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# The Main Inductances of Linear Induction Motor Windings

### E. Matkevicius

Department of Automation, Vilnius Gediminas Technical University, Naugarduko st. 41, LT-03227 Vilnius, Lithuania, phone: +370 2745063, e-mail: edvardas.matkevicius@el.vtu.lt

#### Introduction

Methods of field theory allow to calculate electromagnetic effects of electromagnetic devices with arranged parameters. At present they are considered to be the most accurate methods, allowing to estimate the particular structure of linear induction motor (LIM). In accordance with the initial assumptions considered when formulating the problem it is possible to derive field equations that rate the field variation within one, two or three aspects. That is why there are differentiated one dimensional, two dimensional and three dimensional field problems.

The simplest analytical expressions of various electromagnetic dimensions (B, J, H, E, A) are derived in solving one dimensional electromagnetic field equations. Solutions are nearly always finished forms which allow to explain the physical essence of edges and ends effects, the compensation methods and ways of these effects, allowing to calculate the main working characteristics of LIM, parameters of resultant schemes. One dimensional field solutions depend only on the motion coordinate x and time t. Field dimensions in other directions (y, z coordinates) are considered permanent, i.e. do not depend on them. This one dimensional field method is widely applied in the works of various authors [1-4].

In determining the main inductance of phase winding (first harmonic field inductance) it is necessary to take into consideration the real spatial position of phase windings. If in the air gap of the LIM electromagnetic field the expressions are known and they have been derived when rating the peculiarities of electric machinery with an open magnetic core and when applied the dimensional formulas [5] on field and circuit theory links it is possible to find the expressions of the main inductances [6]. These inductances as the parameters of dynamic equations could be applied for investigations the dynamics of the linear drives [7, 8].

The purpose of the article is to derive analytic expressions of the main inductances of phase windings of flat linear induction motor making use of the solutions of one dimensional electromagnetic field.

### LIM magnetic flux density components

In the air gap of the LIM magnetic flux density towards the direction of longitudinal coordinate (towards the direction of LIM movement) is distributed unevenly. It depends not only on the coordinate but on the velocity of the movement of the secondary element (SE). In the LIM air gap within the boundaries of active zone there are performed three components of the field, each of them is composed of three multiplicand, depending on the SE velocity  $\nu$ , position on the longitudinal axes x and time t:

$$\underline{B}(v,x,t) = \sum_{i=1}^{n,1,2} \underline{B}_{i}(v) \cdot \underline{B}_{i}(x) \cdot \underline{B}_{i}(t) ; \qquad (1)$$

where the indexes n, 1, 2 - accordingly mean normal component, positive and negative components of the travelling field.

Each component is considered to be a complex dimension characterized by the module and phase:

$$\underline{B}_{i}(v) = B_{i}(v)e^{j\varphi_{i}(v)}; \qquad (2)$$

$$\underline{B}_{i}(x) = B_{i}(x)e^{x(a+jb)} = 1e^{x(a+jb)}; \quad (3)$$

$$\underline{B}_{i}(t) = B_{i}(t)e^{j\omega t} = 1e^{j\omega t};$$

$$B_{i}(x) = B_{i}(t) = 1.$$
(4)

Such a division into the components significantly simplifies the derivation of the main inductance expressions. The complex index of the component  $B_i(x)$  is to be the complex roots of the specific equation, which are obtained from the differential equation with partial derivatives:

$$\gamma_{1,2} = \pm (a_{1,2} + jb);$$

where  $\gamma_1$  and  $\gamma_2$  are the complex roots of the specific square equation [1-3].

The specific equation of the analyzed problem is considered to be the same in accordance with the works of various authors [1-4], but the roots of the equation when

solving this equation are derived different, due to the different parameters used and notations. That is why it is reasonable instead of *a* and *b* to insert the dimensions derived in some diverse works.

## Complete magnetic flux

The complete magnetic flux of a coil with any phase winding is obtained when integrating magnetic flux density B according to the coordinate x and when multiplying the result by the number of coil windings and the length of the coil, because transversely the magnetic flux density is considered to be a constant one. From the general expression  $\underline{B}(v,x,t)$  it is seen that on x depends only one component of the sum of each multiplicands. That is why it is necessary to integrate only the component  $B_i(x)$ , but the remaining components stay unchanged. The expression of complete magnetic flux of a any LIM inductor phase coil winding is the following [6]:

$$\underline{\Psi}_{r}(v,x,t) = 2aw_{r}\,\underline{B}_{i}(t)s_{l}\sum_{i}^{n,1,2}\underline{B}_{i}(v)\int_{x}^{x+\beta\tau}\underline{B}_{i}(x)dx; (5)$$

where x is the initial coordinate of the medium line of the coil;  $\tau$  is the length of the pole of inductor winding;  $x + \beta \tau$  is the final coordinate of medium line of the coil;

 $\beta = \frac{y}{\tau}$  is the coefficient of winding step shortening;  $w_r$  is the number of the windings of the coil; 2a is the width of inductor;  $s_l = 1$ , when the winding is one layer and  $s_l = 2$ , when the winding is duplex.

It is possible to move  $B_i(t)$  component behind the symbol of the sum because it is the equal for all the multiplicands.

 $B_i(x)$  general expression of component integral is [6]:

$$\int_{x}^{x+\beta\tau} \underline{B}_{i}(x)dx = \sqrt{\frac{1+e^{-\beta\tau a}(e^{-\beta\tau a} - 2\cos\beta\tau b)}{a^{2} + b^{2}}} \times e^{a(x+\beta\tau)}e^{-j\varphi_{i}(x)};$$
(6)

$$\varphi_i(x) = xb - arctg \frac{b}{a} + arctg \frac{\sin \beta \tau b}{\cos \beta \tau b - e^{-\beta \tau a}}$$
 (7)

When field solutions are derived using vector magnetic potential, then the magnetic field flux density integral is changed by the linear integral with closed circuit. This circuit is comprised by the coil perimeter within the margins of the inductor. So instead of the integral in the formula (5) it is necessary to write such an integral:

$$\oint \underline{A}(x)d\ell = \underline{A}(x+\beta\tau) - \underline{A}(x).$$
(8)

All the values of the magnetic flux density are changed by the values of the vector magnetic potential:

$$\Psi_{r}(v,x,t) = 2aw_{r}\underline{A}_{i}(t)s\sum_{i}^{n,1,2}\underline{A}_{i}(v)\cdot \iint_{I}\underline{A}_{i}(x)d\ell.$$
 (9)

The separate components of vector magnetic potential are the same as the components of the magnetic flux density.

In the expression of any inductor coil of the combined magnetic flux the integral is the following:

$$\bigoplus_{\ell} \underline{A}_{i}(x) dl = e^{(x+\beta\tau)(a+jb)} - e^{x(a+jb)}.$$
(10)

After having made some rearrangements the formula of the integral is the following:

$$\oint_{\ell} \underline{A}_{i}(x) d\ell = e^{a(x+\beta\tau)} \sqrt{1 + e^{-\beta\tau a} \left(e^{-\beta\tau a} - 2\cos\beta\tau b\right)} \times i\alpha_{i}(x)$$

$$\varphi_{i}(x) = xb + arctg \frac{\sin \beta \tau b}{\cos \beta \tau b - e^{-\beta \tau a}}.$$
 (12)

 $\underline{B}_i(v)$ ,  $\underline{A}_i(v)$  are components derived by solving the field equation by means of partial derivatives, after having accepted certain marginal terms. In a general case the solutions may differ due to the diversely selected marginal terms and dissimilar system of approved parameters. It is also necessary in the complex dimensions  $\underline{B}_i(v)$ ,  $\underline{A}_i(v)$  to single out the module and the argument because in the field solutions this dimension is usually submitted partly in the algebraic type or index type.

The integrals of particular components it is possible to calculate when instead of a and b there are inserted their specific dimensions.

For *normal component* of the magnetic flux density:

$$a=0$$
;  $b=-\frac{\pi}{\tau}$ .

After having inserted these dimensions into the general expressions (6), (7) there is received the following expression:

$$\int_{x}^{x+\beta\tau} \underline{B}_{n}(x)dx = \frac{\tau}{\pi}\sqrt{2}\cdot\sqrt{1-\cos\beta\pi} \cdot e^{j\varphi_{n}(x)}; \qquad (13)$$

$$\varphi_n(x) = -\frac{\pi}{\tau}x + \frac{\pi}{2} - arctg \frac{\sin \beta \pi}{\cos \beta \pi - 1}$$
 (14)

It is also possible to find the expression of the integral for *direct moving field* when  $a = -a_1$ ,  $b = -\frac{\pi}{\tau_a}$ :

$$\int_{x}^{x+\beta\tau} \frac{B_{1}(x)dx = e^{-a_{1}(x+\beta\tau)} \times$$

$$\times \sqrt{\frac{1 + e^{a_1\beta\tau} \left(e^{a_1\beta\tau} - 2\cos\beta\tau\frac{\pi}{\tau_e}\right)}{a_1^2 + \left(\frac{\pi}{\tau_e}\right)^2} \cdot e^{j\varphi_1(x)}}; \quad (15)$$

$$\varphi_1(x) = -xN - arctg \frac{\sin\beta\tau N}{\cos\beta\tau N - e^{[M-\eta]\beta\tau}}.$$

$$For reverse vector magnetic potention  $a = M + n \text{ and } b = N$ :$$

$$\varphi_{1}(x) = \frac{\pi}{\tau_{e}} x - arctg \frac{\pi}{a_{1}\tau_{e}} - arctg \frac{\sin \beta \tau \frac{\pi}{\tau_{e}}}{\cos \beta \tau \frac{\pi}{\tau_{e}} - e^{\beta \tau a_{1}}}$$
(16) 
$$\frac{\oint_{\ell} \underline{A_{2}}(x) d\ell = e^{(M+\eta)(x+\beta \tau)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e^{-(M+\eta)\beta \tau} \left(e^{-(M+\eta)\beta \tau} - 2\cos \beta \tau N\right)} \times \sqrt{1 + e$$

and for the reverse field, when  $a = a_2$ ,  $b = \frac{\pi}{a_2}$ :

To the reverse field, when 
$$a = a_2$$
,  $b = \frac{\pi}{\tau_e}$ :
$$\varphi_2(x) = xN + arctg \frac{\sin \beta \tau N}{\cos \beta \tau N - e^{-(M+\eta)\beta \tau}}$$

$$\times \sqrt{\frac{1 + e^{-\beta \tau a_2} \left( e^{-\beta \tau a_2} - 2\cos \beta \tau \frac{\pi}{\tau_e} \right)}{a_2^2 + \left( \frac{\pi}{\tau_e} \right)^2}} \cdot e^{j\varphi_2(x)} ;$$

$$\times \sqrt{\frac{1 + e^{-\beta \tau a_2} \left( e^{-\beta \tau a_2} - 2\cos \beta \tau \frac{\pi}{\tau_e} \right)}{a_2^2 + \left( \frac{\pi}{\tau_e} \right)^2}} \cdot e^{j\varphi_2(x)} ;$$

$$\Rightarrow m_1 = \frac{1}{\sqrt{2}} \sqrt{1 + \sqrt{1 + \varepsilon^2}} ; n = \frac{\varepsilon}{2m} ; \varepsilon = \frac{\mu_0 \gamma_2 \omega s}{\alpha^2} ;$$

$$\Rightarrow m_1 = \frac{1}{\sqrt{2}} \sqrt{1 + \sqrt{1 + \varepsilon^2}} ; n = \frac{\varepsilon}{2m} ; \varepsilon = \frac{\mu_0 \gamma_2 \omega s}{\alpha^2} ;$$

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$$\Rightarrow m_1 = \frac{1}{\sqrt{2}} \sqrt{1 + \sqrt{1 + \varepsilon^2}} ; n = \frac{\varepsilon}{2m} ; \varepsilon = \frac{\mu_0 \gamma_2 \omega s}{\alpha s} ;$$

$$\Rightarrow m_1 = \frac{1}{\sqrt{2}} \sqrt{1 + \sqrt{1$$

According to [3]

$$a_1 = -\frac{1}{\alpha_1}$$
;  $a_2 = \frac{1}{\alpha_2}$ ;

where  $\alpha_1$ ,  $\alpha_2$ ,  $\tau_e$  expressions are the functions of the SE velocity v and characterize the character of the electromagnetic process of LIM.

For normal component of the vector magnetic potential when a = 0,  $b = -\frac{\pi}{2}$ :

$$\oint_{\ell} \underline{A}_{n}(x) d\ell = \sqrt{2} \sqrt{1 - \cos \beta \pi} \cdot e^{j\varphi_{n}(x)};$$
(19)

$$\varphi_n(x) = -x\frac{\pi}{\tau} - arctg \frac{\sin \beta \pi}{\cos \beta \pi - 1}.$$
 (20)

For positive vector magnetic potential  $a = -(M - \eta)$  b = -N

$$\iint_{\ell} \underline{A}_{1}(x) d\ell = e^{-(M-\eta)(x+\beta\tau)} \times \\
\times \sqrt{1 + e^{(M-\eta)\beta\tau} \left( e^{(M-\eta)\beta\tau} - 2\cos\beta\tau N \right)} \times \\
\times e^{j\varphi_{1}(x)}; \tag{21}$$

$$\varphi_{1}(x) = -xN - arctg \frac{\sin \beta \tau N}{\cos \beta \tau N - e^{(M-\eta)\beta \tau}}.$$
 (22)

For reverse vector magnetic potential, when  $a = M + \eta$  and b = N:

$$\oint_{\ell} \underline{A_2}(x) d\ell = e^{(M+\eta)(x+\beta\tau)} \times \\
\times \sqrt{1 + e^{-(M+\eta)\beta\tau} \left( e^{-(M+\eta)\beta\tau} - 2\cos\beta\tau N \right)} \times \\
\times e^{j\varphi_2(x)}; \qquad (23)$$

$$\varphi_2(x) = xN + arctg \frac{\sin\beta\tau N}{\cos\beta\tau N - e^{-(M+\eta)\beta\tau}} \cdot \qquad (24)$$

$$M = m_1\alpha; \quad \alpha = \frac{\pi}{\tau}; \quad \eta = \frac{\mu_0 \gamma_2 \omega v}{2};$$

$$m_1 = \frac{1}{\sqrt{2}} \sqrt{1 + \sqrt{1 + \varepsilon^2}} \quad ; \quad n = \frac{\varepsilon}{2m}; \quad \varepsilon = \frac{\mu_0 \gamma_2 \omega s}{\alpha^2};$$

here  $\gamma_2$  is equivalent SE conductance; s is the slide.

The components of the magnetic flux density which depend on the SE velocity are found in the following way:

normal component:

$$\underline{B}_{n}(v) = \frac{\mu_{0}J_{1}}{kg\sqrt{1+\left\lceil\frac{m(v_{S}-v)}{k}\right\rceil^{2}}} \cdot e^{j\varphi_{n}(v)}; \qquad (25)$$

$$\varphi_n(v) = arctg \frac{k}{m(v_S - v)}, \ m = \frac{\mu_0}{\rho_S g}, \ k = \frac{\pi}{\tau}; \quad (26)$$

where  $\mu_0 = 4\pi 10^{-7} \ H/m$  is magnetic constant;  $J_1$  – inductor linear current density;  $v_s$  – synchronous velocity;  $\rho_{\rm S}$  – SE surface resistance; g – SE width;

Positive traveling edge effect component:

(19) 
$$\frac{B_{1}(v) = J_{1}m\rho_{S} \times}{(20)} \times \sqrt{\frac{1 + \frac{k^{2} + (mv)^{2}}{k^{2} + [m(v_{S} - v)]^{2}} + 2\sqrt{\frac{k^{2} + (mv)^{2}}{k^{2} + [m(v_{S} - v)]^{2}}} \cdot \cos\varphi_{0}}{(mv + a_{1})^{2} + b^{2}}} \times$$
when 
$$\varphi_{1}(v) = \pi + arctg \frac{\sin\varphi_{0}}{\sqrt{\frac{k^{2} + [m(v_{S} - v)]^{2}}{k^{2} + (mv)^{2}}} + \cos\varphi_{0}} - \frac{(27)}{k^{2} + (mv)^{2}} + \cos\varphi_{0}}{\sqrt{\frac{k^{2} + [m(v_{S} - v)]^{2}}{k^{2} + (mv)^{2}}}} + \cos\varphi_{0}}$$

$$\varphi_{0} = \varphi_{n} + arctg \frac{k}{mv}, \quad a_{1} = \frac{1}{\alpha_{1}}, \quad b = \frac{1}{\tau_{e}}; \qquad (29)$$

$$\alpha_{1} = \frac{2\sqrt{2}\sqrt{\sqrt{(mv)^{4} + (4\omega m)^{2} - (mv)^{2}}}}{b - mv\sqrt{2}\sqrt{\sqrt{(mv)^{4} + (4\omega m)^{2} - (mv)^{2}}}}; \quad (30)$$

$$2\pi\sqrt{2}$$

$$\tau_e = \frac{2\pi\sqrt{2}}{\sqrt{\sqrt{(mv)^4 + (4\omega m)^2 - (mv)^2}}};$$
 (31)

where  $\omega$  – angular frequency of power supply source;

negative traveling end effect component:

$$\underline{B}_{2}(v) = \sqrt{\frac{\left(mv + a_{1}\right)^{2} + b^{2}}{\left(mv - a_{1}\right)^{2} + b^{2}}} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2}} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + b^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + a^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + a^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + a^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} + a^{2} \left(1 + e^{-a_{1}L} \left(e^{-a_{1}L} - 2\cos bL\right)\right) \times \left(mv - a_{1}\right)^{2} \times \left$$

Components of vector magnetic potential depend on SE velocity and are derived in the following way:

normal component:

$$\underline{A}_{n}(v) = J_{1m} \frac{\mu_{0}}{\alpha^{2} \sqrt{1+\varepsilon^{2}}} \cdot e^{-j\varphi_{n}(v)}; \qquad (35)$$

$$\varphi_n(v) = arctg\varepsilon$$
; (36)

positive traveling component:

$$\underline{A}_{1}(v) = J_{1m} \frac{\mu_{0}}{2\alpha^{2}} \cdot e^{-2p\tau(M+\eta)} \sqrt{\frac{\left(M-\eta\right)^{2} + \left(N-\alpha\right)^{2}}{\left(M^{2}+N^{2}\right)\left(1+\varepsilon^{2}\right)}} \times \\ \times e^{j\varphi_{1}(v)}; \qquad (37)$$

$$\varphi_{1}(v) = -(\pi + 2p\tau N + arctg\frac{N}{M} + \\ + arctg\varepsilon - arctg\frac{N-\alpha}{M-\eta}); \qquad (38)$$

negative traveling component:

$$\underline{A}_{2}(v) = J_{1m} \frac{\mu_{0}}{2\alpha^{2}} \cdot \sqrt{\frac{\left(M+\eta\right)^{2}+\left(N+\alpha\right)^{2}}{\left(M^{2}+N^{2}\right)\left(1+\varepsilon^{2}\right)}} \cdot e^{j\varphi_{2}(v)}; (39)$$

$$\varphi_{2}(v) = -(\pi + arctg \frac{N}{M} + arctg \varepsilon - arctg \frac{N - \alpha}{M - \eta}). \quad (40)$$

#### The components of the windings main inductances

To derive the analytic expressions of LIM inductor phase windings of the main inductances there are used the solutions of one dimensional electromagnetic field solutions [7], after having rated the end effects.

In such a way are derived all the necessary expressions in order to calculate the complete magnetic flux of any coil, assessing the effect of the ends.

In each expression of a component depending on the linear SE velocity there is longitudinal current density marked by various authors differently but possessing the following equal value:

$$J_1 = J_{1m} = J_m = \frac{3\sqrt{2}w_1k_{ap1}}{p\tau}I_1; \tag{41}$$

where  $w_1$  - the number of turns on the phase inductor windings.

 $J_1$  is possible to move beyond the sum symbol of the expressions (5), (9) and we derive:

$$\Psi_{r}(v,x,t) = 2aw_{r}J_{1}\underline{B}(t) \sum_{i=1}^{n,1,2} \underline{f}_{i}(v) \int_{x}^{x+\beta\tau} \underline{B}_{i}(x)dx; (42)$$

$$\Psi_{r}(v,x,t) = 2aw_{r}J_{1}\underline{A}(t)\sum_{j=1}^{n,1,2}\underline{f}_{j}(v) \oplus \underline{A}_{j}(x)d\ell; \quad (43)$$

The values of the functions  $\underline{f}_{i}(v)$ ,  $\underline{f}_{i}(v)$  are derived after having moved  $J_1$  beyond the sum symbol of the expressions (5), (9).

The main inductances of phase windings of separate coils of LIM is possible to derive using two methods.

According to the first method at first there are determined the main inductances of separate field components for each separate phase coil. After that by summing separate components of inductances for each coil, there is derived general inductance of any phase:

$$L_f = s_l \sum_{l} \sum_{l} L_n + s_l \sum_{l} \sum_{l} L_l + s_l \sum_{l} \sum_{l} L_2 ; \qquad (44)$$

where f = A, B, C –are phases;  $L_n, L_1, L_2$  – are inductances of coils respectively to normal, positive and negative traveling components;  $s_1 = 1$ , when the winding is single layered and  $s_l = 2$ , when the winding is double layered; p, q – the number of pairs of poles and tracks falling to the pole and phase.

The second method is more suitable because it permits to find the complete magnetic flux of the phase first and after that to find separate components of the inductances and general major inductance of the phase. In the equations of dynamics of rectilinear drives with LIM there are used only the main inductances of the phase but not the inductances of separate coils.

The complete magnetic flux of any LIM inductor phase winding is found by adding complete fluxes of separate coils forming general phase winding. After having assessed the fact that the complete flux parts do not depend on the longitudinal coordinate, but which is dependant on the field components namely the normal component and the velocity obtained from the following expression:

$$\Psi_{f} = s_{l} 2aw_{r} J_{1} \underline{B}(t) \cdot [pq \underline{f}_{ni}(v) \int_{x}^{x+\beta \tau} \underline{B}_{n}(x) dx + \sum_{i=1}^{1,2} \underline{f}_{i}(v) \cdot \sum_{i=1}^{p} \sum_{x}^{q} \underline{B}_{i}(x) dx]; \qquad (45)$$

$$\Psi_{f} = s_{l} 2aw_{r} J_{1} \underline{A}(t) \cdot [pq \underline{f}_{nj}(v) \underset{\ell}{\coprod} \underline{A}_{n}(x) d\ell + \\ + \sum_{l} \underline{f}_{j}(v) \cdot \sum_{l} \sum_{\ell} \underset{\ell}{\coprod} \underline{A}_{j}(x) d\ell];$$

$$(46)$$

where pq is the number of one phase coils;  $B(t) = A(t) = e^{j\omega t}$ .

# The main inductance components of the phase winding

After the complete magnetic flux of the phase (42), (43), in the expression of which the current density has been substituted by the current, is divided by the current, in the phase windings:

$$i = \sqrt{2}I_1 e^{j\omega t} \tag{47}$$

and multiplied by  $\beta\tau$ , derived from the general formula used for calculating the main inductance of the phase winding expressed by the magnetic flux density and vector potential:

$$L_{f} = s_{l} \frac{6aw_{l}w_{r}k_{apl}}{p} \beta \cdot [pq\underline{f}_{nf}(v) \int_{x}^{x+\beta\tau} \underline{B}_{n}(x)dx + \sum_{i=1}^{l,2} \underline{f}_{i}(v) \cdot \sum_{i=1}^{p} \sum_{x}^{x+\beta\tau} \underline{B}_{i}(x)dx]; \qquad (48)$$

$$L_{f} = s_{l} \frac{6aw_{l}w_{r}k_{ap1}}{p} \beta \cdot [pq\underline{f}_{nf}(v) \underbrace{\mathbb{1}}_{\ell} \underline{A}_{n}(x) d\ell + \sum_{j=1}^{n} \underline{f}_{j}(v) \cdot \sum_{j=1}^{n} \underbrace{\mathbb{1}}_{\ell} \underline{A}_{j}(x) d\ell]. \tag{49}$$

Separate components of the main inductance of the phase winding are:

$$L_{nf} = s_l \frac{6aw_1^2 k_{ap1}\beta}{p} \underline{f}_n(v) \int_{x}^{x+\beta\tau} \underline{B}_n(x) dx; \quad (50)$$

$$L_{nf} = s_l \frac{6aw_1^2 k_{ap1}\beta}{p} \underline{f}_n(v) \underbrace{\sharp}_{\ell} \underline{A}_n(x) d\ell; \quad (51)$$

$$L_{1f} = s_l \frac{6aw_1^2 k_{ap1}\beta}{p} \underline{f}_1(v) \sum_{i=1}^{p} \sum_{x}^{q} \underline{B}_1(x) dx; \qquad (52)$$

$$L_{1f} = s_{l} \frac{6aw_{1}^{2}k_{ap1}\beta}{p} \underline{f}_{1}(v) \sum_{k=1}^{p} \sum_{\ell=1}^{q} \underline{A}_{1}(x) d\ell; \qquad (53)$$

$$L_{2f} = s_l \frac{6aw_1^2 k_{ap1}\beta}{p} \underline{f}_2(v) \sum_{i=1}^{p} \sum_{x}^{q} \underline{B}_2(x) dx; \qquad (54)$$

$$L_{2f} = s_l \frac{6aw_1^2 k_{ap1}\beta}{p} \underline{f}_2(v) \sum_{\ell} \sum_{\ell} \underbrace{1}_{\ell} \underline{A}_2(x) d\ell; \quad (55)$$

where  $pqw_r = w_1$  – is the number of one phase turns.

After entering the particular values of the symbols of the formulas (50-55)  $L_n$ ,  $L_1$ ,  $L_2$  and evaluating only the modules of the complex dimensions it is possible to get the component expressions of the main inductances.

#### **Conclusions**

After having applied the solutions of the one dimensional electromagnetic field of flat rectilinear induction electro motor:

- 1. There has been composed the methodology according to which by applying one dimensional electromagnetic field solutions it is possible to derive analytic expressions of the main inductance of the phase windings of the LIM.
- 2. There have been derived the analytic, normal, positive, negative traveling field magnetic flux density expressions of the components and vector potential, necessary for calculating complete magnetic flux and inductance.
- 3. There have been derived the specific expressions of component integrals of magnetic flux density and vector potential, depending on the position x and magnetic flux density as well as vector potential components, depending on the expression of velocity v.
- 4. There have been derived general formulas for calculating the main inductance of any phase windings and normal, positive, negative formulas for calculating the separate inductance components.
- 5. The derived analytic inductance expressions may be used as the parameters of LIM mathematical model, in modeling automotive linear electric drives.

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# E. Matkevicius. The Main Inductances of Linear Induction Motor Windings // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 4(68). – P. 63–68.

The article deals with the methodology according to which when using one dimensional electromagnetic field solutions it is possible to derive analytic expressions of the main inductance of LIM inductor phase windings. The derived analytic normal, positive and negative traveling field magnetic flux density and vector potential are required in calculating complete magnetic flux and inductances. There have been presented analytic positive and negative component expressions depending on the position on the longitudinal coordinate x and secondary element velocity v, used in determining the main inductances. The derived analytic expressions of any complete magnetic flux of LIM inductor phase coil and windings are expressed via the magnetic flux density and vector potential. There have been derived a general formula for calculating the main inductance of any phase windings and inductance of separate normal, positive and negative components. These formulas may be applied in calculating the specific linear induction motor parameters in compiling a dynamic drive model and in modeling automatic linear electric drive systems. Bibl. 8 (in English; summaries in English, Russian and Lithuanian).

# Э. Маткявичюс. Основные индуктивности обмоток линейного асинхронного двигателя // Электроника и электротехника. – Каунас: Технология, 2006. – № 4(68). – С. 63–68.

Представлена методика, по которой, используя решения одномерного элекромагнитного поля, можно получить аналитические выражения основных индуктивностей фазных обмоток индуктора линейного асинхронного двигателя (ЛАД). Получены аналитические выражения нормальной, прямобегущей и обратнобегущей составляющих магнитного поля магнитной индукции и векторного потенциала, необходимые для расчета полного магнитного потока и индуктивностей. Также представлены аналитические выражения нормальной, прямобегущей и обратнобегущей составляющих одномерного бегущего элекромагнитного поля, зависящие от положения по продольной координате x и скорости v вторичного элемента, используемые для определения основных индуктивностей. Получены аналитические выражения полного магнитного потока любой катушки фазы и обмотки индуктора ЛАД, выраженные через магнитную индукцию и векторный потенциал. Получены общие формулы расчета основной индуктивности любой фазовой обмотки и отдельных нормальной, прямобегущей и обратнобегущей ее составляющих. Эти формулы могут быть использованы для расчета конкретных параметров линейного асинхронного двигателя, для составления динамической модели привода и моделирования систем автоматизированных линейных электроприводов. Библ. 8 (на английском языке; рефераты на английском, русском и литовском яз.).

# E. Matkevičius. Tiesiaeigio asinchroninio variklio apvijų pagrindiniai induktyvumai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 4(68). – P. 63–68.

Pateikta metodika pagal kurią, panaudojant vienmačio elektromagnetinio lauko sprendinius, galima apskaičiuoti TAV induktoriaus fazinių apvijų pagrindinių induktyvumų analizines išraiškas. Gautos analizinės normalinės, tiesioginės ir atvirkštinės bėgamojo lauko magnetinio srauto tankio ir vektorinio potencialo dedamamųjų išraiškos, reikalingos pilnutiniam magnetiniam srautui ir induktyvumams apskaičiuoti. Pateiktos analizinės vienmačio bėgamojo elektromagnetinio lauko normalinės, tiesioginės ir atvirkštinės dedamųjų priklausančių nuo padėties išilgine koordinate x ir antrinio elemento greičio v išraiškos, naudojamos pagrindiniams induktyvumams nustatyti. Gautos analizinės bet kurios TAV induktoriaus fazės ritės ir apvijos pilnutinio magnetinio srauto išraiškos išreikštos per magnetinio srauto tankį ir vektorinį potencialą. Gautos bendrosios bet kurios fazės apvijos pagrindinio induktyvumo ir atskirų induktyvumo normalinės, tiesioginės ir atvirkštinės dedamųjų skaičiavimo formulės. Šios formulės gali būti panaudotos skaičiuojant konkrečius tiesiaeigio asinchroninio variklio parametrus, sudarant dinaminį pavaros modelį ir modeliuojant automatizuotų tiesiaeigių elektros pavarų sistemas. Bibl. 8 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).