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Iterative Spectrum Analysis of Nonuniformly Undersampled Wideband Signals

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Introduction

To widen the frequency range for fully digital processing of signals, the original analog signals should be digitised in a way ensuring elimination or at least suppression of aliasing. The sampling procedure then, inevitably, has to be nonuniform and the obtained digital signals in this case have to be processed digitally in an appropriate way. It is not so easy to succeed at that. However, as digital processing of Radio Frequency signals is of high practical interest, the problems of this kind have drawn a lot of attention and, in result, much has been already accomplished in this area. The developed Digital Alias-free Signal Processing technology [1] provides hardware and software tools that could be effectively used in a wide application range. Of course, there are still problems and one of them is related to a particular disadvantage of nonuniform sampling. Specifically, the existing fast algorithms, developed for periodically sampled signals, could not be directly used for processing nonuniformly sampled signals. The attempts to use FFT algorithms for estimation of nonuniformly sampled signal parameters in the frequency domain so far have not been really successful. The problem of getting better results in this area is considered and an iterative approach to its solution is suggested. The achievable in the suggested way results are illustrated by an example. A pulse train characterised by a wideband spectrum is digitised, analysed and the obtained results are displayed. They confirm the applicability of the proposed method for DFT calculations implemented as an iterative FFT procedure based on the sampled signal regularization.

Essence of the suggested method

To process nonuniformly sampled signals by using algorithms developed for processing of periodically sampled signals, so-called zero padding method is typically used. According to it, zeroes are inserted into the nonuniform signal sample value sequence at the time instants where there are no signal sample values so that the sampling process is transformed into a periodic one. However zero padding is a crude method and its application leads to introduction of substantial errors. In general, the suggested approach to spectrum analysis and waveform reconstruction is based on the idea that a priori information has to be used as fully as possible. To realise this, the zeroes should be replaced by approximate estimates of the signal sample values. That leads to dramatic reduction of the mentioned errors. Even better results could be obtained if an iterative procedure of spectral analysis and waveform reconstruction is used. It is based on the substitution of the missing uniform sample values at first by zeroes, then the estimated signal values are inserted in those places and after that even more accurately estimated signal sample values are used. Significant improvement of accuracy is obtained in result. Let us consider this method for signal spectral analysis and waveform reconstruction in some details.

Variable threshold DFT iterations

It is essential how the signal value estimation is organized. A step-by-step approach to that task is suggested. The more powerful signal components are estimated first, then the components that are less powerful are estimated and so on. Such an approach makes sense as the relative errors are smaller for the more powerful components. To realize it, threshold levels are introduced. The signal components above the threshold are estimated first. Then the inverse DFT is carried out and the missing sample values are substituted by the corresponding instantaneous values of the roughly reconstructed waveform. Then the obtained signal sample value sequence is used for the repeated DFT. Now it provides spectral estimates significantly more accurate. At the next step, the threshold level is lowered and the components above it are estimated again. The process is continued in this way for a given number of cycles.

At first, the spectral estimates are calculated on the basis of DFT as follows:

$$a_{(0)}(f_k) = \frac{2}{N}\sum_{i=0}^{N-1} x(t_i)\cos(2\pi f_k t_i), \quad (1)$$

$$b_{(0)}(f_k) = \frac{2}{N}\sum_{i=0}^{N-1} x(t_i)\sin(2\pi f_k t_i), \quad (2)$$

$$A_{(0)}(f_k) = \sqrt{[a_{(0)}(f_k)]^2 + [b_{(0)}(f_k)]^2} \ . \quad (3)$$

The index in the round brackets (in this case (0)) shows the number of iteration. The estimate (3) is used to define some initial threshold for further iterative operations.

The first threshold is set up at the level

$$U_0 = \mu_{(0)} \max_k(A_{(0)}(f_k)), \quad 0 < \mu_{(0)} < 1, \quad (4)$$

where the initial relative threshold often is chosen at the level $\mu_{(0)}$ = (0.7…0.9). The signal components exceeding the given threshold $A_{(0)}(f_k) > U_0$ are estimated at the corresponding frequencies.

All estimated signal components with power below the first threshold level are excluded from the spectrogram in the following way:

$$\hat{a}_{(0)}(f_k) = \begin{cases} a_{(0)}(f_k), & if\ A_{(0)}(f_k) > U_0, \\ 0 \quad , & if\ A_{(0)}(f_k) \le U_0, \end{cases} \quad (5)$$

$$\hat{b}_{(0)}(f_k) = \begin{cases} b_{(0)}(f_k), & if\ A_{(0)}(f_k) > U_0, \\ 0 \quad , & if\ A_{(0)}(f_k) \le U_0. \end{cases} \quad (6)$$

Corrections in spectral peak positions

At first DFT is performed for frequencies located on the frequency grid with the interval between the frequencies determined by the signal observation time as usual. If all signal components are at frequencies located exactly on this grid, DFT provides spectral estimates that are accurate enough. However the frequencies of real signal components often are shifted in regard to this grid. Then the positions of these components on the frequency axis have to be estimated more precisely. Otherwise the inverse DFT will result in unacceptable waveform reconstruction errors.

At first a crude estimator is determined for the location of the maximum of the absolute of the discrete Fourier transform $A[k]$ for a number of the frequency components:

$$K_m = \arg \max_k A[k] \ , \quad (7)$$

where $A[k] = \left| \frac{2}{N}\sum_{i=0}^{N-1} x_i e^{-j2\pi(f_a + k\delta f)t_i} \right|$, f_a is the lower boundary of the frequency range where the search for the maximum of the peak is to be performed. δf is the step of the search on the frequency axis. Then the methods for numerical differentiation are used for precise estimation of the frequency at which there is the extreme value of the considered discrete function. The precise estimate of the maximum frequency is

$$f_m = f_a + (K_m - 1 + \xi)\delta f \ , \quad (8)$$

where the correction component ξ is calculated on the basis of the following formula:

$$\xi = \frac{1}{2} \ \frac{A[K_m - 1] - A[K_m + 1]}{A[K_m - 1] - 2A[K_m] + A[K_m + 1]} . \quad (9)$$

Fourier coefficients are estimated for all of the detected frequencies $\{f_m\}$, $m = \overline{1, L}$:

$$\hat{a}_{(0)}(f_m) = \frac{2}{N}\sum_{i=0}^{N-1} x(t_i)\cos(2\pi f_m t_i), \quad (10)$$

$$\hat{b}_{(0)}(f_m) = \frac{2}{N}\sum_{i=0}^{N-1} x(t_i)\sin(2\pi f_m t_i). \quad (11)$$

Waveform reconstruction on the basis of the iterative DFT - inverse DFT cycles

Inverse DFT is performed for all frequencies exceeding the mentioned thresholds.

$$y_{(0)}(i\Delta T) = \sum_{m=0}^{L-1} \hat{a}_{(0)}(f_m)\cos(2\pi f_m i\Delta T) + \sum_{m=0}^{L-1} \hat{b}_{(0)}(f_m)\sin(2\pi f_m i\Delta T), \ i = \overline{0, M-1}, \quad (12)$$

where L – is the number of frequencies with non-zero spectral amplitudes.

The reconstructed vector of waveform samples is given as

$$\vec{y}_{(0)} = [y_{(0)}[0], ..., y_{(0)}[i\Delta T], ..., y_{(0)}[(M-1)\Delta T]]^T . \quad (13)$$

The actually taken signal sample values are inserted in the reconstructed vector of signal waveform sample values for sampling instants on the time grid with discrete step ΔT:

$$\vec{q} = [q_0, q_1, ..., q_i, ..., q_{N-1}]^T , \quad (14)$$

where $q_i = round\left[\frac{t_i - t_0}{\Delta T}\right]$, $i = \overline{0, N-1}$.

The vector $\vec{q}$ shows where $\vec{y}$ is built inside of the initial sample vector $\vec{x}$. The dimension of the vector of the uniform time grid is given as

$$M = 2^p \ge q_{N-1} . \quad (15)$$

The really taken samples are inserted in the reconstructed vector $\vec{y}_{(0)}(\vec{q}) = \vec{x}$ of the waveform.

At the beginning of a current iteration cycle, the indexes in the round brackets (n) and (n+1) show the number of iteration. FFT is performed over the vector

$$\vec{y}_{(n)} = [y_{(n)}[0], ..., y_{(n)}[i\Delta T], ..., y_{(n)}[(M-1)\Delta T]]^T \quad (16)$$

with the really taken samples again inserted in the reconstructed vector $\vec{y}_{(n)}(\vec{q}) = \vec{x}$:

$$Y_{(n)}[k] = \sum_{i=0}^{M-1} y_{(n)}[i] e^{-j\frac{2\pi ik}{M}}, \quad j = \sqrt{-1}, k = \overline{0, M-1} \cdot \quad (17)$$

Then Fourier coefficients:

$$a_{(n)}[k] = \frac{2}{M} real(Y_{(n)}[k]), \quad (18)$$

$$b_{(n)}[k] = -\frac{2}{M} imag(Y_{(n)}[k]) \quad (19)$$

are calculated.

After that the threshold is set up at the next level

$$U_n = \mu_n \max_k(|Y_{(n)}[k]|), \quad 0 < \mu_n << 1, \quad (20)$$

where it is inversely proportional to the iteration number $\mu_{(n)} = \mu_{(n-1)} / n$ or exponential dependent on the iteration number $\mu_{(n)} = \mu_{(n-1)} / 2^n$, where n is the number of iteration.

At each iteration, the frequencies with amplitudes exceeding the corresponding threshold $|Y_{(n)}[k]| > U_n$ are identified

$$Y_{(n+1)}[k] = \begin{cases} Y_{(n)}[k], & if\ |Y_{(n)}[k]| > U_n, \\ 0, & if\ |Y_{(n)}[k]| \le U_n \end{cases} \quad (21)$$

and they are processed in the same way in the next iteration cycle. The waveform reconstruction is performed as the inverse FFT

$$y_{(n+1)}[i] = \frac{1}{M} \sum_{k=0}^{M-1} Y_{(n+1)}[k] e^{j\frac{2\pi ik}{M}}, \quad i = \overline{0, M-1} \quad (22)$$

while the initial samples are inserted in the reconstructed vector of the waveform $\vec{y}_{(n+1)}(\vec{q}) = \vec{x}$.

Some experimental results

The described iterative algorithm was applied for spectrum analysis and waveform reconstruction of wideband signals nonuniformly sampled with use of the latest DASP digitiser [2]. Figure 1 illustrates the case of analysing a pulse sequence.

Fig. 1. 100 MHz pulse sequence analyzing

The displayed pulse process is the signal analysed and reconstructed in this case. Note how wide is the signal spectrum. Nevertheless it was processed in an alias-free way under the conditions where the mean signal sampling rate is equal to 53.469 MHz. The signal analysis bandwidth is 0.05-669.3 MHz, that corresponds to equivalent periodic sampling rate 1338.6532 MHz. The spectrum clearly shows all expected harmonic components of the signal, including two aliasings from 700 and 800 MHz harmonic components that exceed 669.3 MHz Nyquist's limit; spurious components in fact are not detected. As for the reconstructed waveform, it is fully defined by the spectrum.

Figures 2-3 show the analysis results for two-tone test signals. Spectrums of these signals are similar to the previous one in sense of spurious component absence. Figure 3 directly shows that even signal component so small as –54 dB can be clearly detected on the background of noises.

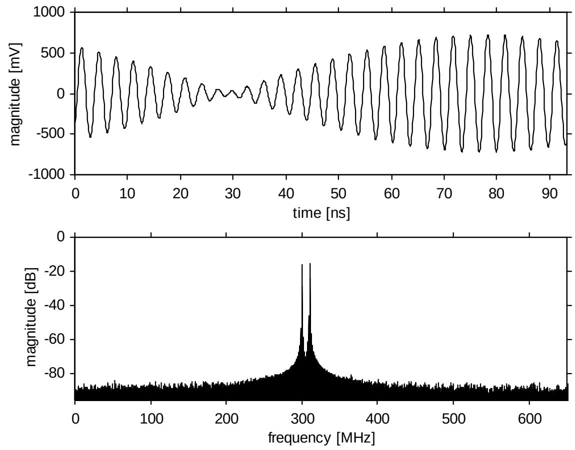

Fig. 2. Two-tone signal analysis when the tones are similar in amplitude

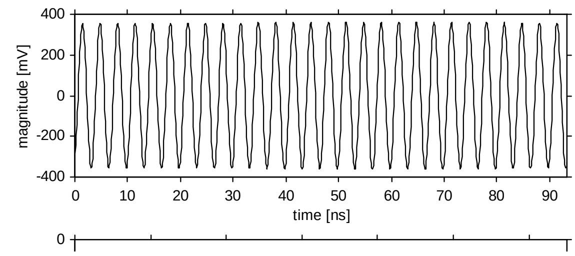

Fig. 3. Two-tone signal analysis when the tones differ in amplitude considerably

Conclusion

1. As can be see from the given experimental results, the suggested iterative direct and inverse DFT calculations leads to quite good reconstruction of signal waveforms. It

has to be emphasized that waveform reconstruction is essential as they could be resampled periodically at much higher frequencies than used for original data collection. After that there are no problems of processing them in a classical way by the well-developed regular DSP algorithms.

2. The information carried by the nonuniform signal sample value sequences might be sufficient not only for providing elimination of aliases but it could be also successfully used, under certain conditions, for obtaining accurate spectral estimates and recovered waveforms. However it should be emphasized that not only the iterative approach is responsible for that. The most important part of the whole process is sampling.

3. However, it should be emphasized, that this high performance has been achieved in alias-free way in the whole frequency range up to 669 MHz.

References

1. **Bilinskis I., Mednieks I.** Introduction to Digital Alias-free Signal Processing / Institute of Electronics and Computer Science. – Riga: “Dasp Lab Ltd.”; London, 2001.
2. **Artyukh Yu., Bilinskis I., Boole E., Rybakov A., Vedin V.** Wideband RF signal digititising for high purity spectral analysis. // Proceedings of the 2005 International Workshop on Spectral Methods and Multirate Signal Processing (SMMSP 2005). – Riga, Latvia, 2005. – P.123–128.

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I. Bilinskis, A. Rybakov. Iterative Spectrum Analysis of Nonuniformly Undersampled Wideband Signals // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 4(68). P. 5–8.

The problem of how to use already existing fast algorithms, first of all FFT, for alias-free estimation of nonuniformly sampled wideband signal parameters in the frequency domain is considered and an iterative approach to its solution is suggested. It is applicable for signal analysis in a wide frequency range with the highest frequency considerably exceeding the mean sampling rate. The essentials of signal processing according to the developed method are discussed. The results achievable in the suggested way are illustrated by an example. Ill.3, bibl.2 (in English; summaries in English, Russian and Lithuanian).

И. Билинскис, А. Рыбаков. Итерационный спектральный анализ нерегулярно дискретизованных широкополосных сигналов при низкой средней частоте выборки // Электроника и электротехника. – Каунас: Технология, 2006.– № 4(68).– С. 5–8.

Рассматривается проблема использования существующих быстрых алгоритмов обработки сигналов, в первую очередь БПФ, для оценки параметров нерегулярно дискретизованного широкополосного сигнала. Предложен свободный от спектральных наложений итерационный метод спектрального анализа. Метод применим для спектрального анализа сигнала и реконструкции его формы в широком частотном диапазоне, максимальная граница которого в несколько раз превышает среднюю частоту дискретизации. Возможности предложенного метода иллюстрируются примерами. Ил.3, библ. 2 (на английском языке; рефераты на английском, русском и литовском яз.).

I. Bilinskis, A. Rybakov. Iteracinė netolygiai diskretizuotų ir mažo vidutinio atskaitų skaičiaus plačiajuosčių signalų spektrinė analizė // Elektronika ir elektrotechnika.– Kaunas: Technologija, 2006. – Nr. 4(68). – P. 5–8.

Tiriama, kaip panaudoti esamus greitus signalų apdorojimo algoritmus, pirmiausia GFT, netolygiai diskretizuoto plačiajuosčio signalo parametrams įvertinti. Pasiūlytas iteracinis spektrinės analizės metodas, kuris nepasižymi spektro sanklota. Metodas pritaikomas signalo spektrinei analizei atlikti ir jo formai rekonstruoti plačiame dažnių diapazone, kurio maksimali riba keletą kartų viršija vidutinį diskretizacijos dažnį. Pasiūlytojo metodo galimybės iliustruojamos pavyzdžiais. Il. 3, bibl. 2 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).