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Peculiarities of Digital Infinite Impulse Response Filters Design

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Introduction

Signal filtering is essential in control, telecommunication, data gathering, measurement and other systems. Pure analog systems are replaced with mixed signal systems, because of the prices and power consumption of microcontrollers and digital signal processors are dropping. It tends growing signal processing sizes in the systems digital part.

Digital signal processing is critical in the systems, which require physical space and power consumption minimization. The example of such system is the system of physiological parameters monitoring in a real time [1]. With an eye to systems physical space minimization, the main signal processing part is handed in a digital way.

Infinite impulse response digital filters are more efficient in comparison with finite impulse filters and application of these filters is limited with non linear phase and in a rare case – non stability. Infinite impulse response digital filters are chosen in many applications there the linear phase is not necessary, because of calculation efficiency.

Demand of software for easy and flexible digital filters design is growing with growing digital filter application sphere.

Transfer function realization

There are several ways for digital infinite impulse response transfer function realization. Each of realization has its own noises level and different sensitivity for coefficients quantization. It is very important when digital filters are used in fixed point digital signal processors. These processors are chosen in many applications because of low power consumption and low prices.

First realization of digital infinite impulse response filters is:

$$H(z)=\frac{Y(z)}{X(z)}=\frac{\sum_{i=0}^{N}a_i z^{-i}}{\sum_{i=0}^{N}b_i z^{-i}}; \qquad (1)$$

where $b_0=1$.

Output is picked up, if first direct realization is chosen:

$$y(n)=\sum_{i=0}^{N}a_i x(n-i)-\sum_{i=1}^{N}b_i y(n-i). \qquad (2)$$

Second direct realization is derived by introducing intermediate expression G(z):

$$H(z)=\frac{Y(z)}{G(z)}\cdot\frac{G(z)}{X(z)}; \qquad (3)$$

where

$$\frac{Y(z)}{G(z)}=\sum_{i=0}^{N}a_i z^{-i} \qquad (4)$$

and

$$\frac{G(z)}{X(z)}=\frac{1}{1+\sum_{i=1}^{N}b_i z^{-i}}. \qquad (5)$$

Output is picked up:

$$y(n)=\sum_{i=0}^{N}a_i g(n-i) \qquad (6)$$

and

$$g(n)=x(n)-\sum_{i=0}^{N}b_i g(n-i). \qquad (7)$$

Direct filter transfer functions realizations are very sensitive for coefficients quantization, particularly for higher order filters. So, second order filter cascade realization was chosen:

$$H(z)=\frac{Y(z)}{X(z)}=\frac{G_1(z)}{X(z)}\cdot\frac{G_2(z)}{G_1(z)}\cdots\frac{Y(z)}{G_K(z)}= \\ =H_1\cdot H_2\cdots H_K=\prod_{i=1}^{K}H_i(z). \qquad (8)$$

H_i – second order sections:

$$H_i(z)=\frac{a_{0i}+a_{1i}z^{-1}+a_{2i}z^{-2}}{1+b_{1i}z^{-1}+b_{2i}z^{-2}}. \qquad (9)$$

n/2 quadratic sections for a cascade realization are necessary for even order filter. If filter is odd order, there are need of (n-1)/2 quadratic sections and one first order section. n is filter order.

The analog filter transfer function

Digital infinite impulse response filters are designed according to duplicate analog filter transfer functions, by doing shift from Laplase to z transformation. So, transfer function has to be made of normalized (cutoff frequency ω_c=1) analog lowpass filter. Cascade filter realization is chosen. So, it has to be made second order transfer function sections for even order filter. One first order section has to be made if filter order is odd. Ad hoc stabile poles and zeroes are calculated.

Stabile Butterwoth filter poles could be found:

$$s_K = -\sin\frac{(2K+1)\pi}{2n} + j\cos\frac{(2K+1)\pi}{2n}; \qquad (10)$$

where n – filter order, K=0,1,2...n-1.

Butterworth transfer function is made if poles are known:

$$H(s) = (-1)^n \prod_{K=0}^{n-1} \frac{s_K}{s - s_K} \; . \qquad (11)$$

Poles are symmetrical across real axis. For making second order section poles s_i and $s_{n-1-i,}$ are multiplied. Here i=0,1,..,n/2-1. So the imaginary part is eliminated. Section of quadratic normalized Butterworth transfer function practical could be made:

$$H_i(s) = \frac{1}{s^2 + 2s_{rei}\cdot s + 1}; \qquad (12)$$

where s_{rei} – i Butterworth pole real part, i=0,1..n/2-1.

Chebyshev poles could be found:

$$s_K = \sin\left((2K+1)\frac{\pi}{2n}\right)\cdot\sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\cos\left((2K+1)\frac{\pi}{2n}\right)\cdot\cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right); \qquad (13)$$

$$\varepsilon = \sqrt{10^{0.1Ap} - 1}; \qquad (14)$$

where n – filter order, A_p – passband ripple.

Chebyshev filter transfer function could be made if filter order is odd:

$$H(s) = -\prod_{K=0}^{n-1} \frac{s_k}{s - s_K}; \qquad (15)$$

if filter order is even:

$$H(s) = \frac{1}{\sqrt{1+\varepsilon^2}} \prod_{K=0}^{n-1} \frac{s_k}{s - s_K}. \qquad (16)$$

Made functions are normalized lowpass filter functions. To get filter with another cutoff frequency (not 1 rad/s), frequency transformation has to be made.

Analog filter frequency transformation

Frequency transformation is made for filter cuttoff frequency denormalization and for replacing lowpass filter with highpass, bandpass or bandstop [4].

Laplase operator s is replaced for lowpass filter denormalization this way:

$$s \to \frac{s}{\omega_a}\;; \qquad (17)$$

where ω_a – passband cutoff corner frequency.

For replacing lowpass filter to highpass, with cutoff frequency ω_a, Laplase operator must be replaced with:

$$s \to \frac{\omega_a}{s}\;. \qquad (18)$$

Lowpass filter transformation to bandpass is made:

$$s \to \frac{s^2 + \omega_v^2}{Bs}; \qquad (19)$$

where ω_v – geometrical mean of upper ω_{au} and lower ω_z corner frequencies:

$$\omega_v = \sqrt{\omega_{au}\omega_z}\;. \qquad (20)$$

B – bandwidth of the filter is

$$B = \omega_{au} - \omega_z\;. \qquad (21)$$

For lowpass filter transformation, Laplase operator is replaced:

$$s \to \frac{Bs}{s^2 + \omega_v^2}\;. \qquad (22)$$

Next step in digital filter design is transfer function z transformation.

Digital filter design according analog filter transfer functions

It is necessary at first get transfer function of analog filter with needed characteristics and then transform it to z plane for infinite impulse response filter design. Ad hoc step invariant, impulse invariant and bilinear transformations could be used. It is hard to implement impulse invariant or step invariant transformations and they could be used just for a lowpass and bandpass filter design. Therefore, bilinear transformation was chosen, which could be used in lowpass, highpass, bandpass and bandstop filter design.

To get digital filter transfer function, H(z), Laplase operator in analog denormalized transfer function should be replaced with:

$$s \to \frac{2}{T}\cdot\frac{z-1}{z+1}; \qquad (23)$$

where T – period of discretization.

Frequency characteristics are warped by shifting from s to z transform. Therefore frequencies in a transfer functions should be recalculated:

$$\omega_a = \frac{2}{T}\tan\frac{\omega_s}{2T}; \qquad (24)$$

where ω_a – analog frequency, which is used for filter denormalization. ω_s – digital frequency. T – period of discretization.

Transfer function of digital second order filter is [3]:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{a_{i0} + a_{i1}z^{-1} + a_{i2}z^{-2}}{1 + b_{i1}z^{-1} + b_{i2}^{-2}}; \qquad (25)$$

After bilinear transformation and after readjustment for a digital second order filter coefficients to get form as in 25, Butterworth lowpass filter numerator coefficients are calculated by derived formulas:

$$a_{i0} = \frac{\omega_a^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2}\;; \qquad (26)$$

$$a_{i1} = \frac{2\omega_a^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2} \; ; \qquad (27)$$

$$a_{i2} = \frac{\omega_a^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2} \; . \qquad (28)$$

Denumerator coefficients:

$$b_{i1} = \frac{2\omega_a^2 - 8f_d^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2} \; ; \qquad (29)$$

$$b_{i2} = \frac{4f_d^2 - 2kf_d\omega_a + \omega_a^2}{4f_d^2 + 2kf_d\omega_a + \omega_a^2} ; \qquad (30)$$

where ω_a – according 24 recalculated cutoff frequency; f_d – discretization frequency.

Quantity k could be found: $k = 2 \cdot \left| s_{rei} \right|$. (31)

S_{rei} – real part of pole i.

The same operations were made to get Butterworth highpass, and Chebyshev, Cauer lowpass and highpass digital filter coefficients expressions.

To derive bandpass filter calculation formulas operator s were replaced:

$$s \to B\left(\frac{1 - 2Az^{-1} + z^{-2}}{1 - z^{-2}}\right); \qquad (32)$$

where

$$A = \frac{\cos\dfrac{2\pi(f_a + f_z)}{2f_d}}{\cos\dfrac{2\pi(f_a - f_z)}{2f_d}} . \qquad (33)$$

Quantity B:

$$B = \cot\left(\frac{2\pi(f_a - f_z)}{2f_d}\right); \qquad (34)$$

where f_a – upper cutoff frequency, f_z – lower cutoff frequency, f_d – discretization frequency.

For finding bandstop digital filter transfer function operator s should be replaced with:

$$s \to B\left(\frac{1 - z^{-2}}{1 - 2Az^{-1} + z^{-2}}\right). \qquad (35)$$

In this case A is calculated from 33 and B

$$B = \tan\left(\frac{2\pi(f_a - f_z)}{2f_d}\right). \qquad (36)$$

Readjustments were made to get digital transfer function denumerator coefficient by z in zero degree value become equal one after z transformation. Expression was obtained after this readjustment of all other coefficient calculation. Fourth order digital bandpass and bandstop filter sections are obtained from quadratic analog filter sections.

Obtained coefficients expressions were used in software for digital filter design development.

Digital filter design software

Software for digital filter design was developed in C++ programming language. System for object – oriented software development – C++ Builder was used for these purposes. User interface of this program is presented in figure 1.

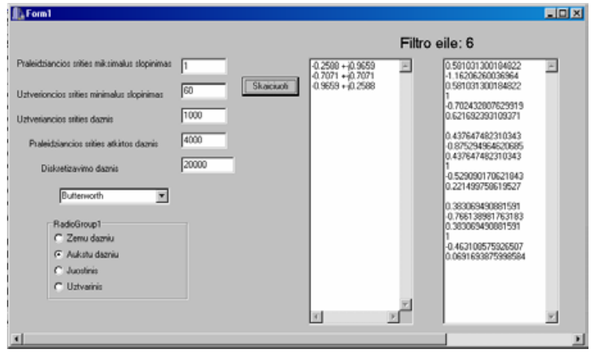

Fig. 1. Interface of software for digital filter design

User has to set desirable filter characteristics, such as: maximal passband attenuation or ripple, minimal stopband attenuation, and frequencies of passband edge stopband edge and discretization frequency for digital filter design. Filter type: Butterworth, Chebyshev or Cauer has to be chosen by scrollable list component ComboBox. Filter frequency characteristic: lowpass, highpass, bandpass or bandstop has to be chosen by alternative choice component – RadioGroup. After filter parameters is chosen, filter design is made by pushing one button. Filter coefficients are shown in the C++ Builder text display component Memo. The output of these coefficients is made in a text file as well. Output of stabile poles is made into another text component Memo. Output of normal poles, zeroes and digital filter coefficients is made in dynamic arrays.

Verification of software

Verification was made by doing mathematical modeling. Ad hoc routine of MathLab 6.5 - Simulink was used. The scheme of modeling blokes used in verification is presented in figure 2. Here number of Discrete filter blocks depends on filter order.

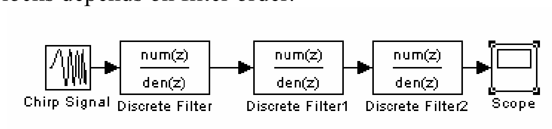

Fig. 2. Block scheme of verification

Output of Chirp Signal block is unity amplitude, linearly growing in time frequency sine wave. Designed filter coefficients are brought into Discrete Filter block. Output signal is graphically showed in a Scope block.

Output of modeling results example is shown in figure 3 for designed Chebyshev lowpass filter with such characteristics:

Passband ripple – 2dB;
Stopband minimal attenuation – 60 dB;
Passband cutoff edge frequency – 2 KHz;
Stopband edge frequency – 8 KHz;
Discretization frequency – 30 KHz.

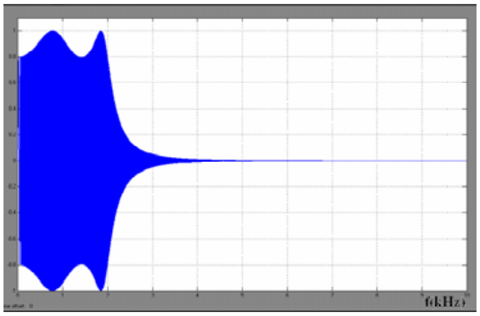

Fig. 3. Chebyshev lowpass filter modeling results

Obtained passband ripple is 2.04 dB, attenuation at 8 KHz frequency – 72 dB. Verification was made for different discretization frequencies and different filter characteristics. Made results prove adequacy of set and obtained characteristics.

Analog active filter design

There was applied method for digital filters design from analog filters transfer functions. Analog filter normalized poles, zeroes and transfer function coefficients are obtained in digital filter design process. They are also used for active analog filters design. So, the part of this software could be used for analog active filter design – for finding of R, C active filter element values.

The future work is to develop complex digital infinite impulse response and analog active filter design software using normal poles and zeroes and transfer functions coefficients finding algorithms.

Conclusions

1. Cascade filter realization is chosen to minimize sensitivity of coefficient quantization. It minimizes limitations for filter using in fixed form digital signal processors.
2. Bilinear z transform was chosen because of possibility to apply it in miscellaneous frequency characteristic filter design.
3. Results of filter mathematical modeling showed adequacy of set and obtained filter characteristics.
4. Algorithms for finding of normalized poles, zeroes and transfer functions coefficients could be used for active analog filter design.

Literature

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R. Lukočius, J. A. Virbalis. Skaitmeninių begalinės delsos filtrų projektavimo ypatumai // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 1 (65). – P. 52 – 55 .

Skaitmeninis filtravimas ypač aktualus sistemom, kuriom svarbus sistemos fizinis dydis ir energijos sąnaudos. Tokios sistemos pavyzdys – fiziologinių parametrų sekimo įprastinėje veikloje sistema. Begalinės delsos impulsiniai filtrai dėl savo skaičiavimo efektyvumo plačiausiai taikomi ten, kur nebūtina tiesinė dažninė fazės charakteristika. Aptartos pagrindinės skaitmeninių filtrų perdavimo funkcijų realizacijos galimybės ir pateiktų realizacijų koeficientų jautrumo kvantacijoms palyginimai. Taip pat pateikta analoginių filtrų normalizuotų polių bei perdavimo funkcijų koeficientų nustatymo seka. Parodyta skaitmeninių filtrų projektavimo pagal analoginių filtrų funkcijas seka. Vadovaujantis šiuo metodu C++ programavimo kalba sukurta programinė įranga, skirta skaitmeniniams filtrams projektuoti. Pateikti programa suprojektuotų filtrų patikrinimo, atliekant matematinį modeliavimą, rezultatai. Il. 3, bibl. 4 (anglų kalba; santraukos lietuvių, anglų ir rusų k.)

R. Lukočius, J. A. Virbalis. Peculiarities of Digital Infinite Impulse Response Filters Design // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 1 (65). – P. 52 – 55.

Digital filtering is very important in systems, which require minimal physical space and low power consumption. Example is system of physiological parameters monitoring in a real time. Infinite impulse response filters are widely used where linear phase are not critical, because of calculation efficiency. There are presented digital filter transfer functions realizations and given comparison of sensitivity of coefficients quantization for any actual realization. Algorithm for calculating normal poles and zeroes of analog filter and coefficients of transfer function subjected. Order for digital filter design from analog normalized transfer filter function shown. Software, according this method was made in C++ programming language, intended for digital filter design. The results are presented of this software designed filters testing. Ill. 3, bibl. 4 (in English; Summaries in Lithuanian, English, Russian).

Р. Лукочус, Ю. А. Вирбалис. Особенности проектирования цифровых рекурсивных фильтров. // Электроника и электотехника. – Каунас: Технология, 2006. - № 1 (65). – С. 52 – 55 .

Цифровые фильтры очень важны для систем, в которым большое значение имеют такие требования как миниатюризация и затраты энергии. Пример такой системы – система мониторинга физиологицеских параметров в реальном времени. Рекурсивные фильтры благодаря эфективности вычислений широко применяются там, где линейная фазная характеристика не нужна. Здесь представлены возможные реализации передаточных функций цифровых фильтров, сравнена чуствительность квантациям коэффициентов представленных реализаций. Также представлена последовательность для вычисления нормализированных полюсов аналоговых фильтров и для нахождения коэффициентов передаточной функции. Паказана последовательность для проектирования цифровых фильтров применяя передаточные функции аналоговых фильтров. Применяя этот метод в языке C++, создано программное обеспечение для проектирования цифровых фильтров. Представлены результаты тестирования этой программой спроектированных фильтров. Ил. 3, библ. 4 (на английском языке; рефераты на литовском, английском и русском яз.).