русском и литовском яз.).

P. Tarvydas. Briaunų baigtinių elementų taikymas elektromagnetiniams laukams modeliuoti erdvėje // Elektronika ir elektrotechnika – Kaunas: Technologija, 2007. – Nr. 4(76). – P. 29–32.

Nagrinėjami vis plačiau elektroniniams įtaisams modeliuoti naudojami briaunų baigtiniai elementai. Pateikiamas jų palyginimas su mazgų baigtiniais elementais, apibūdinami jų pranašumai ir trūkumai, pateikiamos jų taikymo rekomendacijos. Pateikiamos formos funkcijos, taikomos stačiakampiam baigtiniam elementui aprašyti erdvėje. Analizuojamos įvairių baigtinių elementų analizės programų galimybės. Aptariamos programoje, ANSYS/Emag realizuoto briaunų baigtinio elemento *SOLID117*, skirto žemųjų dažnių elektromagnetiniams laukams modeliuoti, savybės. Sudarytas apibendrintas elektroninių įtaisų modeliavimo, naudojant briaunų baigtinius elementus, algoritmas. Il. 5, bibl. 7 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).