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Theoretical Evaluation of Space Constants of Electrotonic Decay in Resistive Anisotropic Media: Two-dimensional case

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Introduction

For modeling the electrocardiosignal genesis and excitation wave spread in myocardium [1-3], values of the parameters of the passive electric properties of myocardium (such as resistivity of the intracellular medium, electrogenic membrane, intercellular contacts, space constants of electrotonic decay) are essential. However, modeling makes sense only provided the used values of parameters are most precise. One of possibilities to find the parameters is by measuring experimental space constants of electrotonic decay (λ_{xe} , λ_{ye}) (i.e. the distance at which the amplitude of the electrotonic potential decreases by 2.71 times) with further analysis of data by mathematical models of resistive (*R*) media. Such media are described by differential equations of second order with partial derivatives, and the analytical solutions may only be obtained in the presence of either spherical or cylindrical symmetry (for a point-shaped source of current) [4-7]. If myocardium is a complex anisotropic structure and the current is delivered to the intracellular medium by circle-shaped suction-electrodes (i.e. there is no spherical or cylindrical symmetry in a normalized system of coordinates), the analysis of experimental data is made basing on models of isotropic medium and theoretical conclusions derived on such basis [8-10].

In the present study we established the dependencies of experimental values of space constants of electrotonic decay along (λ_{xe}) and across (λ_{ye}) the fiber orientation upon the size of the current electrode, electrotonic anisotropy and the distance between the current electrode and the electrotonic potential recording site, we assessed the relation between the measured and actual values of electrotonic anisotropy and calculated the possible errors in the evaluations of the actual values of the space constants of electrotonic decay, when myocardium is modeling by two-dimensional anisotropic medium [11]. The preliminary results were published in [12].

The results of modeling of electrotonic potential distribution in two-dimensional resistive medium.

In the experiment, intracellular current is delivered by suction electrodes [13], the inner electrical structures of which is unknown. Let the current electrode in the metric system of coordinates *x*, *y* be a circle or a disk, the radius of which is r_o. Using the principle of superposition, well known in electrostatics, let us divide the current electrode into elementary point sources, for each analytical solution is known. The solution for the electrotonic potential distribution in two-dimensional anisotropic medium is presented as a sum of electrotonic potentials, caused by *M* point-shaped current sources [11]. In this paper we will confine ourselves to three ways of arranging the elementary point sources on the perimeter or on surface of the current electrode [11, 14]. For the calculation of the characteristics of electrotonic potential distribution in the resistive anisotropic medium, computer programs were developed.

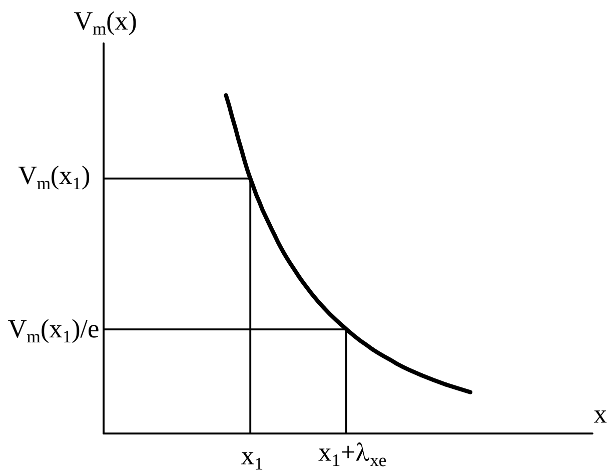

Fig. 1. Determination of experimental space constant of electrotonic decay at point $x=x_1$. $V_m(x)$ – the amplitude of electrotonic potential. λ_{xe} – a distance at which the electrotonic potential decrease in 2.71 times

We calculated the dependence of normalized experimental space constants of electrotonic decay $L_x = \lambda_{xe} / \lambda_x$ and $L_y = \lambda_{ye} / \lambda_y$ on the current electrode radius r_o, electrotonic anisotropy A_e ($A_e = \lambda_x / \lambda_y$), distance between the recording point and the center of the current electrode in X and Y directions ($X = x/\lambda_x$, $Y = y/\lambda_y$) and on the mode of dividing the current electrode into elementary point-shaped sources. Also the eventual errors of the evaluation of true space constants of electrotonic decay are estimated. Besides, we assumed that $\lambda_x = 1mm$. In Fig. 1 it is illustrated how the value of λ_{xe} in point $x = x_1$ is determined: an experimental space constant of electrotonic decay λ_{xe} at point x_1 – is the distance at which electrotonic potential $V_m(x_1 + \lambda_{xe})$ decreases in 2.71 times.

The calculation results are presented as families of curves in the two-dimensional system of coordinates.

As can be seen from the calculation results (Fig. 2), with the increase of the electrotonic anisotropy, the values of L_x increase and asymptotically approach to 1, i.e. $\lambda_{xe} \to \lambda_x$, regardless of the mode of dividing the current electrode into point-shaped sources.

Besides, the values of L_x are higher when the radius of the current electrode is bigger. The dependence of L_y on the same parameters is more complex. For smaller current electrode ($r_o = 0.1$mm) with the increase of the electrotonic anisotropy, the values of L_y practically remain stable (the mode *1* of positioning the point-shaped sources) or increase (the mode *2* and mode *3*). When the current electrode is bigger ($r_o = 1.0$ mm), increasing of A_e slightly reduces the values of L_y, irrespective of the mode of positioning of the elementary point-shaped sources. For stable values of A_e, r_o, and the mode *1* of positioning of the point-shaped sources, the values of L_x are higher, and the values of L_y are respectively always smaller than in case of the mode *2* or mode *3* (see Fig. 2d).

The values of L_x were calculated on the surface of the current electrode, in point (r_o/λ_x,0), and L_y values were calculated on the surface of the current electrode, in point ($0, r_o/\lambda_y$) (*X*, *Y* coordinates system).

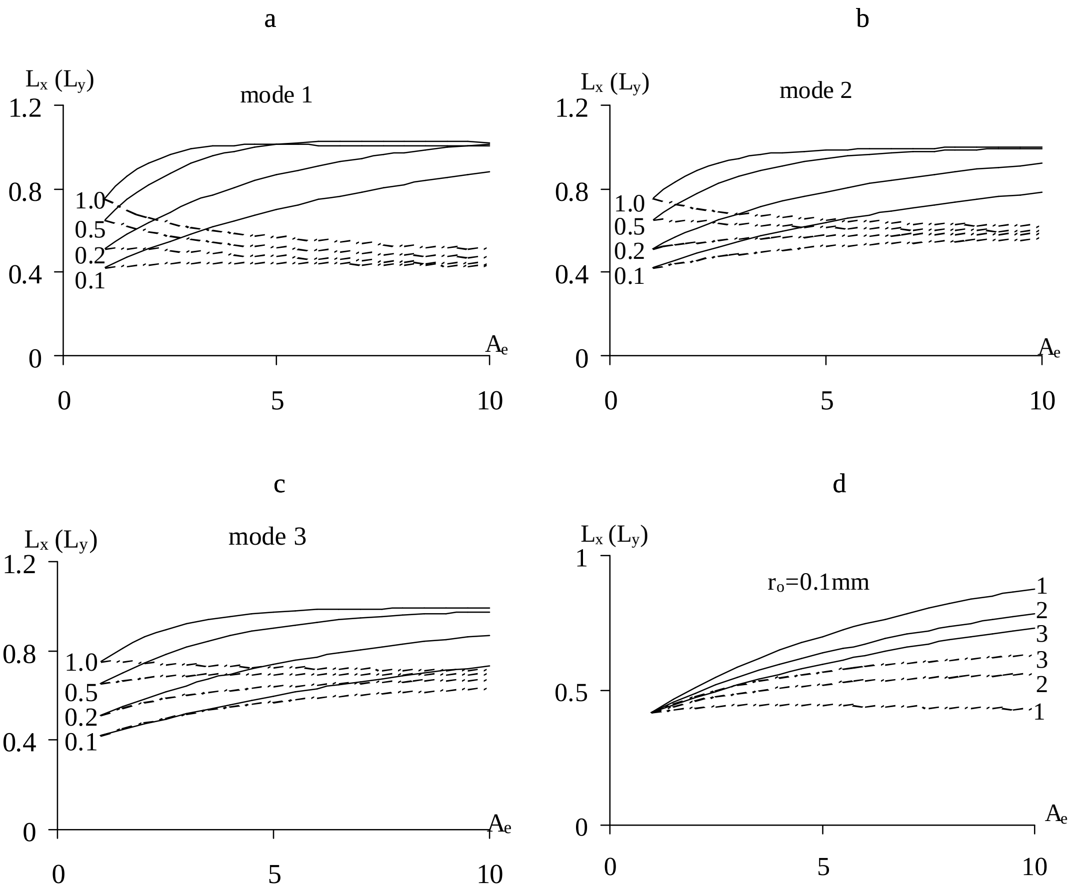

Fig. 2. The dependence of normalized space constants of electrotonic decay L_x (continuous curves) and L_y (dotted curves) on electrotonic anisotropy and the mode of dividing the current electrode into point-shaped sources. In parts **a**, **b** and **c** the numbers *0.1*, *0.2*, *0.5*, *1.0* next to curves are the radii of current electrode (in mm). **d** – numbers *1*, *2*, *3* next to curves refer to the mode of dividing the current electrode into point-shaped sources. The current electrode in the *x*, *y* coordinates system is a circle with the radius (r_o) equal to *0.1*, *0.2*, *0.5* and *1.0* mm, and λ_x =1.0 mm

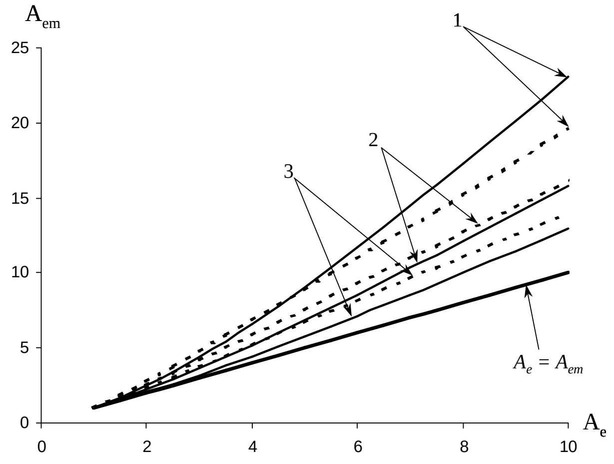

Fig. 3. Relation between the experimental anisotropy A_{em} and true anisotropy A_e, for different radii of current electrode (continuous thin curves for r_o=0.2 mm and dotted curves for r_o=1.0 mm). Numbers *1*, *2* and *3* next to curves show the mode of dividing of the current electrode into point-shaped sources. In all cases λ_x=1.0 mm

It may be seen from the families of curves (Fig. 2) that the normalized space constants of electrotonic decay $L_x = \lambda_{xe}/\lambda_x$ and $L_y = \lambda_{ye}/\lambda_y$ to a different extent depend on the size of the current electrode and the mode of positioning the point-shaped sources of current. For any fixed values of r_o and A_e and the same mode of positioning of point-shaped sources, $L_x > L_y$, therefore then $\lambda_{xe}/\lambda_x > \lambda_{ye}/\lambda_y$ and $\lambda_{xe}/\lambda_{ye} > \lambda_x/\lambda_y$, i.e. the experimental electrotonic anisotropy ($A_{em} = \lambda_{xe}/\lambda_{ye}$) is bigger than A_e and should be dependent on the above factors. This dependence is provided in Fig. 3.

As can be seen from the diagram, A_{em} is always higher than A_e, and in mode *1* of positioning of point-shaped sources the experimental anisotropy is increased to a higher extent than in the mode *2* and mode *3*. Data of the dependence of A_{em}, L_x and L_y on the mode of dividing of the electrode into point-shaped sources and the size of the electrode (Fig. 2, Fig. 3) were obtained by performing calculations on the surface of the current electrode. Fig. 4 and Fig. 5 provides the dependence of normalized space constants of electrotonic decay (L_x, L_y) on the distance between the center of the current electrode and the point of measurement/calculation of potential (i.e. moving away from a current electrode) for different sizes of the current electrode, electrotonic anisotropy and mode of dividing of current electrode into point-shaped sources. The case when $r_o = 0$ corresponds to the case of isotropic medium ($A_e = 1$). Moving away from the current electrode, L_x and L_y increase. As compared with the case of point-shaped source of current (when $r_o = 0$), the obtained values of L_x are greater, and L_y are smaller.

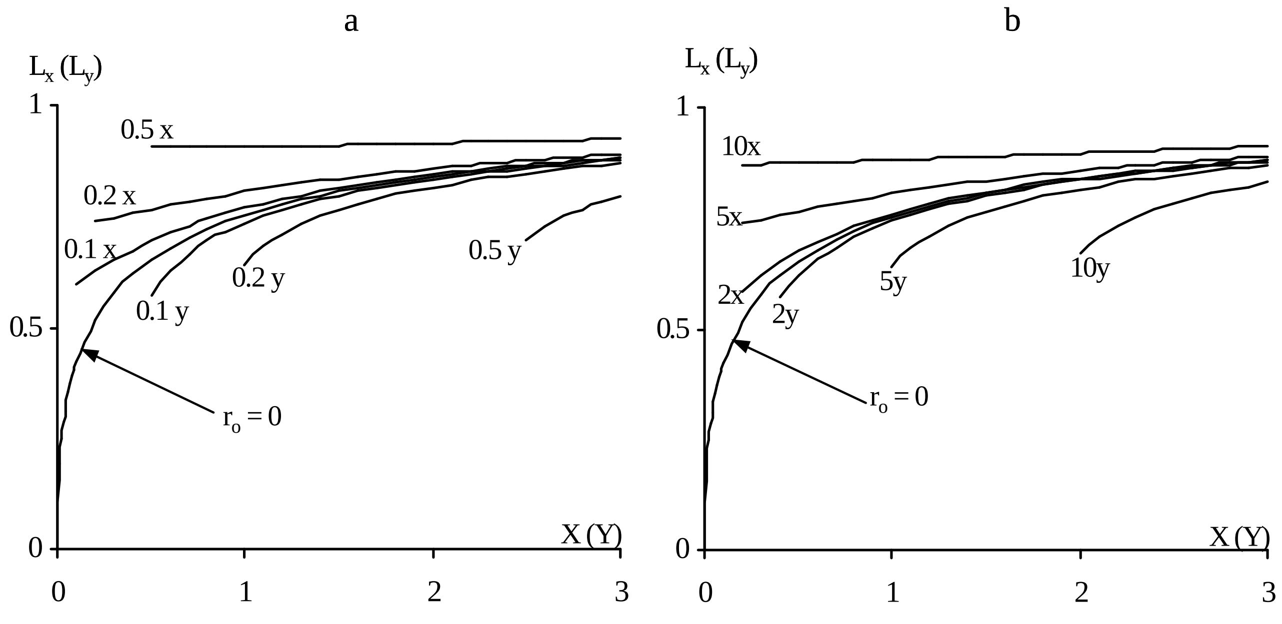

Fig. 4. Dependence of L_x and L_y on electrode radius r_o, anisotropy and on the distance between the current electrode and the potential calculation point in the normalized coordinates system *X*, *Y*. "*x*" next to curve indicate that this curve is the dependency of L_x on *X*, "*y*" - that this curve is the dependency of L_y on *Y*. All curves are calculated when λ_x=1.0mm. **a:** dependencies of L_x and L_y on r_o, when A_e=5, and the current electrode is divided into point-shaped sources by mode *3*. Numbers *0.1*, *0.2* and *0.5* next to curves show the value of r_o in mm. **b:** dependence of L_x and L_y on A_e, when r_o=0.2mm, and the current electrode is divided into point-shaped sources by mode *3*. Numbers *1*, *2* and *5* next to curves show A_e value

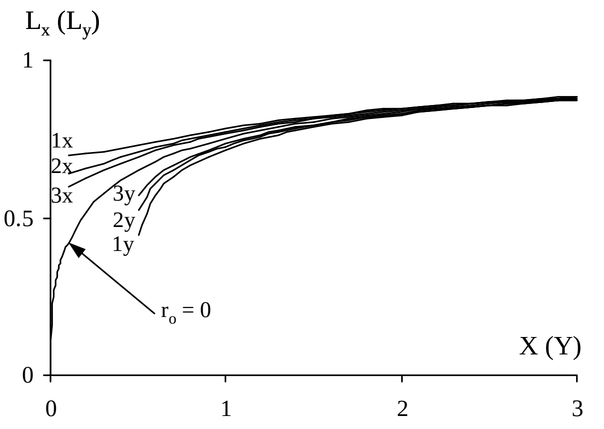

Fig. 5. Dependence of L_x and L_y on the mode of dividing and on the distance between the current electrode and the potential calculation point in the normalized coordinates system *X*, *Y*. "*x*" next to curve indicate that this curve is the dependency of L_x on *X*, "*y*" - that this curve is the dependency of L_y on *Y*. All curves are calculated when λ_x=1.0mm, A_e=5, r_o=0.1mm. Numbers *1*, *2* and *3* next to curves show the mode of dividing the current electrode into point-shaped sources

For stable values of A_e and r_o (see fig. 5), the obtained values of L_x are greatest in mode *1* and smallest in mode *3*, while values of L_y are the smallest in mode *1* and the greatest in mode *3*. For stable values of A_e, in the presence of higher values of r_o, we obtain higher values of L_x and lower values of L_y.

Errors of the evaluation of space constants of electrotonic decay

When the distribution of electrotonic potential is measured in experimental conditions, the parameters of anisotropic medium – experimental space constants of electrotonic decay λ_{xe}, λ_{ye} – are obtained. For evaluation of true values of electrotonic decay (λ_x, λ_y) from experimentally measured λ_{xe} and λ_{ye}, normalized half-axis of disk-shaped current electrode – r_o/λ_x and r_o/λ_x – are necessary, which are unknown. Therefore in practice for evaluation of true values of space constants of electrotonic decay (λ_x, λ_y) from experimentally measured λ_{xe} and λ_{ye}, the model of two-dimensional medium with point-shaped current source is applied. In cause of application of inadequate model the possibility of errors arises.

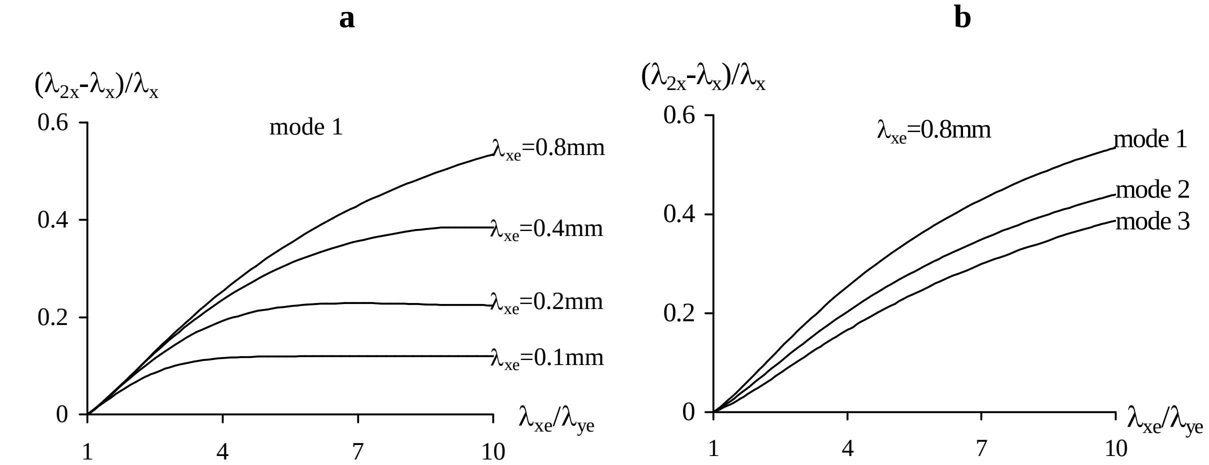

Fig. 6. Dependence of relative errors of space constants of electrotonic decay (λ_x) on λ_{xe}, electrotonic anisotropy ($\lambda_{xe}/\lambda_{ye}$), mode of division of current electrode into point-shaped sources. For all cases radius of current electrode is fixed (r_o=0.15mm), λ_{xe} and λ_{ye} are measured at the same distance (0.35mm) from the current electrode center. λ_{2x} – space constant of electrotonic decay obtained with aid of two-dimensional medium model, when current source is point-shaped. **a:** a current electrode is divided into point-shaped sources by mode *1*; **b:** comparison of errors for different modes of division of current electrode into point-shaped sources

When the model of two-dimensional isotropic medium is applied, the values λ_{2x} and λ_{2y} will be obtained, instead of true space constants of electrotonic decay λ_x and λ_y. In *X*–axis direction the relative error (*RE*) is $RE = (\lambda_{2x} - \lambda_x)/\lambda_x$, while in direction of *Y*-axis – $RE = (\lambda_{2y} - \lambda_y)/\lambda_y$. In Fig. 6 and 7 the magnitude of possible errors is presented when λ_{xe} and λ_{ye} were measured in two-dimensional anisotropic medium at 0.35 mm from a current electrode of radius 0.15 mm, while "true" space constants of electrotonic decay are evaluated by using the model of two-dimensional anisotropic medium with point-shaped current source. When electrotonic anisotropy (A_{em}) increases, a relative error increases in all modes of division of current electrode into point-shaped current sources: for mode *1* the errors are maximal, while for mode *3* the errors are minimal. The errors of evaluation of λ_x can reach up to +50 percents (see Fig. 6), while errors of evaluation of λ_y are little and do not exceed –4% (see Fig. 7).

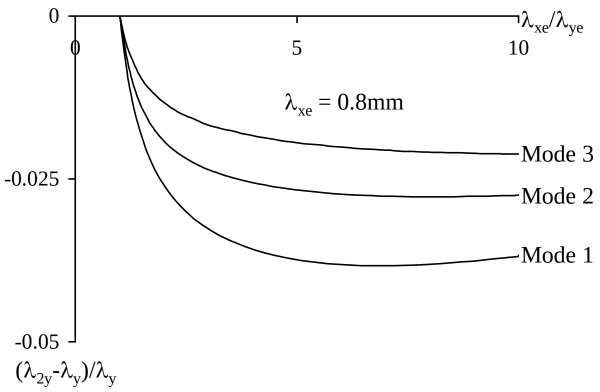

Fig. 7. Dependence of relative errors of λ_y on electrotonic anisotropy ($\lambda_{xe}/\lambda_{ye}$), on mode of division of current electrode into point-shaped sources, when radius of current electrode is fixed (r_o=0.15mm), λ_{xe} and λ_{ye} are measured at the same distance (0.35mm) from the current electrode center. λ_{2y} - space constant of electrotonic decay obtained with aid of two-dimensional medium model, when current source is point-shaped. The calculations were performed when λ_{xe}=0.8mm

Some aspects of resistive media models application for experimental data estimation

In the experimental recordings of the electrotonic potential distribution in the myocardial tissue, a circle-shaped suction electrode with internal perfusion of isotonic *KCl* [13] is applied. When microelectrode as the current electrode is applied, the amplitude of the electrotonic potential moving away from the current electrode abruptly decreases and it is impossible to precisely measure the values of λ_{xe} and λ_{ye}.

Some authors tried to apply the camera partition method to thin cylindrical-shaped pieces of the tissue excised from myocardium [15, 16]. However, when the electrotonic anisotropy of the tissue is great, the values of the measured true space constant of electrotonic decay strongly depend on the direction of excision.

By using a suction electrode as a current electrode, we avoid the above-mentioned drawbacks, but other problems arise. Due to the fact that the current flows not only into intracellular space but in intercellular clefts too, furthermore, the ratio of these currents is unknown, we cannot adequately mathematically describe a current electrode electrical structure. Therefore, in the calculations we used the superposition principle according which the current electrode is divided into evenly distributed point-shaped sources, and a potential generated at some site of the medium is equal to the sum of the potentials generated by these point-shaped sources. However, due to the anisotropy of the myocardial tissue intracellular and extracellular space [17], we cannot state that point-shaped sources are distributed evenly. Therefore, we chose three modes for current electrode division into point-shaped current sources and discovered that this factor has a noticeable influence.

Conclusions

1. Using the superposition principle of electrostatics the two-dimensional resistive media models were created and analytical solutions of an electrotonic potential in respect to distance were obtained, when a current electrode is circle- or disk-shaped. Computer programs for the calculation of the normalized experimental space constants (L_x, L_y) were created and their evaluation errors were obtained.
2. When the distance between the current electrode center and the L_x, L_y recording site is fixed, then for bigger current electrode radius and/or electrotonic anisotropy, L_x value is bigger and L_y value is smaller. When the distance between a current electrode and the potential recording site increases, L_x and L_y increases too.
3. In two-dimensional resistive media, L_x, L_y depend on current electrode internal structure and always L_x is greater than L_y. The greatest difference between L_x and L_y is observed in case of regular distribution of point sources on perimeter of current electrode in metric coordinate system. The smallest difference – when point sources are distributed evenly on the surface of the current electrode.

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Computer programs have been developed to calculate the distribution of electrotonic potential in a two-dimensional anisotropic medium when the source of current is point-shaped, circle-shaped and disk-shaped. Families of curves have been obtained to demonstrate the dependence of the normalized experimental space constants of electrotonic decay on anisotropy, distance between the electrotonic potential recording site and the source of current, the size of the source of current. Ill. 7, bibl. 17 (in English; summaries in English, Russian and Lithuanian).

Р. Ветейкис. Теоретическая оценка постоянных длины электротонического затухания в омических анизотропных средах: двумерный случай // Электроника и электротехника. – Каунас: Технология, 2009. – № 1(89). – С. 59–64.

Разработаны компьютерные программы для расчета распределения электротонического потенциала в двумерной анизотропной среде, когда источник тока имеет форму точки, окружности и диска. Получены семейства кривых, описывающие зависимости нормализованных экспериментальных констант электротонического затухания от анизотропии, размеров токового электрода, расстояния между точкой регистрации электротонического потенциала и электродом. Ил. 7. библ. 17 (на английском языке; рефераты на английском, русском и литовском яз.).

R. Veteikis. Elektrotoninių gesimo konstantų teorinis įvertinimas ominėse–talpinėse anizotropinėse terpėse: dvimatės terpės atvejis // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2009. – Nr. 1(89). – P. 59–64.

Sukurtos kompiuterinės programos elektrotoninio potencialo pasiskirstymui dvimatėje anizotropinėje terpėje apskaičiuoti, kai srovės šaltinis yra taškinis, apskritimo arba disko formos. Gautos kreivių šeimos, atvaizduojančios matuojamųjų normalizuotų elektrotoninio gesimo konstantų priklausomybę nuo anizotropijos, atstumo tarp potencialo registravimo vietos ir srovės šaltinio dydžio. Il. 7, bibl. 17 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).