

## Operation of Electric Motor with Elastic Load

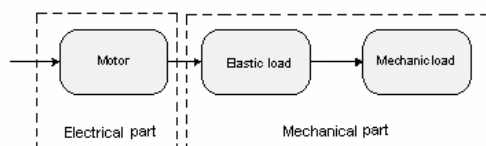
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### Introduction

Electric drive is an electromechanical system, which performs the conversion of electrical energy to mechanical energy or vice versa for running various processes such as: production plants, transportation of people or goods, home appliances, pumps, air compressors, computer disc drives, robots, music or image players, etc. To couple electrical motor with mechanical load, the mechanical drives are used. The basic types of mechanical drives are: geared transmission – provides specific fixed type ratios; belt drives - provide flexibility in the positioning of the motor; chain drives – provide infinitely variable speeds; traction drives – provides adjustable speed with relatively high speed.

Thus, electromechanical system consists of two parts: electrical and mechanical (Fig. 1).



**Fig. 1.** Structure of electromechanical system

Rotating parts of the motor and mechanical load are the main components of the mechanical part of drive. Usually they have different parameters of the movement, therefore intermediate mechanical chains such as shafts, reducers, belt-drives and screw-drives as well as clutches are used. Always there is a proper parameter of interest, for example, inertia, speed of rotation, elasticity of mechanical chains, characterizing the movement of these parts or entire system.

Two-mass or three-mass systems are characterized with oscillations and vibrations of controlled parameters. Advanced control methods are used to reduce that [1–8]. One-mass system at constant control action – torque is considered in [9, 10].

Operation of two-mass system of finite stiffness when torque of a motor is control action and varies by exponential law is explored in [12].

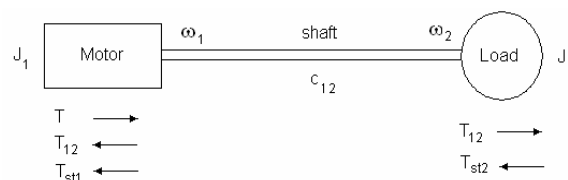
The article discusses the movement of mechanical part, which consists of actuator (motor) and elastic

mechanical load and dependences of controlled parameters upon elasticity and possible clearance in detail, when torque, developed by the motor is not constant but varies in time by exponential law.

### Mechanical Part of Electromechanical System

Mechanical part consists of all system chains linked between them mechanically and revolving with different speeds. To simplify solution of the problem usually the simplest case is considered, when electric motor and the load are connected with absolutely stiff shaft and both parts rotate with the same speed. Such system is called one-mass system. It is characterized by torque, developed by the motor  $T$ , mechanical load torque  $T_{12}$ , total inertia  $J$  and speed of rotation  $\omega$ . This is the simplest well known model of electromechanical system. Two stages of the motor starting are considered: initial, while the deformation of elastic shaft or other chains take part and the other end of shaft does not rotate yet, and work stage – when rotor starts to revolve.

Electromechanical system which is composed of elements with finite stiffness is more complex (Fig. 2). The finite stiffness shaft coupling motor with mechanical load is specific element which appears in the system. In this case instantaneous speeds of the shaft ends are different and the system behavior obtains new features. Electromechanical system, which consists of two masses rotating with different speeds  $\omega_1$  and  $\omega_2$  coupled with the finite stiffness shaft, is called two-mass electromechanical system.



**Fig. 2.** Block diagram of two mass mechanical part – the motor and mechanical load; where  $J$  – inertia;  $\omega_1$  – instantaneous rotation speed of motor shaft;  $\omega_2$  – instantaneous rotation speed of driven machine shaft;  $T$  – electromagnetic torque of a motor;  $T_{st}$  – load torque;  $M_{12}$  – torque of elasticity;  $c_{12}$  – stiffness of elastic mechanical part

Direction of the movement of the drive depends on the acting forces and torques as well as its character depends on properties of electrical circuit and mechanical chains.

Several elements of mechanical parts, for example, shaft can have finite stiffness and the total system can have kinematical clearance. According to this, electromechanics deals with one-mass, two-mass or several-mass mechanical part of a system.

In the same way more complex structure can be elaborated if more nonlinear elements of electromechanical system have to be considered.

### Differential Equations and Structure of Two Mass Mechanical Part

Movement of mechanical part of electromechanical system in general form is described by Lagrange's equation [10]:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i; \quad (1)$$

where  $L = W_k - W_p$  – Lagrange's function;  $Q_i$  – force, depending on elementary works of external forces and their possible displacement  $\partial q_i$ .

In general case, electromechanical systems deal with systems elements of finite stiffness, possible clearance and non-linearity of motor and load characteristics. To obtain analytical solutions of high order nonlinear differential equations with varying coefficients or sets of those, they must be solved, but it is impossible to solve these equations by any known methods. To get the results, methods of mathematical and computer modeling must be used.

In two-mass system the finite stiffness and non linearity of the shaft caused by kinematical clearance is considered. The set of operational equations of this system is written in this way [9,10]:

$$\begin{cases} T - T_{st1} - T_{12} = J_1 s \omega_1; \\ -T_{st2} + T_{12} = J_2 s \omega_2; \\ s T_{12} = c_{12} (\omega_1 - \omega_2). \end{cases} \quad (2)$$

In dynamic system torque of elasticity  $T_{12}$  linearly depends on torsion angle  $\Delta\varphi$  (Fig. 3).

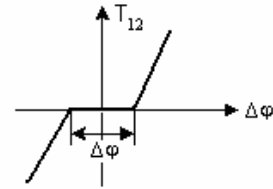


Fig. 3. Dependence of elasticity torque against torsion angle  $\Delta\varphi$

Set of operational equations [10] of that system can be written as:

$$\begin{cases} T - T_{12} - T_{st1} = J_1 s \omega_1; \\ T_{12} - T_{st2} = J_2 s \omega_2; \\ T_{12} = c_{12} \left( \varphi_1 - \varphi_2 - \frac{\Delta\varphi}{2} \right), \quad \text{if } |\varphi_1 - \varphi_2| > \frac{\Delta\varphi}{2}; \\ T_{12} = 0 \quad \text{if } |\varphi_1 - \varphi_2| \leq \frac{\Delta\varphi}{2}. \end{cases} \quad (3)$$

A new element considering kinematical clearance is added to the structure of the system (Fig. 4).

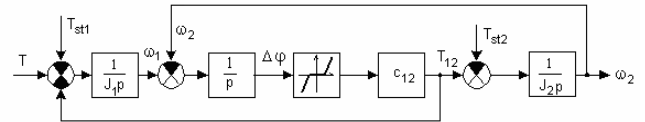


Fig. 4. Block diagram of two-mass system with kinematical clearance

Developed block diagrams give possibility to obtain dependences of responses of two-mass system on parameters of the system elements.

### Simulation of Two-Mass System

It is convenient to investigate dynamic characteristics of two-mass system with MATLAB software and SIMULINK. Application of SIMPLORER to SIMULINK interface can be discussed also, but the SIMULINK software [11] was used to develop a model of this system. It gives possibility to consider nonlinearities of any type. The model developed for two-mass system without the clearance is given in Fig. 5.

All developed model consists of three coupled models: motor, elastic mechanical load and motor load models. It is possible to set up various models of a motor and its load. Below results of simulation are presented at motor torque  $T$  assumed as exponential time function.

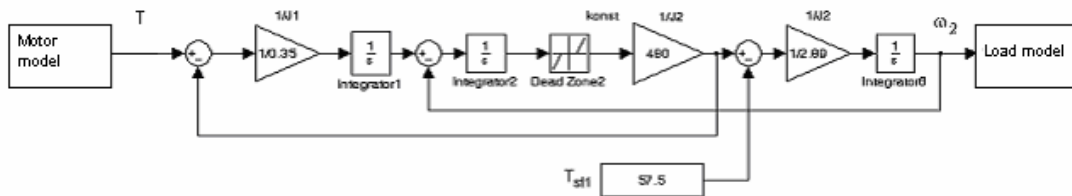


Fig. 5. Model of two-mass system without clearance

The presented system requires to be supplemented with the structures of torque  $T$ , acting at the system input and load. Electromagnetic torque is the input of the system. Its dependence against time can be described in different ways: by exponential, dead beat and oscillating response. Fig. 6 presents one of possible variants, when torque of the motor changes in time by exponential law.

The clearance in this model is expressed by dead zone. Just electromagnetic transients happen in the system, while dead zone is not passed. After passing the dead zone, the second mass starts to influence the dynamics of the system. The response of the system rotation speed and output torque is presented in Fig. 7.

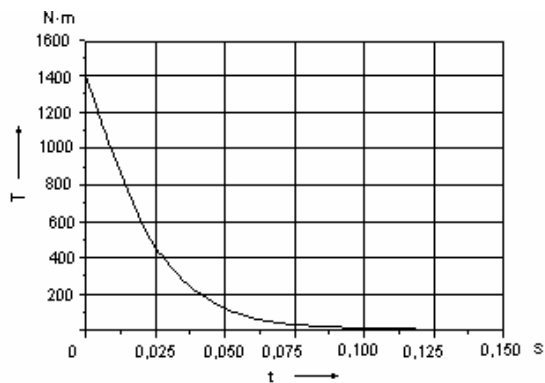


Fig. 6. Dependence of the input torque against time

The character of speed change can be explained by influence of elastic shaft deformation and clearance. At the beginning of the process, speed of the motor increases until developed electromagnetic torque becomes equal to the torque of elasticity  $T_{12}$ . The speed of load at this time interval is equal to zero. If the active load present, this speed can be even negative, i.e. the motor speed changes its direction.

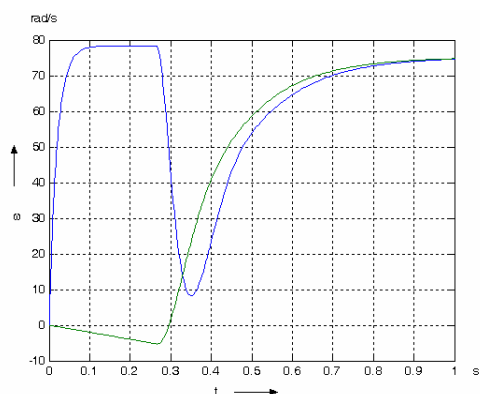


Fig. 7. The response of rotation speed of dynamic system with clearance

When the electromagnetic torque overcomes the torque of elasticity, the motor starts to rotate on no-load, while clearance exists. When clearance is passed, motor starts to operate with load and its speed rapidly decreases. At this time the torque, developed by motor, becomes greater and the speed begins to increase and to turn shaft of the load. After some time it reaches steady-state value.

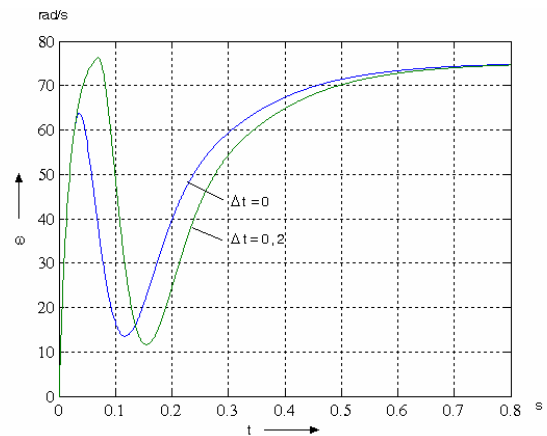


Fig. 8. The response of developed torque of dynamic system with clearance

Figures 7 – 8 show the influence of finite stiffness and clearance of mechanical chains to operation of electromechanical system. Specifically, the greater clearance causes the greater increments of rotation speed, greater accelerations and greater dynamic forces which delay transient process.

## Conclusions

1. Finite stiffness causes oscillations of rotation speed. Amplitude of oscillations is greater if stiffness of elastic mechanical part is smaller. At stiffness coefficient equal to infinity, the ideal transient process, corresponding to the dead beat response is obtained.
2. The change in speed of the system with clearance at the beginning of starting process corresponds to the rule of no-load electric drive rotation speed while under elastic torque the load appears. During this part of transient process the motor rotation speed increases exponentially.
3. Due to clearance the output shaft speed lags the motor shaft speed.
4. Clearance in two-mass electromechanical system causes dynamical torques.

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**A. Smilgevičius, R. Rinkevičienė, Z. Savickienė. Operation of Electric Motor with Elastic Load // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 6(70). – P. 15–18.**

Electromechanical system as object of research comprises two parts: electrical and mechanical. Converter of electric energy and control system compose an electrical part, all other linked between them moving masses form mechanical part. Several elements of mechanical parts, for example, shaft can be of finite stiffness and the total system can have kinematical clearance. According to this, electromechanics deals with one-mass, two-mass or several-mass mechanical part of a system. In the paper two-mass systems with the finite stiffness and non linearity of the shaft caused by kinematical clearance is considered. The structures of electromechanical system with finite stiffness of the shaft and clearance are presented, computer models of those systems are developed and simulation at different controlling inputs and different parameters of the systems is carried out. Two stages of the motor starting are considered: initial, while the deformation of elastic shaft or other chains takes part and the other end of shaft does not rotate yet, and work stage – when rotor starts to revolve. Ill. 8, bibl. 12 (in English; summaries in English, Russian and Lithuanian).

**A. Смильгявичюс, Р. Ринкявичене, З. Савицкене. Работа электрического двигателя с упругой нагрузкой // Электроника и электротехника. – Каунас: Технология, 2006. – №. 6(70). – С. 15–18.**

Электромеханическая система, как объект исследования, состоит из двух частей: электрической и механической. Электрическую часть составляет электромеханический преобразователь энергии и система его управления, а механическую – все связанные между собой движущиеся массы. Некоторые механические части, например, вал, могут быть конечной упругости или система может иметь кинематический зазор. По этим признакам различают одномассовую, двухмассовую и многомассовую систему. Исследуются два этапа пуска двигателя: начальный, когда происходит деформация упругого вала и его другой конец еще не вращается, и рабочий этап, когда ротор начинает вращаться. Начальный этап пуска соответствует работе двигателя с упругой нагрузкой. Представлены структуры электромеханической систем, компьютерные модели и результаты имитации при разных воздействиях управления. Ил. 8, библи. 12 (на английском языке; рефераты на английском, русском и литовском яз.).

**A. Smilgevičius, R. Rinkevičienė, Z. Savickienė. Elektros variklio veika su tampriąja apkrova // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 6(70). – P. 15–18.**

Elektromechaninė sistema, kaip tyrimo objektas, yra sudaryta iš dviejų dalių: elektrinės ir mechaninės. Elektromechaninis energijos keitiklis ir jo valdymo sistema sudaro elektrinę dalį, o visos tarp savęs sujungtos judančios masės – mechaninę dalį. Kai kurios mechaninės sistemos dalys, pavyzdžiui, velenas gali būti baigtinio standumo arba sistemoje gali būti kinematinį laisvumą. Pagal tai elektromechanikoje skiriama vienmasė, dvimāsė ar daugiamāsė sistemos mechaninė dalis. Straipsnyje nagrinėjama dvimāsė sistema su baigtinio standumo vėlenu ir netiesišku, kurį sudaro kinematinis laisvumas. Nagrinėjami du variklio paleidimo etapai: pradinis, kol vyksta tampraus veleno ar kitos dalies deformacija ir kitas veleno galas dar nesisuka, ir darbinis etapas – kai rotorius ima sukstis. Pirmasis etapas atitinka variklio veiką su tampriąja apkrova. Pateiktos tokios elektromechaninės sistemos struktūros, kompiuteriniai modeliai ir imitacijos rezultatai esant skirtingiems valdymo poveikiams. Il. 8, bibl. 12 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).