ELECTRONICS AND ELECTRICAL ENGINEERING

ISSN 1392 – 1215 2006. Nr. 2(66)

ELEKTRONIKA IR ELEKTROTECHNIKA

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Synchronization Quality Analysis with Consideration of High Cumulants of Phase Error Distribution

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Introduction

Phase-locked-loop frequency control device serves as optimal filter for the harmonic signal with phase and frequency fluctuations against white noise. This device is inherent component of modern signal receiver and determines quality and possibility of receiving. Phase lock analysis under the noise impact complicates the necessity of equations numerical solution in partial derivative or determination of great number of random process realizations. That is why development of simplified methods of synchronization devices analysis in the noises as well as integrity of those methods is significant for the noise-stable synchronization devices design.

Latest research analysis

The most favorable simplified method of random processes analysis is cumulant analysis [1] with application of which the analytical expressions for synchronization error in the phase-locked-loop frequency control device of the first [2] and second [3] type were received and also inaccessible for the classical methods of bound noise stability analysis [4] as well as resultant action of deterministic and random noise [5]. However these results were received under the assumption of Gauss distribution of phase error, in other words with taking into account only first two cumulants. The analysis of high cumulants influence and detailed comparison with classical methods wasn't performed.

Problem formulation

The aim of this paper is to show up the influence of phase error non-Gaussian distribution on the synchronization quality with cumulants of high order consideration. For the purpose of detailed comparison with exact methods as investigation object the locker of the first order was selected. For this locker the analytical expression of phase error is known.

Investigation summary

The following equation may be used as a mathematical model of dynamic non-filter phase lock device under the noise influence:

$$dx/d\tau = \gamma - \sin x + \sqrt{N} n_0(t) = f(x) + \sqrt{N} n_0(t) \qquad (1)$$

τ – normalized time; x – synchronization phase error; γ – normalized initial frequency mismatch; N – noise and signal ratio in the locking range; n_0 – δ-correlated random process with single distribution of power; f(x) – non-linear transformation function. In sequence with [1] the evolution equations of the first four cumulants in the kurtosis approximation are

$$\begin{cases} d\kappa_1 / d\tau = \langle f(x) \rangle; \\ d\kappa_2 / d\tau = 2\langle x, f(x) \rangle + N; \\ d\kappa_3 / d\tau = 3\langle x, x, f(x) \rangle; \\ d\kappa_4 / d\tau = 4\langle x, x, x, f(x) \rangle, \end{cases} \qquad (2)$$

κ_1, κ_2 cumulants – mean value and deviation of phase error ; κ_3, κ_4 – high cumulants (skewness is equal to $\kappa_3/\kappa_2^{3/2}$, kurtosis is equal to κ_4/κ_2^2), <…> – cumulant Malachov's brackets. When we open the cumulant brackets:

$$\begin{cases} \langle x, f(x) \rangle = \langle f^{I} \rangle \kappa_2 + \langle f^{II} \rangle \kappa_3 / 2 + \langle f^{III} \rangle \kappa_4 / 6; \\ \langle x, x, f(x) \rangle = \langle f^{I} \rangle \kappa_3 + \langle f^{II} \rangle (\kappa_4 + 2\kappa_2^2)/2 + \langle f^{III} \rangle \kappa_2 \kappa_3 + \\ \langle f^{IV} \rangle (\kappa_2\kappa_4/3 + \kappa_3^2/4) + \langle f^{V} \rangle \kappa_3\kappa_4/6 + \langle f^{VI} \rangle \kappa_4^2/36; \\ \langle x, x, f(x) \rangle = \langle f^{I} \rangle \kappa_4 + 3\langle f^{II} \rangle \kappa_2\kappa_3 + \langle f^{III} \rangle (3\kappa_2\kappa_4/2 + \\ + 3\kappa_3^2/2 + \kappa_2^3) + \langle f^{IV} \rangle (5\kappa_2\kappa_4/4 + 3\kappa_2^2\kappa_3/2) + \langle f^{V} \rangle (\kappa_4^2/4 + \\ + \kappa_2^2\kappa_4/2 + 3\kappa_2\kappa_3^2/4) + + \langle f^{VI} \rangle (\kappa_2\kappa_3\kappa_4/2 + \kappa_3^3/8) + \\ + \langle f^{VII} \rangle (\kappa_2\kappa_4^2/12 + \kappa_3^2\kappa_4/8) + + \langle f^{VIII} \rangle \kappa_3\kappa_4^2/24 + \\ + \langle f^{IX} \rangle \kappa_4^3/216. \end{cases} \qquad (3)$$

After consideration of $f(x) = \gamma - \sin(x)$ function derivative recurrence rate the equation (3) becomes next:

$$\begin{cases} \langle x, f(x)\rangle = \langle \sin(x)\rangle \kappa_3/2 + \langle \cos(x)\rangle(\kappa_4/6 - \kappa_2); \\ \langle x, x, f(x)\rangle = \langle \sin(x)\rangle(\kappa_4/2 + \kappa_2^2 - \kappa_2\kappa_4/3 - \kappa_3^2/4 + \\ + \kappa_4^2/36) + +\langle \cos(x)\rangle(\kappa_3 - \kappa_2\kappa_3 + \kappa_3\kappa_4/6); \\ \langle x, x, f(x)\rangle = \langle \sin(x)\rangle(3\kappa_2\kappa_3 + 5\kappa_2\kappa_4/4 - \\ - 3\kappa_2^2\kappa_3/2 + \kappa_2\kappa_3\kappa_4/2 + \kappa_3^3/8 - \kappa_3\kappa_4^2/24) - \\ - \langle \cos(x)\rangle(\kappa_4 - 3\kappa_2\kappa_4/2 - 3\kappa_3^2/2 - \kappa_2^3 + \kappa_4^2/4 + \\ + \kappa_2^2\kappa_4/2 + 3\kappa_2\kappa_3^2/4 - \kappa_2\kappa_4^2/12 - \\ - \kappa_3^2\kappa_4/8 + \kappa_4^3/216), \end{cases} \quad (4)$$

where <sin(x)>, <cos(x)> – appropriate functions averaging due to accepted assumption about phase error distribution W(x). For taking into account the high cumulants influence we accept that W(x) is a kurtosis model distribution, given by the Edeworth row of the fourth order:

$$W_4(x) = W_2(x) - \frac{\kappa_3}{6}\frac{d^3 W_2(x)}{dx^3} + \frac{\kappa_4}{24}\frac{d^4 W_2(x)}{dx^4}, \quad (5)$$

$$W_2(x) = \frac{1}{\sqrt{2\pi\kappa_2}} \exp\left(-\frac{(x-\kappa_1)^2}{2\kappa_2}\right)$$ – normal distribution.

After substitution of appropriate derivatives the equation (5) becomes next:

$$W_4(x) = \sum_{n=0}^{4} a_n (x - \kappa_1)^n \cdot W_2(x), \quad (6)$$

where

$$a_0 = 1 + \frac{\kappa_4}{8\kappa_2^2}; \; a_1 = -\frac{\kappa_3}{2\kappa_2^2}; \; a_2 = -\frac{\kappa_4}{4\kappa_2^3};$$

$$a_3 = \frac{\kappa_3}{6\kappa_2^3}; \; a_4 = \frac{\kappa_4}{24\kappa_2^4}.$$

With consideration of equation (6) the expressions for mean value of sine and cosine of phase error in the kurtosis approximation are received.

$$\begin{cases} \langle \sin(x)\rangle = [(1 + \kappa_4/24)\sin\kappa_1 - \kappa_3\cos\kappa_1/6] \times \\ \times \exp(-\kappa_2/2); \\ \langle \cos(x)\rangle = [(1 + \kappa_4/24)\cos\kappa_1 - \kappa_3\sin\kappa_1/6] \times \\ \times \exp(-\kappa_2/2). \end{cases} \quad (7)$$

By substitution of (4) and (7) equations in the equation (2) we received the system of ordinary differential equations of $x(\tau)$ random process cumulants evolution in the kurtosis approximation.:

$$d\boldsymbol{X}/d\tau = \boldsymbol{F}(\boldsymbol{X}, \gamma, N), \quad (8)$$

$\boldsymbol{X}$= colon (κ_1, κ_2, κ_3, κ_4) – cumulant's vector; $\boldsymbol{F}$ – non-linear vector function.

In the partial case $\kappa_3=\kappa_4=0$ the equation (8) is transformed into the known evolution equations of mean value and deviation in the gauss approximation:

$$d\kappa_1/d\tau = \gamma - \sin(\kappa_1)\exp(-\kappa_2/2); \; d\kappa_2/d\tau = N - 2\kappa_2\cos(\kappa_1)\exp(-\kappa_2/2). \quad (9)$$

Computational complexity of equation (8) integration is 8 times higher in comparison with the same procedure due to equation (9), but approximately 2 times smaller then during the stochastic equation (1) analysis or Foker-Plank equation analysis. Equation (8) solution gives an opportunity to observe transient processes under the variation of statistical characteristics of W(x) distribution and to find stationary distribution $W_{st}(x)$.

On the Fig. 1 the time dependences of phase error standard deviation as the main parameter of synchronization quality for two value sets of (γ, N) parameters is presented. For the comparison, the Tichnov's stationary distribution standard deviation is presented on the Fig. 1.

$$W_{st}(x) = A\exp(-2(\gamma x + \cos x)/N), \quad (10)$$

$A^{-1} = \int_{-\infty}^{\infty} \exp(-2(\gamma x + \cos(x))/N)dx$ – normalizing factor (line 3).

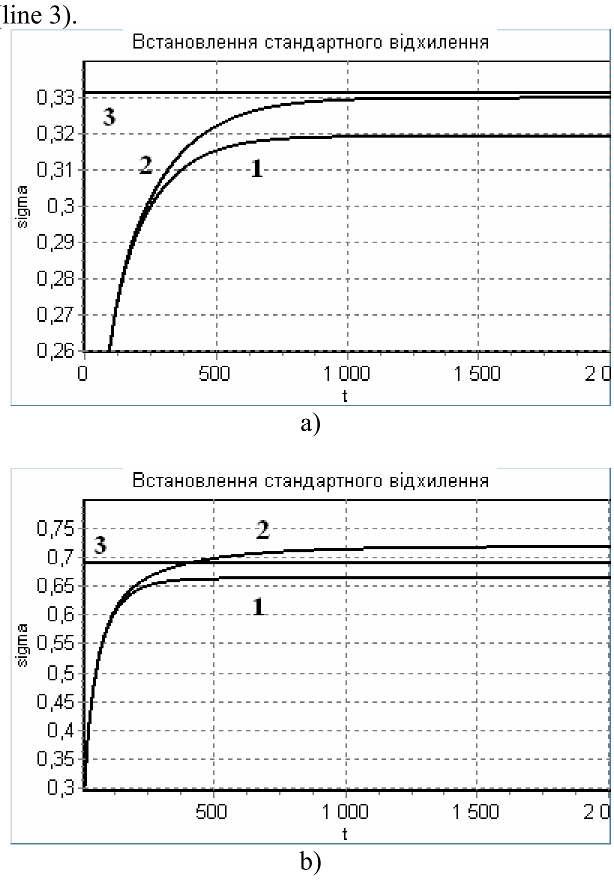
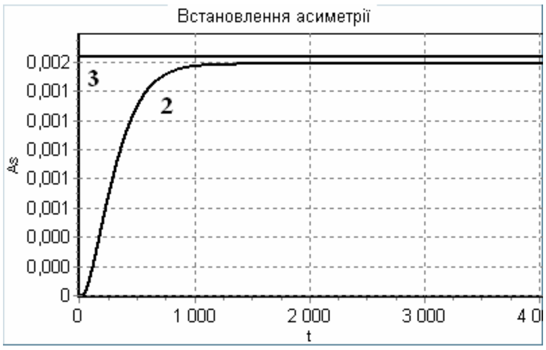


Fig. 1. Phase error standard deviation determination: 1 – gauss approximation, 2 – kurtosis approximation, 3 – exact stationary value;. a) γ=0,1; N=0,7. b) γ=0,6; N=0,15

The analytic for of W(x, τ) distribution isn't available. Under the selected parameters values standard deviation determination error by simplified methods is

10% in case of gauss approximation (curve 1) and 2.5 % in kurtosis approximation case (curve 2) . In two cases gauss approximation is underscore while kurtosis approximation is underscore or high estimate.

On the Fig. 2 the same diagrams of third cumulant κ_3 determination are presented. Estimation error in case of kurtosis approximation is equal to 15%, underscore or high estimate rates are possible. Time required for κ_3 cumulant determination is greater then κ_2 cumulant determination time, that is peculiarity of synchronization system with many balance states. In the single balance state systems, due to [1], deviation is the most slowly determined parameter.



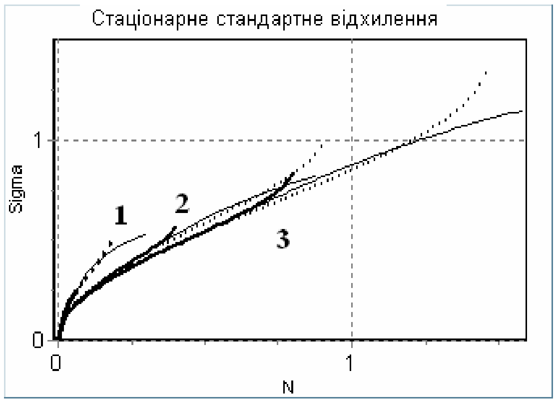
a)

b)

Fig. 2. Determination of phase error distribution skewness: 2 – kurtosis approximation, 3 – exact stationary value; a) γ=0,7; N=0,05. b) γ=0,6; N=0,2

Significant decrease of calculation complexity give an opportunity to perform variants selection of phase lock with noise in range of $\gamma \in [0,1]$ with step 0,1, $N \in [0, 1,6]$ with step 0,02. Under condition $\gamma>1$ synchronization is impossible even at noise absence. During N>1,5 equations (8) and (9) become unstable independently of γ value. Phase error standard deviation stationary value calculation results are presented on the Fig.3a for the several cases of initial disagreement. Comparison shows that kurtosis approximation provide more precise estimation of standard deviation, but set of equations of the fourth order (8) lose stability under the much more lower noise levels than set of the second order equations(9). It's related to inflated estimation of deviation in the kurtosis estimation.

Graphs of ratio error of synchronization quality estimation under all possible parameter combinations are presented on the Fig.3b. These graphs indicate that considering high order cumulants of phase error distribution under the application bounds gives an opportunity to increase approximate method precision up to tenth and hundredth parts of percent in comparison of dozens percents in Gauss approximation.



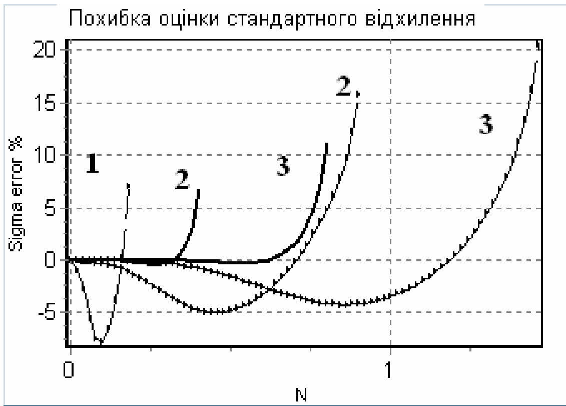


Fig. 3. Phase error standard deviation stationary values (a) and estimation ratio error (b) : straight line – exact value, dot line – gauss approximation , thick line – kurtosis approximation: 1- γ=0,8; 2- γ=0,4; 3- γ=0

Conclusions

Presented calculations have shown that high order cumulants consideration of phase error distribution in the synchronization system gives an opportunity to refine accuracy of synchronization quality estimation. Advantages of described simplified approach over precise method are definite decreased calculation expenses, possibility of analysis of transient process of phase error statistical characteristics determination, possibility of simultaneously consideration of noise and internal or external interference influence.

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Presented 2006 01 04

A. Bondarev, O. Lazko, L. Nedostup. Synchronization Quality Analysis with Consideration of High Cumulants of Phase Error Distribution // Electronics and Electrical Engineering.– Kaunas: Technologija, 2006. – No. 2(66). - P. 17–20.

Presented calculations have shown that high order cumulants consideration of phase error distribution in the synchronization system gives an opportunity to refine accuracy of synchronization quality estimation. Advantages of described simplified approach over precise method are definite decreased calculation expenses, possibility of analysis of transient process of phase error statistical characteristics determination, possibility of simultaneously consideration of noise and internal or external interference influence. Development of these advantages provide opportunities for further application of described analysis technique. Ill. 3, bibl. 5 (in English; summaries in English, Russian and Lithuanian).

А. Бондарев, О. Лазько, Л. Недоступ. Анализ качества синхронизации с учетом высших кумулянтов распределения фазовой погрешности // Электроника и электротехника.– Каунас: Технология, 2006.– № 2(66).– С. 17–20.

Проведенные расчеты показали, что учет негаусовости распределения фазовой погрешности в системе синхронизации разрешает значительно повысить точность оценки качества синхронизации. Преимуществами описанного упрощенного метода перед точными являются значительное уменьшение вычислительных затрат, возможность анализа переходных процессов установления статистических характеристик распределения фазовой погрешности, возможность одновременного учета влияния шумов и внутренней или внешней помехи. Полученные границы стойкости систем кумулянтных уравнений разрешают оценить предельную помехоустойчивость синхронизации. Развитие этих преимуществ и представляет перспективы дальнейшего использования описанного метода анализа. Ил. 3, библ. 5 (на английском языке; рефераты на английском, русском и литовском яз.).

A. Bondarev, O. Lazko, L. Nedostup. Sinchronizacijos kokybės analizė, įvertinant aukštuosius fazinių paklaidų pasiskirstymo kumuliantus // Elektronika ir elektrotechnika.– Kaunas: Technologija, 2006. – Nr. 2(66). - P. 17–20.

Atlikti skaičiavimai parodė, kad gausiniam skirstiniui fazinės paklaidos žymiai viršija sistemos sinchronizacijos kokybės tikslumą. Pasiūlyto supaprastinto metodo esmė ta, kad gerokai sumažėja programavimo išlaidos, iš vienos pusės, ir pereinamųjų procesų statistinių charakteristikų fazinių paklaidų skirstinių analizės galimybė, iš kitos pusės. Šis metodas taip pat leidžia įvertinti išorinių bei vidinių triukšmų įtaką. Apskaičiuotos stabilumo ribos įvertina ribinį sinchronizacijos atsparumą triukšmams. Šie privalumai leidžia teigti, kad sukurtas analizės metodas gali būti panaudotas kaip perspektyviausias tęsiant tyrimus. Il. 3, bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

DOI: 10.5755/j02.eie.10584