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Electrical Artery Model for the Evaluation of Non-Linear Compliance

I. Kupciunas, A. Kopustinskas
Department of Electrical and Control Equipment, Kaunas University of Technology,
Studentų str. 50-303, LT-51368 Kaunas, Lithuania, phone +370 37351409, e-mail: irmantas_k@yahoo.com

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Introduction

Electrical analogy, i.e. when blood flow is considered to be analogous to the flow of electric current and voltage-current relation is used to describe pressure-flow relation in human arteries, is one of the principal methods employed in many human circulation models. It is widely used for both – lumped [1] and distributed [2] representation.

In our recent study [3] we've already showed that arterial compliance can be estimated using electrical two element model along with non-invasive ultrasound measurement technique. Arterial compliance is one of the major indices characterizing arterial stiffness. Changes of arterial compliance indicate degradation of arterial function and associate primarily with increased pulse pressure and heart deficiency. Direct real time ultrasound measurements of local arterial radius, intima-media (I-M) thickness and blood velocity allowed us to increase the plausibility of arterial compliance investigations [4].

Blood circulation in human arteries is a rather complex process, which incorporates many important and often non-linear effects. Thus, classical two element representation (Fig. 1. A) can be insufficient to reflect the real process. Drawback of the earlier used artery model [3] is underestimating inertance or inertness of blood flow. It is believed, that blood inertance is responsible for many important blood flow effects, i.e. resonance, reverse flow and etc. Supposedly, inertance together with non-linear compliance enables more accurate representation and provides quantitatively and qualitatively improved diagnostic information.

The aim of this study is to improve the reliability of evaluation of the non-linear compliance incorporating inductance element into the electrical analogy artery model, applying modern non-invasive measurements.

Method of the investigation

Newly improved model has four elements, which represent substantial features of blood-flow and artery behaviour (Fig. 1. B). Artery's ability to accumulate blood-flow and to distend during systole and conversely – recoil during diastole is represented by a pressure-dependent compliance (C_P). The non-linear relationship between pressure-dependent compliance and blood pressure P is given by

$$C_P(P(t)) = a \cdot e^{-b \cdot P(t)}, \quad (1)$$

where a and b are two parameters that determines the nature of non-linearity.

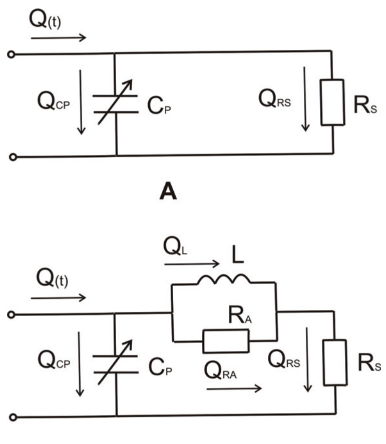

Fig. 1. Hemodynamical representation of two (A) and four (B) element artery models' in systole

Respectively, peripheral resistance (R_S) and artery (characteristic) impedance (R_A) are introduced to reflect resistive properties of peripheral arteries and artery itself. Peripheral resistance is defined as a ratio of mean blood pressure and mean blood flow rate

$$R_S = \frac{\overline{P}}{\overline{Q}}. \quad (2)$$

Characteristic impedance of the artery is determined by taking the slopes of the pressure and flow waves during the early part of the ejection period, P and Q, and calculating their ratio

$$R_A = \frac{\Delta P / \Delta t}{\Delta Q / \Delta t} = \frac{\Delta P}{\Delta Q}. \quad (3)$$

Finally, blood mass accelaration and decelaration effect is introduced with inertance (L) element, which relates the pressure drop (ΔP) with the rate of change of flow (dQ/dt)

$$L = \Delta P \cdot \left(\frac{dQ}{dt}\right)^{-1}. \quad (4)$$

Blood flow rate is calculated using blood velocity (V) and artery internal radius (r) [3]

$$\overline{Q} = \overline{V} \cdot \overline{S} = \overline{V} \cdot \pi \cdot \overline{r}^2. \quad (5)$$

During diastole there is none blood-flow from the heart ($Q(t)=0$) and the model can be electrically represented as a discharge of a capacitor (Fig. 2).

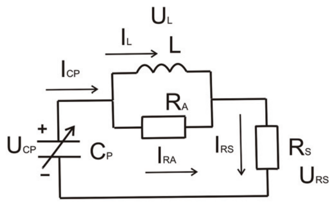

Fig. 2. Electrical analogy of artery model in diastole

Considering the Ohm's law the following can be written

$$I_{CP} = I_L + I_{RA} = I_{RS}. \quad (6)$$

Currents and voltages for the different elements can be defined:

$$-I_{CP} = C\frac{dU_{CP}}{dt}, \quad (7)$$

$$I_L = \frac{1}{L}\int U_L dt + A, \quad (8)$$

$$I_{RA} = \frac{U_{CP} - U_{RS}}{R_A}, \quad (9)$$

$$U_{RS} = I_{CP} \cdot R_S = R_S C_P \frac{dU_{CP}}{dt}, \quad (10)$$

$$U_L = U_{CP} - I_{CP} R_S. \quad (11)$$

Substituting (10) into (9) and then (7), (8) and (9) into (6) the following can be written

$$-C\frac{dU_{CP}}{dt} = \frac{U_{CP}}{R_A} - \frac{R_S C_P}{R_A}\frac{dU_{CP}}{dt} + \frac{1}{L}\int U_L dt + A. \quad (12)$$

Derivative of above equation can be obtained

$$-C\frac{d^2U_{CP}}{dt^2} = \frac{1}{R_A}\frac{dU_{CP}}{dt} - \frac{R_S C_P}{R_A}\frac{d^2U_{CP}}{dt^2} + \frac{1}{L}U_L. \quad (13)$$

After substitution of (11), equation (13) can be rearranged as follows

$$LC_P\left(-1 + \frac{R_S}{R_A}\right)\frac{d^2U_{CP}}{dt^2} -$$

$$-\left(R_S C_P + \frac{L}{R_A}\right)\frac{dU_{CP}}{dt} - U_{CP} = 0. \quad (14)$$

It can be assumed that value of the first member of equation (14) is greatly smaller then the values of the rest members. Moreover, it is acceptable to consider U_{CP} change over time character to be close to linear. Consequentially, second derivative of equation (14)

$$\frac{d^2U_{CP}(t)}{dt^2} \approx 0. \quad (15)$$

Considering (15) equation (14) can be simplified

$$\left(R_S C_P + \frac{L}{R_A}\right)\frac{dU_{CP}(t)}{dt} - U_{CP}(t) = 0. \quad (16)$$

Returning to hemodynamic representation and substituting (1) into (16), equation for the model in diastole is obtained

$$\left(R_S \cdot a \cdot e^{-b \cdot P(t)} + \frac{L}{R_A}\right)\frac{dP(t)}{dt} - P(t) = 0. \quad (17)$$

To estimate pressure-dependent compliance (C_P) equation (17) is written in the following digital form

$$\left(R_S \cdot a \cdot e^{-b \cdot P(n)} + \frac{L}{R_A}\right)(P(n+1) - P(n)) + T_S P(n) = 0, \quad (18)$$

where T is the sampling period, $t=nT$, $n=1\ldots N$, N – the length of the digitized pressure function.

The aim of equation (18) solution is to define parameters a and b, characterizing the nature of compliance non-linearity. We can write the following set (N-1) of equations for the whole blood pressure time function:

$$\left(R_S \cdot a \cdot e^{-b \cdot P(1)} + \frac{L}{R_A}\right)(P(2) - P(1)) + T_S P(1) = 0, \quad (19)$$

$$\left(R_S \cdot a \cdot e^{-b \cdot P(2)} + \frac{L}{R_A}\right)(P(3) - P(2)) + T_S P(2) = 0, \quad (20)$$

$$\ldots$$

$$\left(R_S \cdot a \cdot e^{-b \cdot P(N)} + \frac{L}{R_A}\right)(P(N) - P(N-1)) +$$

$$+T_S P(N-1) = 0. \quad (21)$$

To estimate parameters a and b blood pressure curve fitting technique is realizable employing least mean root square method. Studies have shown that parameter a values ranges from 0 to 5 and parameter b – from 0 to 0,03 [5]. MATLAB algorithm was created to find optimal a and b values, i.e. the values, which minimizes the error

$$MIN\left|\sqrt{\sum_{n=1}^{N}\left\{\left\langle R_S a_i e^{-b_i P(n)} + \frac{L}{R_A}\right\rangle [P(n+1)-P(n)] + TP(n)\right\}^2}\right|_{\substack{a_i = 0...5 \\ b_i = 0...0.03}} . \qquad (22)$$

Experimental investigation

Experimental investigation of overall 15 patients, divided into healthy (11 normal carotis communis arteries) and unhealthy (4 pathological carotis communis arteries) groups, was performed in Clinic of Cardiology of Kaunas University Hospital. For the estimation of the blood flow rate (Q), ultrasound M and D images were acquired *in vivo* using a commercial GE Vingmed ultrasound system, equipped with high-resolution 10 MHz transducer. Typical blood pressure time function [6] has been calibrated with conventional sphygmomanometer measured systolic P_S and diastolic P_D blood pressures and measured duration of cardiac cycle (Fig. 3).

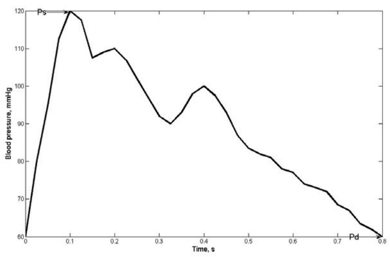

Fig. 3. Typical calibrated blood pressure time function

According to (2), (3) and (4) calculated mean artery model parameters are given in Table 1.

Table 1. Mean model parameters for healthy and unhealthy groups and their difference in percentage

Patient group	Mean peripheral resistance R_S, mmHg/ml	Mean artery impedance R_A, mmHg/ml	Mean inertance L, mmHg s^2/ml
healthy	1,88	2,39	0,0855
unhealthy	5,14	1,99	0,0488
Difference between groups, %	63,4	16,7	42,9

For the pressure-dependent compliance calculation, we used mean value of the estimated instantaneous inertance over diastolic period for each patient (Fig. 4).

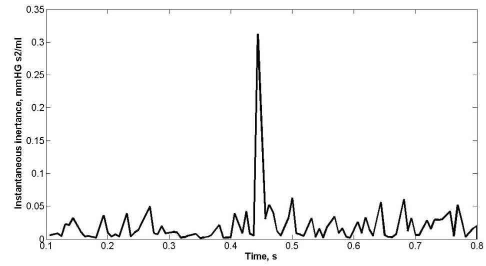

Fig. 4. Instantaneous inertance time function for the patient No. 1 in unhealthy group

Pressure-dependent compliance estimation results were compared to the results, acquired with previously introduced two element model [3] (Table 2).

Compliance pressure functions estimated for unhealthy group, using both, four and two element model, are presented in Fig. 5.

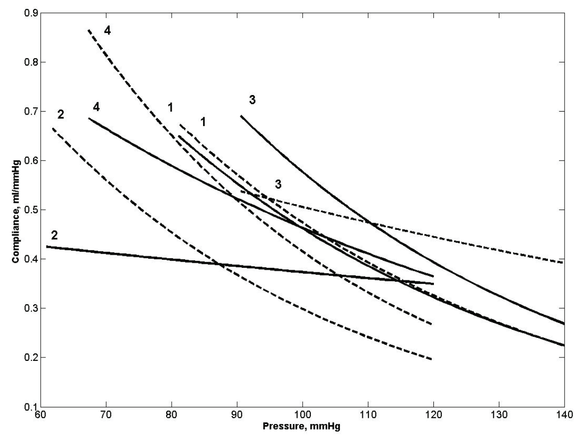

Fig. 5. Pressure-dependent compliances estimated for unhealthy group, using two (dashed line) and four (solid line) element models

Inadequancy for each value of the compliance pressure function, using four and two element artery models, has been calculated. Mean, minimal and maximal values for both groups are presented in Table 3. Mean inadequancy value for compliance at mean pressure $C_P(P_{mean})$ is also given.

Table 2. Pressure-dependent compliance estimation results for unhealthy group, using 4 and 2 element artery models

Patient No.	Four element model					Two element model				
	Coefficient a, ml/mmHg	Coefficient b, m1/mmHg	$C_P(P_s)$ at systolic pressure P_s, mmHg	$C_P(P_d)$ at diastolic pressure P_d, mmHg	$C_P(P_{mean})$ at mean pressure P_{mean}, mmHg	Coefficient a, ml/mmHg	Coefficient b, m1/mmHg	$C_P(P_s)$ at systolic pressure P_s, mmHg	$C_P(P_d)$ at diastolic pressure P_d, mmHg	$C_P(P_{mean})$ at mean pressure P_{mean}, mmHg
1	2,8	0,018	0,65; 140	0,225; 80	0,445; 102	3,08	0,019	0,673; 140	0,225; 80	0,456; 102
2	0,52	0,003	0,425; 120	0,35; 60	0,392; 85	2,44	0,021	0,665; 120	0,196; 60	0,441; 82
3	3,9	0,019	0,691; 140	0,269; 50	0,485; 109	0,96	0,0064	0,538; 140	0,392; 50	0,47; 112
4	1,54	0,012	0,686; 120	0,365; 66	0,537; 88	3,91	0,022	0,865; 120	0,266; 66	0,574; 86

Table 3. Compliance inadequancy for 4 and 2 element artery models

Indices	Healthy group			Unhealthy group		
	Mean	Min	Max	Mean	Min	Max
Inadequancy for C_P, %	6,67	0,01	32,4	12,7	0,02	43,9
Inadequancy for $C_P(P_{mean})$ at mean pressure, %	3,96	-	-	5,77	-	-

It is obvious, that for the same patient calculated two element and four element compliance pressure functions are intersecting almost at their mean values (Fig. 5). This is reflected by inadequancy for the pressure-dependent compliance at mean pressure index, which is only 5,77 % for unhealthy group and 3,96 % for healthy group. However, existing difference between differently calculated functions is rather characterized by mean inadequancy for C_P, which varies from 6,67 to 12,7 % for healthy and unhealthy group, respectively. Moreover, inadequancy for some compliance pressure function points can reach as high as 32,4 % and 43,9 % for healthy and unhealthy groups, respectively. Rather large inadequancy between two models is also evident in different slopes of compliance pressure functions for the same patient.

Conclusions and discussion

Two parameters (coefficients a and b) characterized artery stiffness state and can be further used for diagnostic purposes. Pressure-dependent compliance also reflects compliance dynamics over the whole heart cycle. Due to it's relationship with geometrical parameters, i.e. artery cross-sectional area and length, inertance effect plays a larger role in larger blood-vessels. Therefore, evaluation of inertance in such arteries as, aorta, carotid and etc, should be of great importance. Above assumptions are validated with a mean value of inadequancy for C_P - 9,69 % (for both groups), calculated for four element and two element models.

Direct non-invasive *in vivo* ultrasound measurements of local artery internal radius changes and blood flow rate increase the plausibility of arterial compliance estimation. Locally performed measurements allow us to expect that estimated compliance will characterize more local features of the artery, than those of the whole arterial tree, usually obtained with classic Windkessel models.

It can be presumed, that use of the calibrated typical pressure time function immerse some kind of the inaccuracy to peripheral resistance and inertance calculation and, finally, compliance estimation. However, at this time, accurate pressure time function measurement is inevitably associated with one or another invasive technique, which is often complicated and undesirable nor in clinical study nor in routine medical practice.

A large population experimental *in vivo* study is necessary before model employment in routine medical practice.

Acknowledgement

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Arterial compliance is one of the major indices characterizing arterial stiffness. Electrical analogy artery model, which incorporates elements, characterizing all four main effects of the blood flow, and enables evaluation of pressure-dependent compliance using non-invasive techniques, was proposed in this study. Assumption that, blood inertance together with non-linear compliance enables more accurate representation and provides quantitatively and qualitatively improved diagnostic information, was validated in initial experimental *in vivo* study of human carotid artery. Mean value of inadequancy for C_P - 9,69 % was obtained comparing proposed four and two element models. Ill. 5, bibl. 6, tabl. 3 (in English; abstracts in English and Lithuanian).

I. Kupčiūnas, A. Kopustinskas. Elektrinės analogijos arterijos modelis netiesiniam kompliansui įvertinti // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 1(117). – P. 95–98.

Arterijos kompliansas yra vienas iš svarbiausių parametrų, apibūdinančių arterijų standumą. Sudarytas elektrinės analogijos arterijos modelis atspindi visus keturis pagrindinius kraujotakos procesus ir leidžia neinvaziniais metodais įvertinti nuo slėgio priklausantį kompliansą. Prielaida, kad netiesinio komplianso ir kraujo inertiškumo įvertinimas įgalina tiksliau apibūdinti kraujotaką ir gauti papildomą kokybinę ir kiekybinę diagnostinę informaciją, patvirtinta atlikus *in vivo* žmogaus karotidinės arterijos pradinį eksperimentinį tyrimą. Nustatyta vidutinė 9,69 % C_P verčių neatitiktis pasiūlytiems keturių ir dviejų elementų modeliams. Il. 5, bibl. 6, lent. 3 (anglų kalba; santraukos anglų ir lietuvių k.).