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Decomposition of Random Sampling Point Processes

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Introduction

Whenever wideband signals have to be digitised at high frequencies, it makes sense to consider the randomised signal sampling option. This kind of sampling provides for suppression of the frequency overlapping or the aliasing effect and, therefore, it could be used for signal sampling at high frequencies exceeding the mean sampling rate. However there are also special requirements to be taken into account. The point is that successful application of this kind of nonuniform sampling requires usage of special algorithms for signal processing that are matched to the specifics of such sampling.

The quality of nonuniformly sampled signal processing typically depends on the degree of sampling-specific error suppression. It means that the impact of the sampling nonuniformities on these errors is of special interest. In general, they depend on the incompletely suppressed aliasing, taking place whenever a signal is sampled in this way. A typical spectrogram, obtained in the case where a single tone signal the frequency f_0 is sampled according to the most often used additive random sampling at the mean sampling rate f_s, is shown in Fig. 1.

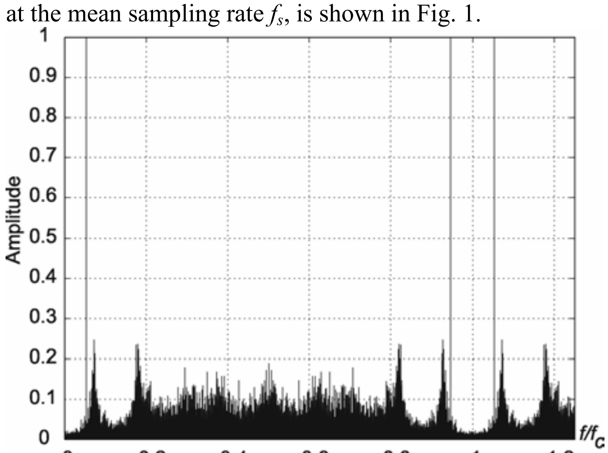

Fig. 1. Typical spectrogram of a pseudorandomly sampled single tone signal according to the additive sampling model

As can be seen, aliasing is suppressed however the peaks at the aliasing frequencies are fuzzy. Increased errors are observed not only at the frequencies f_0, $f_s \pm f_0$; $2f_s \pm f_0$; … at which aliasing normally occurs but also in the immediate vicinity of them. That complicates the task of taking out the aliases. The most popular method for suppressing the errors, due to the insufficiently suppressed aliases, is based on applying the statistical averaging procedure. While it helps, the quality of signal processing achievable in this way is not good enough. That is why more sophisticated algorithms have been developed and are used. Although under certain conditions they provide for high performance signal processing, the problem of effective taking out errors due to aliasing is still of high practical interest.

A new approach to the analysis and synthesis of pseudo-randomised sampling point processes, offering the opportunity of reducing various sampling processes to a common base is suggested. It is based on decomposition of various sampling point processes into a number of periodic sampling point processes with pseudo-random skips of the sampling points. This approach helps to reveal the error dependency on the sampling nonuniformities and, consequently, it helps to find effective means for avoiding these errors.

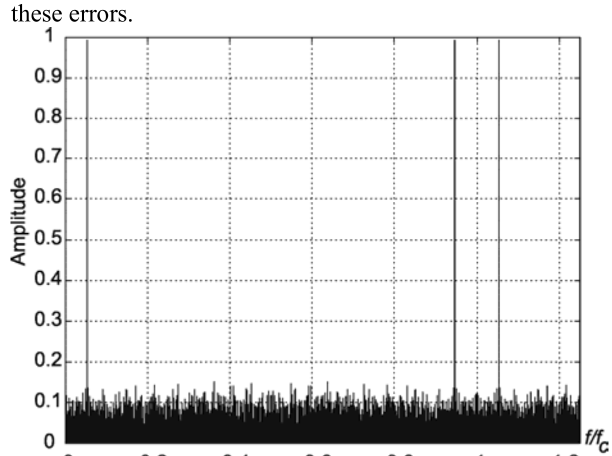

Fig. 2. Spectrogram of the earlier analysed signal obtained in the case where it has been sampled according to a periodic sampling point process with random skipping of sampling points

The spectrogram given in Fig. 2 shows why such decomposing of the sampling point processes into a number of periodic sampling point processes with randomly skipped sampling points is attractive. This spectrogram has been obtained for the same signal for which the spectrogram is given in Fig. 1. However, now the spectrum of the signal looks different. There is full-scale aliasing and it occurs only at well-defined proper aliasing frequencies. Similar spectrograms could be obtained for the cases where this signal is sampled according to all of the periodic components with random skips of the overall sampling process. The aliases in all of these spectrograms would be at the same frequencies but other parameters of them would be different as they depend on the phases of such periodic sampling processes with random skips. Therefore if such spectrograms are obtained for a number of phase-shifted periodic sampling processes with random skips, then the overlapping of the frequencies could be resolved. In other words, the aliases then could be taken out. That is why the possibility of decomposing pseudo-randomised sampling point processes into a number of these-shifted periodic processes with random skips is of high application value. This paper is aimed to drawing the attention to this approach.

Decomposition approach

Essentials of pseudo-randomised signal sampling have been studied with the focus on features of the most often used sampling point processes and the obtained results are reported. Specifically, it is suggested to represent additive sampling and periodic sampling with jitter as composed from periodic processes with randomly skipped points that are variously shifted in phase. It has been found that it is useful to decompose the periodic sampling point process with random skips into a number of component sets at various frequencies. Each of the component sets at a specific frequency in turn contains a number of components shifted in phase for different phase angles.

Once such a decomposition task is approached, it is easy to see that the basic point process with the period δ could be decomposed into two periodic sampling point processes with pseudorandom skips having two times larger period 2δ and the phase shift between them equal to half of this period. Therefore there are at least two equivalent representations of the basic sampling point process. Actually there are more variable parameter decompositions. Their specifics depend on the type of the sampling point process decomposed.

Decomposition of additive sampling point process

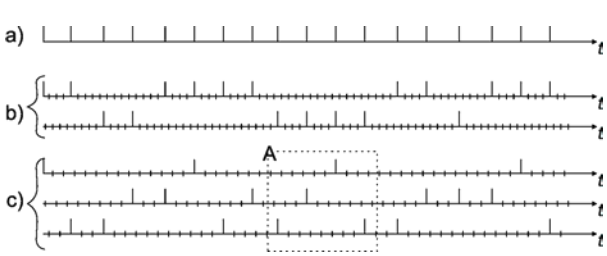

Fig. 3. Decomposition of sampling point process with random skips in the case where the pattern of the skipped points meets the requirements of the additive random sampling

Fig. 3 illustrates this kind of sampling point decompositions in the particular case where the pattern of the skipped points of the initial stream of sampling instants $\{t_k\}$ meets the requirements of the additive random sampling defined as

$$t_k = t_{k-1} + \tau_k, \quad \tau_k \in [0, \quad T_s), \quad k = 0, 1, 2, \ldots, \quad (1)$$

where $\{\tau_k\}$ is a random digital variable with zero mean value and the smallest digit equal to δ.

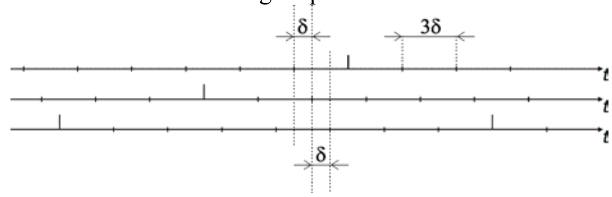

Fig. 3-A. More detailed view of the decomposition 3 (c) showing the actual phase differences

The decomposition of the basic sampling point process with period equal to δ (Fig. 3 (a)) into two phase shifted sampling processes with twice larger period equal to 2δ is shown in Fig. 3 (b). The same approach can be used to decompose the first sampling process into three components at the frequency of basic randomly decimated periodic process equal to $1/3\delta$. This decomposition will have three components shifted in phase for 0, $2\pi/3$ and $4\pi/3$ radians, as shown in Fig. 3 (c). Apparently this decomposition approach makes it possible to decompose the initial basic sampling point process also at frequencies $1/4\delta$, $1/5\delta$, $1/6\delta$, …, $1/n\delta$ each containing 4, 5, 6, …, n phase-shifted components, respectively. Theoretically, only the number N of the taken signal sample values limits the number of possible decompositions. Indeed, at each decomposition, all of the signal sample values are subdivided non-equally between the n components of the particular *n-th* decomposition.

Decomposition of a periodic sampling point process with jitter

Fig. 4 illustrates decomposing of periodic sampling points with jitter defined as

$$t_k = kT + \tau_k, \quad T > 0, \quad k = 0, 1, 2, \ldots, \quad (2)$$

where $\{\tau_k\}$ is a multitude of independent identically distributed random variables with zero mean and T is the mean period of sampling.

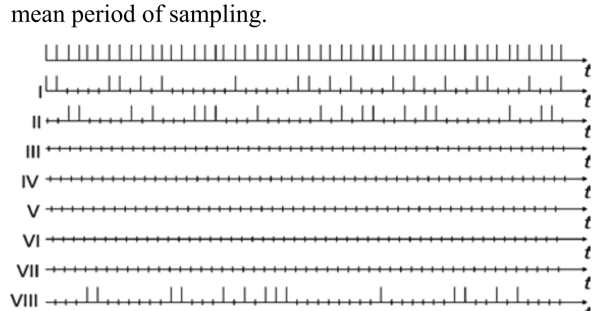

Fig. 4. Decomposition of periodic sampling point process with jitter

Usually random variables $\{\tau_k\}$ are distributed uniformly within a smaller or larger part of the period T. As can be seen, the features of the obtained decomposition in this case differs from the characteristic features of the decompositions obtained when the primary sampling point process belongs to the class of the additive pseudorandom sampling. While the additive sampling points are decomposed approximately equally into all phase-shifted components, the decomposition of the periodic sampling point process with jitter looks quite differently. The sampling points are not distributed between the decomposition components equally in this case. Quite contrary. There are no sampling points belonging to a number of these components at all. In the case illustrated in Fig. 4, only the components with Roman numbers I, II and VIII contain sampling points. The difference in the considered two types of decompositions is clearly displayed in the histograms given in Fig. 5.

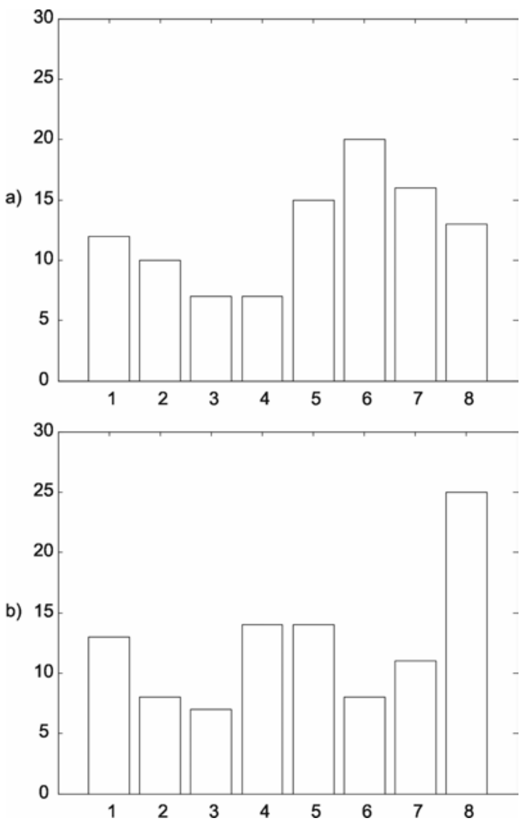

Fig. 5 (a), (b). Histograms showing how the sampling instants are distributed between the components of the decomposition in the cases of additive sampling

They show how the sampling instants of the original sampling point process are distributed between the decomposition components in both considered cases. The point is that the regularity of the sampling instant distribution between the decomposition components is an important characteristic related to suppression of aliasing. The background noise, present in the diagrams shown in Fig. 1 and 2, actually are residues of incompletely compensated aliasing taking place at many frequencies in the case where DFT is performed for nonuniformly sampled signals.

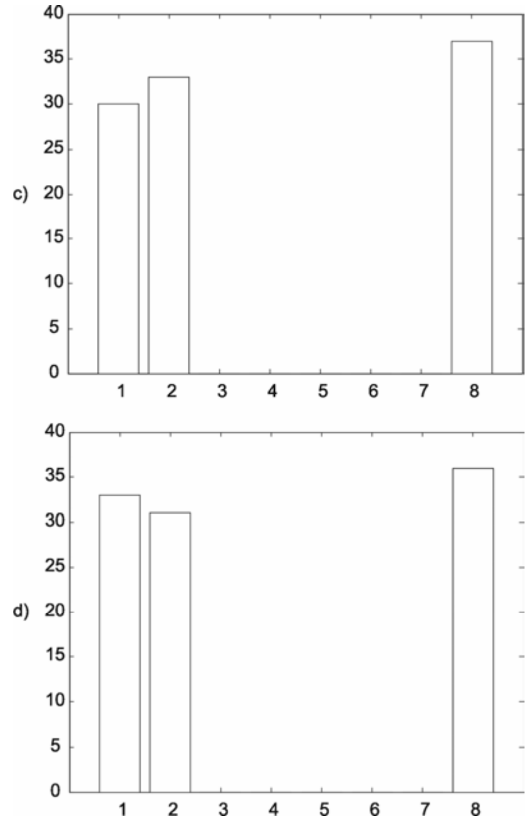

Fig. 5 (c), (d). Histograms showing how the sampling instants are distributed between the components of the decomposition in the cases of periodic sampling with jitter

Compensation of the phase-shifted particular aliases plays a significant role in the whole multi-frequency aliasing mechanism.

The particular errors in the estimates of Fourier coefficients actually are residues of incompletely compensated aliases. And this in turn happens because the components of particular decompositions contain unequal number of signal sample values. Therefore analysis of the decomposition results obtained in a specific case of a particular sampling point process provides information revealing the quality of this sampling process.

The significance of the mentioned essential compensation effect could be demonstrated on the example of processing periodically sampled signals. The point is that a periodic sampling point stream could be decomposed in the explained way as well. However the obtained decompositions then would have different characteristics. For instance, the total number of the signal sample values then would be subdivided between the decomposition components equally. It could be easily checked that the aliases occurring in this case at a particular decomposition frequency would be then fully compensated.

On the other hand, the uniformity of the histograms showing the distribution of the sampling instants between the decomposition components, although crucial, does not guarantee by itself complete suppression of the aliasing effect. The additional essential point to be taken into account is the pattern of the randomly skipped sampling instants characterising each component of the decomposition. While the impact of the random pattern of these missing sample values on the errors corrupting the spectral estimates of signals is out of the scope for this paper, the attention is drawn to the fact that the discussed approach to the decomposition of various pseudorandomised sampling processes proves to be convenient also for revealing this impact of the randomness present in the periodic sampling instant processes.

Comparison of the two given types of decompositions, characterising additive and periodic sampling with jitter, shows that the regularity of the decomposition in the first case is much better than in the second case. And it is a well-known fact that only the first type of the sampling point processes provides for suppression of aliasing.

The spectrogram given in Fig. 6 shows how the spectrum of the same signal, used for calculations of spectrograms in Figure 1 and 2, looks when it is calculated in the case where the signal has been sampled according to the model of periodic sampling with jitter. As can be seen, when decomposition components contain unequal number of signal sample values, aliasing is not suppressed.

Conclusions

It is suggested to represent additive sampling and periodic sampling with jitter as composed from variously shifted in phase periodic processes with randomly skipped sampling points. That is of considerable practical interest as this approach makes it possible to process signals digitally under the conditions of well-defined aliasing, differing for various phase shifts of the sampling process components.

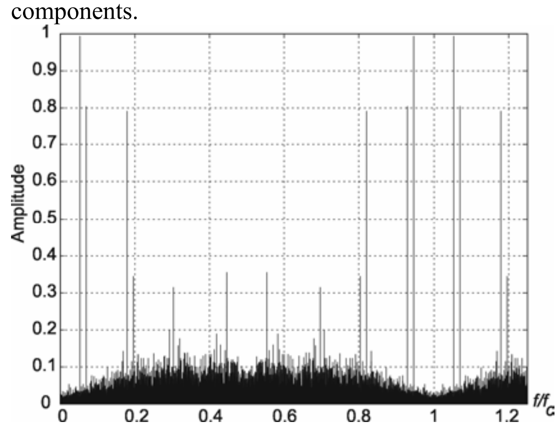

Fig. 6. Spectrogram of a single tone signal obtained in the case periodic sampling with jitter

The described approach could be used for development of methods and techniques for high performance digital alias-free processing of wideband signals at frequencies several times exceeding the sampling rate.

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I. Bilinskis, Z. Ziemelis. Decomposition of Random Sampling Point Processes // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 5(69). – P. 45–48.

An approach to digital processing of pseudo-randomly sampled signals, based on decomposition of the employed sampling point processes, is proposed and discussed. Specifically, it is suggested to represent additive sampling and periodic sampling with jitter as composed from variously shifted in phase periodic processes with randomly skipped sampling points. That makes it possible to estimate signal parameters under the conditions of well-defined aliasing differing for various phase shifts of the sampling process components. The described approach provides the basis for development of various algorithms for alias-free processing of pseudo-randomly sampled wideband signals in a wide frequency range more effectively dealing with the problem of the alias elimination. Ill. 6, (in English; summaries in English, Russian and Lithuanian).

И. Билинскис, З. Зиемелис. Декомпозиция случайных точечных процессов дискретизации // Электроника и электротехника. – Каунас: Технология, 2006. – № 5(69). – С. 45–48.

Рассмотрен подход к цифровой обработке псевдослучайно дискретизованных сигналов, основанный на декомпозиции точечных процессов дискретизации. Предложено представлять аддитивный и периодический процесс выборки с флюктуациями совокупностью сдвинутых по фазе периодических процессов со случайно пропущенными моментами выборки. Это позволяет оценивать параметры сигнала при условиях четко определенного наложения спектров, отличающегося для различных составляющих процесса дискретизации. Описанный подход обеспечивает основу для развития различных алгоритмов обработки псевдослучайно дискретизованных сигналов в широком диапазоне частот, более эффективно решая проблему устранения наложения спектров. Ил.3 (на английском языке; рефераты на английском, русском и литовском яз.).

I. Bilinskis, Z. Ziemelis. Atsitiktinių taškinių diskretizacijos procesų dekompozicija // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 5(69). – P. 45–48.

Analizuojamas pseudoatsitiktiniu principu diskretizuotų signalų skaitmeninis apdorojimas, pagrįstas taškų diskretizacijos procesų dekompozicija. Pasiūlyta adityvųjį ir periodinį diskretizacijos procesus vaizduoti kaip signalą, susidedantį iš paslinktos fazės periodinių procesų su atsitiktinai praleistais išrinkimo momentais. Tai leidžia įvertinti signalo parametrus esant tiksliai apibrėžtai spektrų sanklotai, kuri skiriasi esant skirtingiems diskretizacijos proceso dedamųjų fazės poslinkiams. Aprašytasis metodas įgalina kurti skirtingus pseudoatsitiktinai diskretizuotų signalų algoritmus plačiame dažnių diapazone, efektyviau sprendžiant spektrų sanklotos pašalinimo problemą. Il. 6 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).