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Methods for Determination of Winding Factors of Alternating-Current Electric Machines

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Introduction

Slot-shaped nature of magnetic circuit of stator and rotor of alternating-current machines, uneven distribution of their windings and other factors form conditions for creation of non-sinusoidal rotating magnetic fields. Such instantaneous functions of periodic rotating magnetomotive force which are not distributed according to the sinusoidal law may be expanded into its rotating space harmonics. Each space harmonic of this rotating magnetomotive force induce harmonics of electromotive force of the same order in the windings of stator and rotor, the sum of which constitutes the curves of non-sinusoidal electromotive force.

To achieve the distribution of rotating magnetomotive force in the air gap of electric machines and the form of the curve of electromotive force induced in the windings as close as possible to sinusoidal, some certain measures are taken: the span of the windings is shortened ($y < \tau$), windings are distributed ($q > 1$), skewed slots are made in the magnetic circuit [1 - 2]. All these measures reduce the harmonics of rotating magnetomotive forces and induced electromotive forces. The reduction of these variable magnitudes is expressed using winding factors $k_{\mathrm{w}\nu} = k_{\mathrm{y}\nu}\, k_{\mathrm{p}\nu}\, k_{\mathrm{d}\nu}$, here $k_{\mathrm{y}\nu}$ – pitch shortening factors; $k_{\mathrm{p}\nu}$ – winding distribution factors; $k_{\mathrm{d}\nu}$ – slot skewness factors; ν – number of harmonic number.

Skewed slots of squirrel cage motors are typically formed in the magnetic circuits of rotors, and straight ones are made in magnetic circuits of stator. Therefore in all these cases slot skewness factors of three-phase windings laid into the slots of stator are $k_{\mathrm{d}\nu} = 1$.

Not all mentioned measures can be applied to improve the shape of the curves of rotating magnetomotive forces of all types of the three-phase windings and the shapes of electromotive forces. None of the measures is suitable for single-layer concentrated three-phase windings, and for this reason factors of such windings for all available harmonics of magnetomotive or electromotive force ν are equal to one $\left(k_{\mathrm{w}\nu} = 1\right)$.

Single-layer preformed and concentric three-phase windings are manufactured as distributed ($q > 1$), but their spans can be equal only to the pole pitch ($y = \tau$), thus factors of these windings $k_{\mathrm{w}\nu}$ match their distribution factors $k_{\mathrm{p}\nu}$ ($k_{\mathrm{w}\nu} = k_{\mathrm{p}\nu}$).

Two-layer preformed, concentric and single-layer chain three-phase windings are also manufactured as distributed ($q > 1$), furthermore, their span is shortened ($y < \tau$). Factors of these windings consist of the product of winding distribution and pitch factors ($k_{\mathrm{w}\nu} = k_{\mathrm{p}\nu}\, k_{\mathrm{y}\nu}$).

Methods for determination of winding factors of alternating-current electric machines

On the basis of earlier proposed thoughts it can be stated, that winding factors not only assess the reduction of harmonics of rotating magnetomotive forces and induced electromotive forces, but also reflect the relative magnitudes of amplitude values of the harmonics of these variable quantities in respect of corresponding amplitudes of the same harmonics in concentrated three-phase winding.

Functions of instantaneous periodic rotating magnetomotive force of concentrated three-phase winding are square-shaped.

Since this square-shaped function is symmetrical according to the coordinate axes *F*, *x*, therefore it has only uneven space harmonics without multiples of three (Table 1)[3-5].

Table 1. The results of harmonic analysis of instantaneous rotating magnetomotive force function of concentrated three-phase winding

ν	1	5	7	11	13	17	19
$F_{1\nu}$	0,955	-0,1910	0,1364	-0,0868	0,0735	-0,0562	0,0503
$f_{1\nu}$	1	0,20	0,1429	0,0909	0,0769	0,0588	0,0526

In this table ν – the number of the harmonic; $F_{1\nu}$ – relative amplitude value of the ν-th harmonic of the rotating magnetomotive force; $f_{1\nu}$ – the relative value of the ν-th harmonic of rotating magnetomotive force $\left(f_{1\nu} = F_{1\nu} / F_{11}\right)$.

When electric currents are flowing over single-layer three-phase chain windings, it creates rotating magnetomotive force, for which instantaneous periodic functions are symmetric in respect of coordinate axes *F*, *x*. Such functions (when their harmonic analysis is performed) have odd and even harmonics without multiples of three [4-5]. Therefore winding factors of odd and even harmonics of magnetomotive or electromotive force should be determined for the windings of this type (Fig. 1).

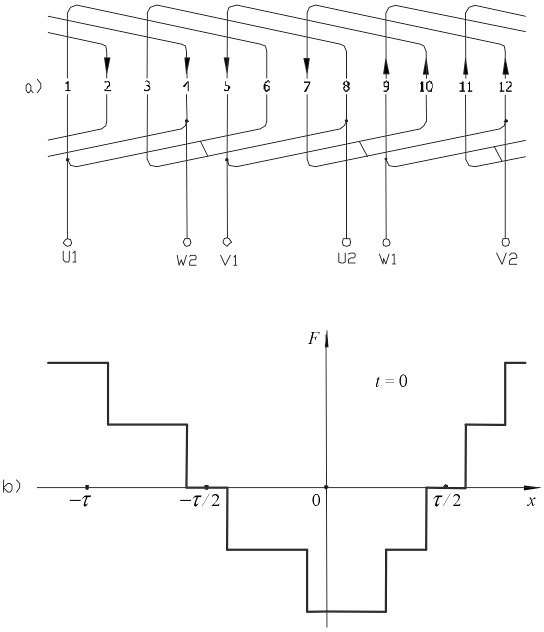

Fig. 1. Development of electrical circuit of single-layer three-phase chain winding, in which $q = 2$ (a) and the distribution of its rotating magnetomotive force in the moment of time $t = 0$ (b)

A single-layer three-phase chain winding, for which $2p = 2$; $q = 2$; $y = 5$ is analyzed (Fig. 1). Harmonic analysis of the instantaneous stair-shaped rotating magnetomotive force functions of these windings in respect of asymmetric coordinate axes is performed according to the following analytical expressions [3-5]:

$$F_{\nu'} = \frac{4}{\pi \nu'} \sum_{i=1}^{k} F_{\mathrm{m}i} \sin\left(\nu' \beta_i / 2\right) \cos\left(\nu' \gamma_i\right); \qquad (1)$$

$$F_{\nu''} = \frac{4}{\pi \nu''} \sum_{i=1}^{k} F_{\mathrm{m}i} \sin\left(\nu'' \beta_i / 2\right) \sin\left(\nu''' \gamma_i\right); \qquad (2)$$

here k – number of rectangles in the half-period of the rotating magnetomotive force; $F_{\mathrm{m}\ i}$ – height of the *i*-th rectangle of stair-shaped magnetomotive force half-period; $\beta_{\ i}$ – width of the *i*-th rectangle of the stair-shaped magnetomotive force, expressed in electric degrees of the fundamental space harmonic; γ_i – the asymmetry of the *i*-th rectangle of the half-period of stair-shaped magnetomotive force in respect of reference axis, expressed in electrical degrees of the fundamental space harmonic; ν' – number of the order of odd harmonics; ν'' – number of the order of even harmonics.

According to expressions (1) and (2) and predefined parameters of the half-period of rotating magnetomotive force (Fig. 1, b) ($k = 2$; $F_{\mathrm{m}1} = 0{,}433$; $F_{\mathrm{m}2} = 0{,}433$; $\beta_1 = 150°$; $\beta_2 = 60°$) its harmonic analysis was accomplished (Table 2).

Table 2. Harmonic analysis results of the rotating magnetomotive force function of the single-layer three-phase chain winding

ν	1	2	4	5	7	8
$F_{2\nu}$	0,799	-0,1194	-0,1034	0,0428	0,0306	0,0517
$f_{2\nu}$	1	0,1494	0,1294	0,0536	0,0383	0,0647

Table 2 (continued)

10	11	13	14	16	17	19
0,0239	0,0726	-0,0614	0,01705	0,0258	-0,0126	-0,0113
0,0299	0,0909	0,0769	0,0213	0,0323	0,0158	0,0141

Single-layer chain three-phase winding is also distributed and short-span winding, additionally by considering that in this winding the sections in the group are spaced apart in two slot spans ($2\alpha = 2\pi/(3q)$), the following analytical expression was derived to determine its winding factors:

$$k_{\mathrm{w}_2\nu} = k_{\mathrm{p}\nu}\, k_{\mathrm{y}\nu} = \frac{\sin\left(\nu \dfrac{\pi}{3}\right) \sin\left(\nu \dfrac{\pi\, y}{6q}\right)}{q \sin\left(\nu \dfrac{\pi}{3q}\right)}. \qquad (3)$$

Winding factors calculated according to this expression are presented in Table 3.

Table 3. Factors of single-layer three-phase chain winding, for which $q = 2$; $y = 5$

ν	1	2	4	5	7	8
$k_{\mathrm{w}\nu}$	0,8365	0,250	0,433	0,224	0,224	0,433

Table 3 (continued)

10	11	13	14	16	17	19
0,250	0,8365	0,8365	0,250	0,433	0,224	0,224

Factors of the analyzed single-layer three-phase chain winding can be calculated according to formula (4), i.e. on the basis of data indicated in Tables 1 and 2:

$$k_{\mathrm{w}_2\nu} = F_{2\nu} / F_{1\nu}\,. \qquad (4)$$

Calculation results are given in Table 4.

Table 4. Factors of single-layer three-phase chain winding for which $q = 2$; $y = 5$

ν	1	2	4	5	7	8
$k_{\mathrm{w}\nu}$	0,8365	∞	∞	0,224	0,224	∞

Table 4 (continued)

10	11	13	14	16	17	19
∞	0,8365	0,8365	∞	∞	0,224	0,224

Since even harmonics of the rotating magnetomotive force of single-layer concentrated three-phase winding are equal to zero, thus winding factors of even harmonics in Table 4 calculated according to formula (4) are obtained as indefinite. Winding factors of odd harmonics in Table 4 are obtained the same as in Table 3.

What is the possible way to avoid the uncertainty of the winding factors of even harmonics calculated by the second method? For this purpose a two-layer preformed fractional-slot winding is selected for the further analysis (Fig. 2, a), parameters of which are: $2p = 2$; $q = 1/2$; $\tau = 3q = 3/2$; $y = 2$; $\alpha = \pi/\tau = 120°$. This winding has lengthened span and the relative quantity of the effective conductors in its slot is $N^*_{3\mathrm{g}} = N^*_{1\mathrm{g}}/q = 1/(1/2) = 2$, and the relative quantity of the number of turns in one section $N^*_3 = N^*_{3\mathrm{g}}/2 = 2/2 = 1$. The latter quantity is obtained equal to the relative quantity of number of turns of one section of concentrated three-phase winding [5].

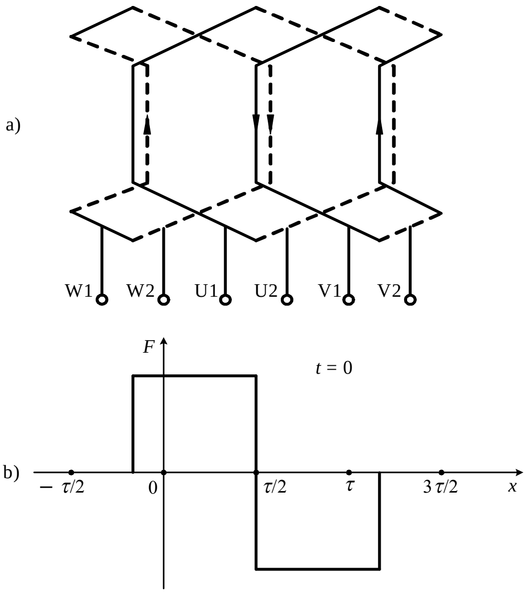

Fig. 2. Development of electrical circuit of two-layer preformed fractional three-phase winding, in which q = 1/2 (a) and the distribution of its rotating magnetomotive force in the moment of time t = 0 (b)

The instantaneous function of the rotating magnetomotive force of the considered winding in certain moments of time, e. g. t = 0, is square-shaped, same as for the concentrated three-phase winding. But this function is asymmetric in respect of coordinate axes (Fig. 2, b). According to expressions (1) and (2) and predefined parameters of the half-period of rotating magnetomotive force (Fig. 2, b) ($k = 1$; $F_\mathrm{m} = 0{,}866$; $\beta = 120°$; $\gamma = -30°$) its harmonic analysis was carried out (Table 5).

Table 5. Harmonic analysis results of the rotating magnetomotive force function of the two-layer preformed fractional-slot three-phase winding, for which q = 1/2

ν	1	2	4	5	7	8
$F_{3\nu}$	0,827	-0,4135	0,207	0,1654	-0,1181	0,1034
$f_{3\nu}$	1	0,50	0,250	0,20	0,1429	0,125

Table 5 (continued)

10	11	13	14	16	17	19
-0,0827	-0,0752	0,0636	-0,0591	0,0517	0,0486	-0,0436
0,10	0,0909	0,0769	0,0714	0,0625	0,0588	0,0526

From the results of harmonic analysis presented in Table 5 it can be stated, that relative quantities $f_{3\nu}$ of odd harmonics of rotating magnetomotive force of two-layer preformed fractional-slot three-phase winding with q = 1/2 are equal to relative quantities of respective odd harmonics of rotating magnetomotive force of concentrated three-phase winding (Table 1). By considering this fact, winding factors of single-layer three-phase chain winding can be determined by using data diven in Tables 2 and 5:

$$k_{\mathrm{w}_2\nu} = \frac{F_{21}}{F_{11}}\frac{f_{2\nu}}{f_{3\nu}} = k_{\mathrm{w}_2 1}\, f_{2\nu} / f_{3\nu} \; . \qquad (5)$$

Winding factors calculated according to formula (5) are summarized in Table 6.

Table 6. Factors of single-layer three-phase chain winding, for which $q = 2$; $y = 5$

ν	1	2	4	5	7	8
$k_{\mathrm{w}\nu}$	0,8365	0,250	0,433	0,224	0,224	0,433

Table 6 (continued)

10	11	13	14	16	17	19
0,250	0,8365	0,8365	0,250	0,433	0,224	0,224

Data given in Table 6 obtained after performing calculations according to expression (5) coincides with results indicated in Table 3.

Analytical expression can not be used to determine winding factors of two-layer preformed fractional three-phase windings, since numbers of sections forming the section groups of these windings are not equal to the number of pole and phase slots q. This problem is solved on the basis of expression (5) by deriving the following formula:

$$k_{\mathrm{w}_i\nu} = \frac{F_{i1}}{F_{11}}\frac{f_{i\nu}}{f_{3\nu}} = k_{\mathrm{w}_i 1}\, f_{i\nu} / f_{3\nu} \, ; \qquad (6)$$

here $k_{\mathrm{w}_i\nu}$ – winding factor of the ν-th harmonic of the i-th two-layer preformed fractional three-phase winding; F_{i1} – relative amplitude value of the first harmonic of rotating magnetomotive force of the i-th two-layer preformed fractional three-phase winding; F_{11} – relative amplitude value of the first harmonic of rotating magnetomotive force of the concentrated three-phase winding; $f_{i\nu}$ – relative quantity of the ν-th harmonic of rotating magnetomotive force of the i-th two-layer preformed fractional three-phase winding; $f_{3\nu}$ – relative quantity of ν-th harmonic of rotating magnetomotive force of the two-layer preformed fractional three-phase winding for which q = 1/2.

Due to the lengthened pitch winding factors of the considered two-layer preformed fractional (q = 1/2) three-phase winding are not equal to one, as in case of concentrated three-phase winding. According to expression (6) winding factors of all harmonics are of the same value and equal to 0,866 $(k_{\mathrm{w}_3\nu} = 0{,}866)$.

Conclusions

1. Winding factors of various types of three-phase windings (except for two-layer preformed fractional-slot

windings) can be calculated according to derived respective analytical expressions, which depending on the type of the windings assess their distribution, distribution and pitch shortening factor or all three measures (distribution, pitch shortening factor and unequal number of turns in sections).

2. Winding factors of the three-phase windings, the instantaneous functions of rotating magnetomotive force created by them are symmetric in respect of coordinate axes, can be determined by dividing their ν-th harmonic quantities of rotating magnetomotive forces by respective harmonic quantities of rotating magnetomotive force of concentrated three-phase winding.

3. Winding factors of two-layer preformed fractional-slot three-phase windings are impossible to determine using analytical expressions due to unequal number of sections in section groups. Winding factors of these windings can be calculated only by using the harmonic analysis results instantaneous functions of their rotating magnetomotive force.

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Methods for determination of winding factors of alternating-current electric machines are discussed. It was shown, that single-layer concentrated and two-layer preformed fractional (q = 1/2) three-phase windings are similar in their nature, since their relative quantities of respective ν-th odd harmonics of rotating magnetomotive forces are equal, even though factors of all harmonics of the first winding are equal to one, and due to the lengthening of the second winding span become equal to 0,866. It was proved by theoretical calculations, that winding factors of odd and even harmonics of single-layer three-phase chain-winding can be determined not only according to newly created analytical expression, which assesses distribution of this winding and pitch shortening factor, but also by using harmonic analysis results of instantaneous functions of rotating magnetomotive force of this winding and two-layer preformed fractional-slot (q = 1/2) three-phase winding. Winding factors of respective harmonics calculated by both methods are equal. Ill. 2, bibl. 5 (in English; summaries in English, Russian and Lithuanian).

Ю. Букшнайтис. Методы определения коэффициентов обмотки электрических машин переменного тока // Электроника и электротехника. – Каунас: Технология, 2009. – № 1 (89). – С. 83–86.

Рассматриваются методы определения обмоточных коэффициентов электрических машин переменного тока. Показано, что однослойная сосредоточенная и двухслойная шаблонная с дробным числом пазов на полюс и фазу (q = 1/2) трёхфазные обмотки являются близкими, ввиду того что их относительные величины соответствующих нечётных гармоник вращающихся магнитодвижущих сил являются равными, хотя обмоточные коэффициенты всем гармоникам первой обмотки равны единице, а другой с целью удлинения шага обмотки – равны 0,866. Теоретические расчёты показали, что коэффициенты нечётных и чётных гармоник однослойной цепной трёхфазной обмотки можно определитъ не только по новообразованному аналитическому выражению, которым оценивается распределение и укорочение шага этой обмотки, но и при использовании результатов гармонического анализа мгновенных функций вращающейся магнитодвижущей силы обеих обмоток. Обоими методами рассчитанные обмоточные коэффициенты соответственных гармоник являются одинаковыми. Ил. 2, библ. 5 (на английском языке; рефераты на английском, русском и литовском яз.).

J. Bukšnaitis. Kintamosios srovės elektros mašinų apvijos koeficientų nustatymo metodai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 1 (89). – P. 83–86.

Nagrinėjami kintamosios srovės elektros mašinų apvijos koeficientų nustatymo metodai. Parodyta, kad viensluoksnė sutelktoji ir dvisluoksnė forminė trupmeninė (q = 1/2) trifazės apvijos yra artimos, nes jų sukamųjų magnetovarų atitinkamų ν-ųjų nelyginių harmonikų santykiniai dydžiai yra vienodi, nors visų harmonikų pirmosios apvijos koeficientai yra lygūs vienetui, o antrosios dėl apvijos žingsnio pailginimo – lygūs 0,866. Teoriniais skaičiavimais įrodyta, kad nelyginių ir lyginių harmonikų viensluoksnės grandininės trifazės apvijos koeficientus galima nustatyti ne tik pagal naujai sudarytą analizinę išraišką, kuri įvertina šios apvijos paskirstymą ir žingsnio sutrumpinimą, bet ir naudojantis šios apvijos ir dvisluoksnės forminės trupmeninės (q = 1/2) trifazės apvijos sukamosios magnetovaros akimirkinių funkcijų harmoninės analizės rezultatais. Abiem metodais apskaičiuoti atitinkamų harmonikų apvijos koeficientai yra vienodi. Il. 2, bibl. 5 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

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