

## Calculation of the Hankel Matrix Ranks of Electric and Haemodynamic Processes in the Heart

**G. Kersulyte, Z. Navickas**

*Kaunas University of Technology,*

*Studentu str. 50, LT-51368 Kaunas, Lithuania, phone: +370 650 81853; e-mail: gintarei2001@yahoo.com*

**A. Vainoras, L. Gargasas**

*Institute of Cardiology, Kaunas University of Medicine*

*Sukileliu str. 17, LT-50009 Kaunas, Lithuania, phone: + 370 687 92517; e-mail: liudas.gargasas@kmu.lt*

### Introduction

The primary function of the heart is supply of blood and nutrients to the body. The regular beating or contraction of a heart moves the blood throughout the body. Each heartbeat is controlled by electrical impulses and in normal heart these electrical impulses occur at regular intervals, normally, the heart beats 60-100 times per minute [4]. When something goes wrong with heart's electrical system, a heart does not beat regularly. An irregular beating results a rhythm disorder, or arrhythmia. Arrhythmias could be frightening, but in many cases, especially in younger persons with normal underlying hearts, they are not life threatening and can be effectively treated with medications.

The design of effective heart disease diagnostic tools is needed to help medical personal to investigate cardio signals in details [7] [9] [10] [17] [21]. Construction of auto regression model for RR sequences has been the most widely accepted technique [14] [15]. Zemaityte, Varoneckas and et al maintained that night sleep with shifts of sleep stages and sustained fluctuations of autonomic functions as temperature, blood pressure, heart rate (HR), etc. can be analyzed as chaotic process [23]. Authors think that analysis of HR in terms of chaotic dynamics might be useful for investigation of autonomic HR control during sleep. Bonaduce et. al accepted, that evaluation of autonomic heart rate (HR) control, measured by means of HR variability, might be important characteristic of cardiovascular function [1] [2]. Irregular ventricular rate can lead progressively to the development of cardiomyopathy with symptoms and signs of heart failure. Because of disturbed haemodynamic, atrial fibrillation and atria flutter are between of most usual causes of thrombi-embolic events [20]. HR Poincare plots might be as easy and informative method for visual presentation of HR method for visual presentation of HR variability changes during process of treatment [16]. Poincare plots based on RR interval sequences were shown

to display patterns specific for normal sinus rhythm and for HR disturbances [19] [22] too. Piskorski and Guzik showed that there was a visible and statistically highly significant asymmetry in Poincare plot, with an upper part, corresponding to decelerations of heart rate, larger than lower part, which corresponds to accelerations. The effect was shown in one hundred 30 min long time series of RR intervals derived from ECG recordings of 100 young (19-32 years old) and healthy adults. After shuffling data to random order asymmetry disappears, this shows that this is a genuine physiological phenomenon rather than an artifact of the method [13].

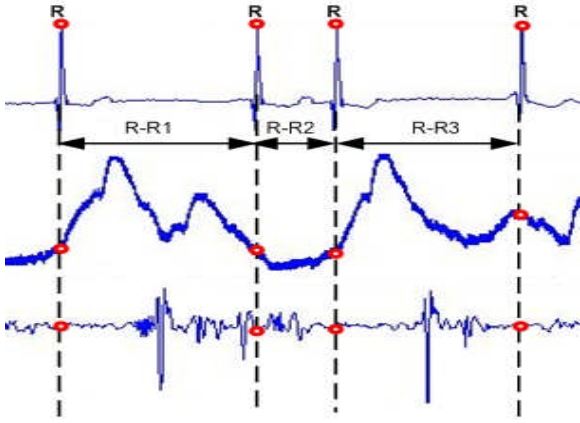
The main idea of this paper is to adapt Hankel matrix ranks to describe interpersonal and intrapersonal relationship of cardio signals. Sequences of RR intervals evaluated with ranks were used for Poincare plots. This technique was applied for cardio signals of 85 persons.

The work is divided in three sections. The first theoretical section describes mathematical reasoning of H ranks evaluation. In the second section computational results are showed. Conclusions delivered in the last section.

### Hankel matrices for system identification

Three cardio signals were analyzed: electrocardiogram (ECG), which reflects electric heart activity; impedance cardiogram (ICG) - reveals haemodynamic properties of the cardiovascular system; seismocardiogram (SCG) that shows changes of heart mechanic activity. These three signals were recorded synchronously, so they describe activity of person heart from three different sides.

Fig. 1 shows how data samples for analysis were collected. RR interval was defined in ECG signal in the first processing step. The others data samples from ICG and SCG signals were extracted according defined RR intervals.



**Fig. 1.** Data samples selected from ECG, ICG and SCG signals according to RR intervals

Usually, in system identification Hankel matrices are formed when given a sequence of output data and a realization of an underlying state-space or hidden Markov model is desired, but in this paper the ranks of Hankel matrix will be used as features for system identification purposes. Each RR interval of cardio-signal (ECG, ICG or SCG) consists of vector  $\vec{p} = (p_1, p_2, \dots, p_n)$ ,  $n > k$ . Then for every  $k$  and fixed  $p_j$  it is possible to construct the following matrix, which is called as Hankel matrix:

$$H_0^k = \begin{pmatrix} p_0 & p_1 & \dots & p_{k-1} \\ p_1 & p_2 & \dots & p_k \\ \dots & \dots & \dots & \dots \\ p_{k-1} & p_k & \dots & p_{2k-2} \end{pmatrix}. \quad (1)$$

A Hankel matrix is a matrix that is symmetric and constant across the anti-diagonals. Many alternative methods existed to test rank of Hankel matrix. Barlett rank of Hankel matrix estimation based on computation of canonical correlations [3]. Schwarz suggested an alternative penalty on increasing number of parameters of criterion in searching of rank [18]. Comba-Mendez G. and Kapetanios G. recently proposed statistical test of rank [5]. But the evaluation of H rank is complicated because of Hankel matrix size is rarely given exactly, therefore it must be computed. In this paper we use a determinant value of Hankel matrix. If for a sequence of numbers  $p_j$  exists a number  $m$  such that condition

$$m = \max_{k \in N} \text{rang} H_0^{(k)} \quad (2)$$

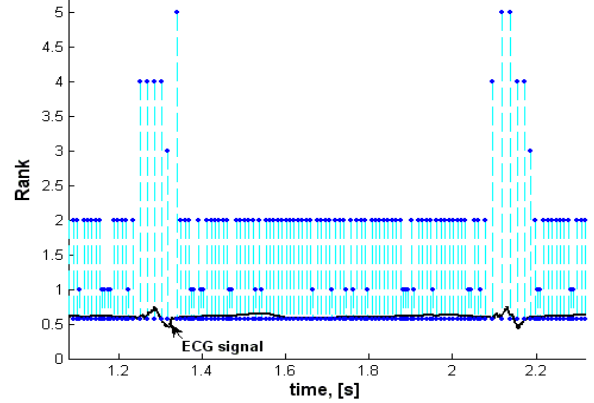
is satisfied, then sequence  $p_j$  has H-rank  $m$ . It is possible to find  $m$ , that  $\det H_0^{(m)} \neq 0$  and  $\det H_0^{(m+r)} = 0, \forall r \in N$  [12]. The computational results of applied mathematical reasoning are shown in the next section.

### Computational results

All signals used in this work are obtained using software of ECG analysis system "Kaunas – Load", developed in the Institute of Cardiology. All signal analysis techniques used in this paper are implemented on a PC using custom software developed in Matlab R2007b.

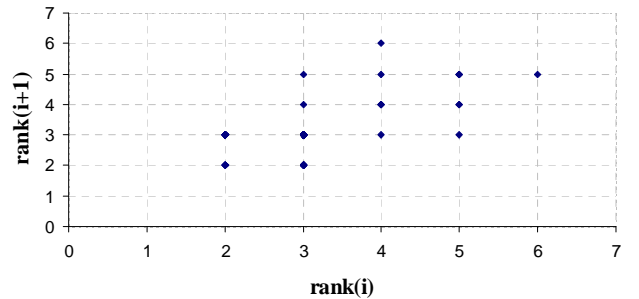
Contingent of study consisted of 78 patients with heart diseases and 7 persons who hadn't any big gripe about health.

As was mentioned, it is possible to construct Hankel matrix of RR and describe ECG, ICG or SCG signal with ranks of Hankel matrix. The higher rank value describes higher signal complexity in certain interval.



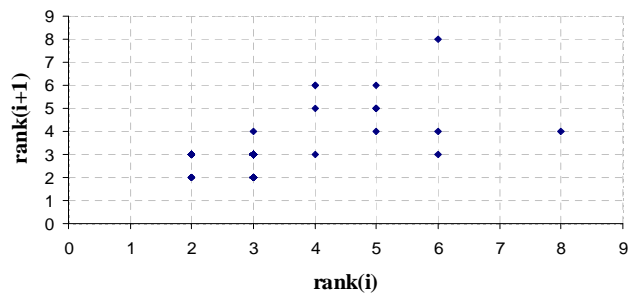
**Fig. 2.** An interval of 1 second of ECG signal evaluated with ranks

From numerical relation between ranks and computation step can be clearly visible that for describing of higher variation of signal the higher rank is needed. ECG signal defined by ranks is showed in Fig. 2, where ranks are on Y axis, and time - on X axis.



**Fig. 3.** An interval of RR of ECG signal evaluated with ranks of Poincare plot

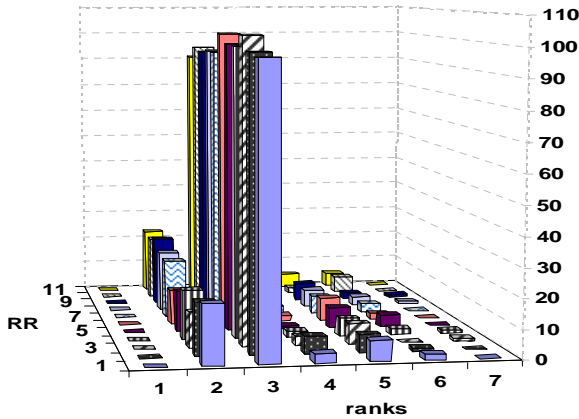
The amount of ranks for expression of all measured ECG signals is different because every ECG signal and length of RR is personal to each person. Sequences of RR intervals ranks were used for Poincare  $\text{rank}_i$  and  $\text{rank}_{i+1}$  plots (Fig. 3, 4).



**Fig. 4.** Poincare plot of RR interval of ECG signal evaluated with ranks

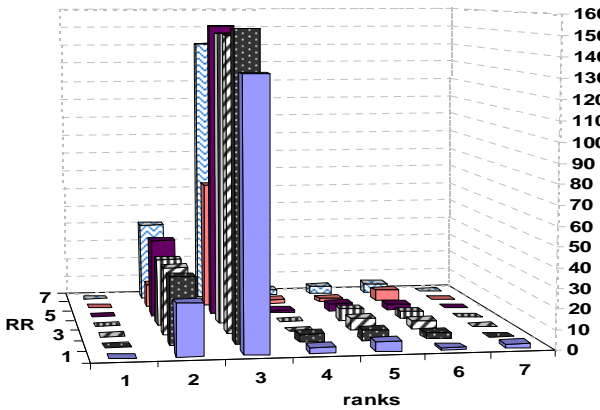
RR ranks Poincare plot are easy and informative method for visual presentation of RR ranks variability and

assessment of intrapersonal characteristics. Poincare plot shows that ECG signal is very complex because this signal is described with many different ranks (Fig. 3, 4).



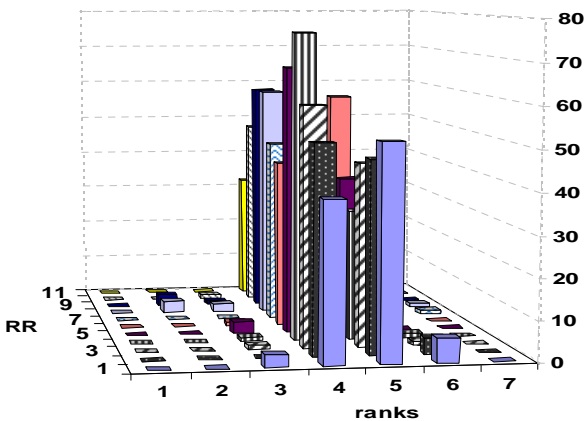
**Fig. 5.** 3D column plot compares 10 second (11 RR intervals) ECG values across RR interval and across number of ranks

Results were visualized 3D column for particular sequence of ranks of RR intervals (Fig. 5, 6).



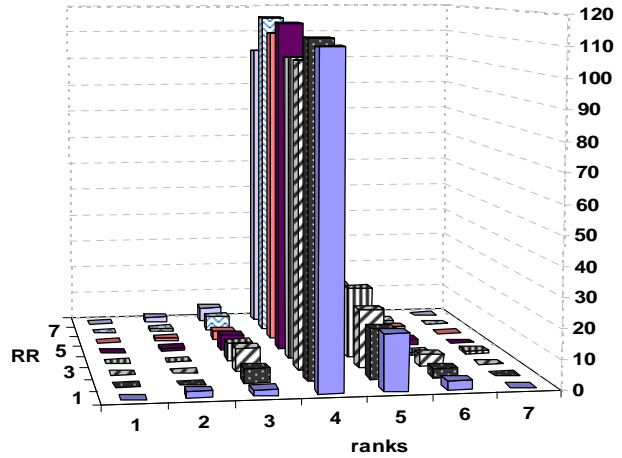
**Fig. 6.** 3D column plot compares 10 second (7 RR intervals) ECG values across RR and across number of ranks

It was noticed that irrespective of RR interval length, the main ranks of ECG are “3” and “2”. Ranks “4”, “5” or “6” characterize QRS complex (Fig. 5, 6).



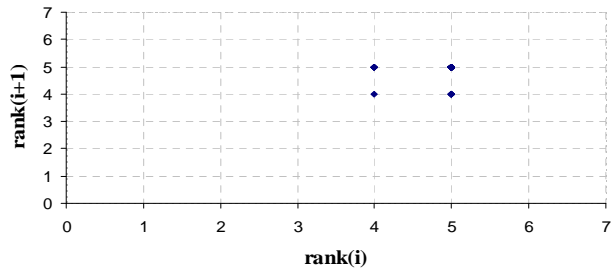
**Fig. 7.** 3D column plot compares 10 second (11 RR intervals) ECG values across RR and across number of ranks of “healthy” person

The main ranks of ECG of “healthy” person are “4” and “5” (Fig. 7, 8). Results of Fig. 7 shows that rank counting of expressing cardio signals with Hankel matrix can be useful for diagnostic purposes.



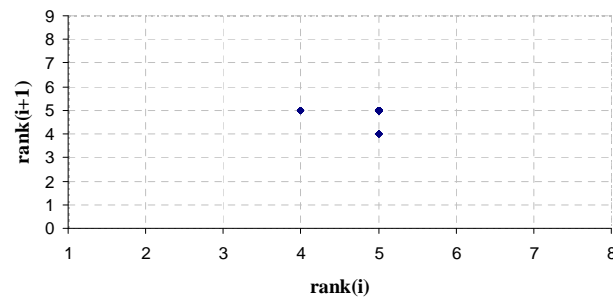
**Fig. 8.** 3D column plot compares 10 second (8 RR intervals) ECG values across RR and across number of ranks of “healthy” person

An ICG signal consists of two major components: one, reflecting respiratory movements and another, reflecting a blood flow in a chest. Decomposition of this signal into these two components could give a possibility to perform the detail analysis of ICG shape [6].

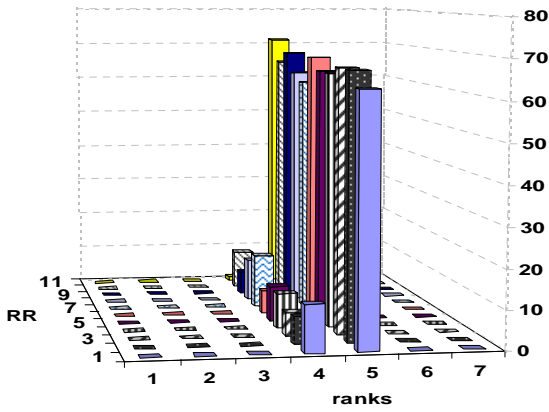


**Fig.9** An interval of RR of ICG signal evaluated with ranks of Poincare plot

Detail shape analysis of signal component caused by blood flow could reveal more important information about pump function of a heart and haemodynamics in general. However, in usual way based on frequency range separation of these two components is not optimal for this signal and does not allow performing the detail analysis [11].

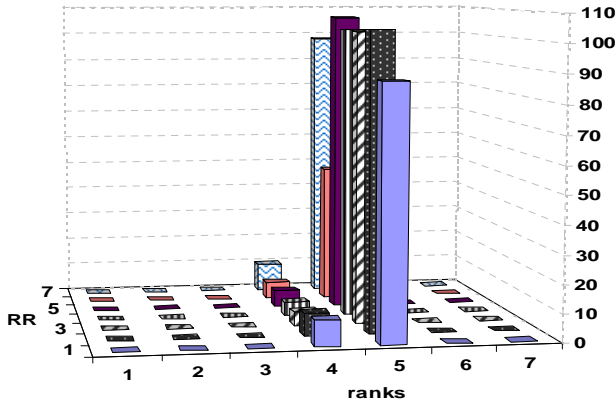


**Fig. 10.** Poincare plot of RR interval of ICG signal evaluated with ranks



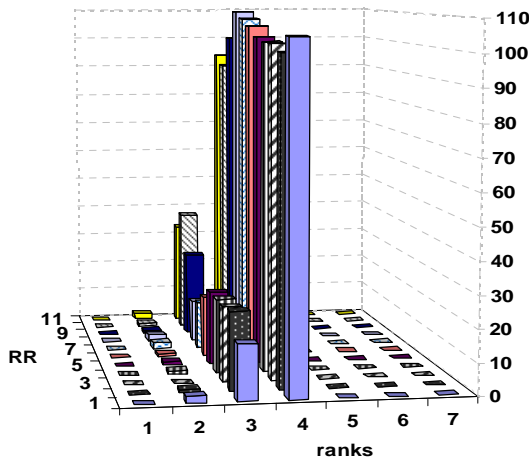
**Fig. 11.** 3D column plot compares 10 second (11 RR intervals) ICG values across RR and across number of ranks

Ranks of RR interval of ICG corresponding to any dot of interest on Poincare plot could be visualized too. In Fig. 9 and Fig. 10, each rank of ICG RR interval was plotted as a function of a previous rank of ICG RR interval. ICG signal of “sick” person consists of two ranks “4” and “5” only.



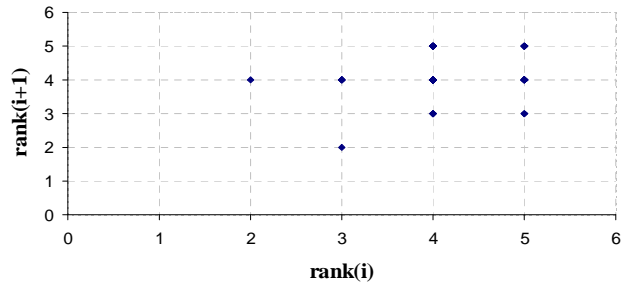
**Fig. 12.** 3D column plot compare 10 second (7 RR intervals) ICG values across RR and across number of ranks

It was noticed that irrespective of RR interval length, main rank of ICG is “5” (Fig. 11, 12). It means that in moment when ICG rank is “5”, ICG signal depends on some body processes, such as breathing, physical readiness or muscle tone, etc.



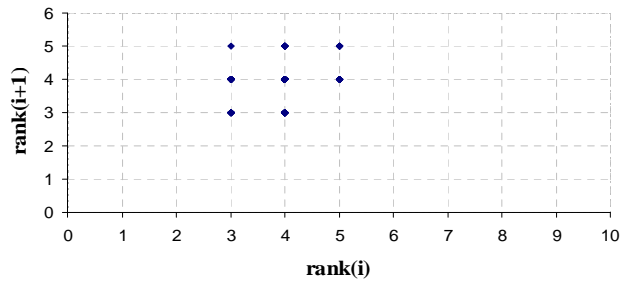
**Fig. 13.** 3D column plot compare 10 second (11 RR intervals) ICG values across RR and across number of ranks of “health” person

Ranks result of “healthy” persons (hadn’t any big gripe about health) were different. The main ranks of ICG of “health” person are “3” and “4” (Fig. 13).



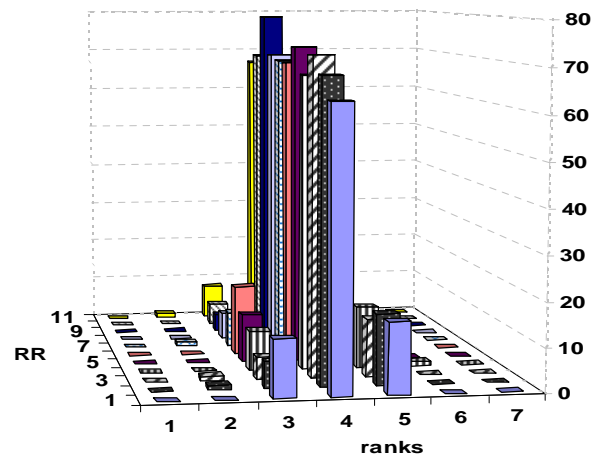
**Fig. 14.** An interval of RR of SCG signal evaluated with ranks of Poincare plot

Seismocardiography is a non-invasive technique developed for recording and analyzing of cardiac vibratory activity [11]. Mathematical model of craniocaudal forces developed by a mechanical activity of heart and blood flow through great arteries has been described in many papers [8].



**Fig. 15.** Poincare plot of RR interval of SCG signal evaluated with ranks

Poincare plot shows that SCG is more complex pattern than ICG (Fig. 9, 14, 15). The main ranks of SCG of “sick” persons are “3”, “4” or “5”.



**Fig. 16.** 3D column plot compares 10 second (11 RR intervals) SCG values across RR and across number of ranks

It was noticed that irrespective of RR interval length, the main rank of SCG is “4” for “sick” person (Fig. 16, 17).

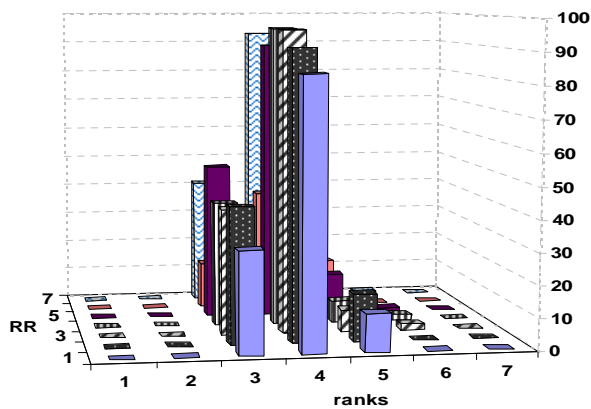


Fig. 17. 3D column plot compares 10 second (7 RR intervals) SCG values across RR and across number of ranks

Ranks results of “healthy” persons were not different. The main ranks of SCG of “health” person are “3” and “4” too (Fig. 18).

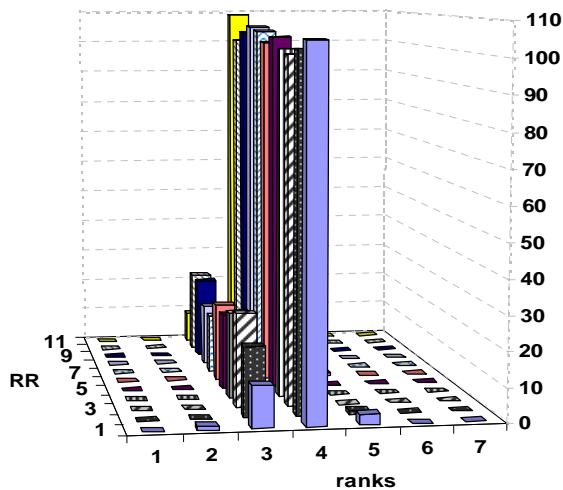


Fig. 18. 3D column plot compares 10 second (11 RR intervals) SCG values across RR and across number of ranks of “healthy” person

## Conclusions

The amount of ranks for expression of all measured signals is different because every signal and length of RR is personal to each person. Poincare plot of RR ranks is an easy and an informative method for visual presentation of RR ranks variability and assessment of intrapersonal characteristics. Results show that expressing of cardio signals with Hankel matrix could be useful for diagnostic purposes, because ranks counted in each RR interval separate “healthy” and “sick” persons groups.

## Acknowledgements

This work was supported by Lithuanian State Science and Studies Foundation (Contract B-02/2009)

## References

- Alia S. Khaled, Mohamed I. Owis, Abdalla S. A. Mohamed. Employing time-domain methods and poincare plot of heart rate variability signals to detect congestive heart failure // BIME journal. – Dec. 2006. – Vol. 06, Issue 1. – P. 35–41.
- Bonaduce D., Petretta M., Marciano F., Vicario M. L., Apicela C., Rao M. A., Nicolai E., Volpe M. Independent and incremental prognostic value of heart rate variability in patients with chronic heart failure // Am. Heart J. – Aug. 1999. – No. 138(2 Pt 1). – P. 273–84.
- Barlett M. Multivariate analysis // Journal of the royal statistical society. – Series B, No. 9. – P. 176–197.
- Craig A., McPherson M. D., Lynda E., Rosenfeld M. D. Heart rhythm disorders. Chapter 16. – Yale University School of medicine heart book. – P. 195–204.
- Comba-Mendez G., Kapetanios G. Testing the rank of the Hankel matrix: a statistical approach / ECB working paper. – March 2001. – No. 45. – P. 1–25.
- Drėgūnas K., Povilonis E. Cardiac output and hemodynamic monitoring system “Heartlab” // Biomedical engineering. Proceedings of International Conference. – Kaunas. – 1999. – P. 100–105.
- Gargasas L., Janusauskas A., Lukosevicius A., Vainoras A., Ruseckas R., Korsakas S., Miskinis V. Development of methods for monitoring of electrocardiograms, impedance cardiograms and seismocardiograms // Studies in health technology and informatics. – ISSN 0926-9630. – 2004. – Vol. 105. – P. 131–141.
- Halcik J., Mudr J. Mathematical Model of Seismocardiogram // World Congress on Medical Physics and Biomedical Engineering. – 2006. – Vol. 5, Track 21. – P. 3415–3418.
- Keršulytė G., Navickas Z., Blužas J., Gargasas L., Vainoras A., Ruseckas R., Sadauskas S., Naudžiūnas, A. Polycardiosignals coherence evaluation results for patients with cardiopulmonary diseases. Electronics and Electrical Engineering 2007, nr. 5(77). ISSN 1392-1215 p. 41-44.
- Keršulytė G., Navickas Z., Vainoras A., Gargasas L., Jaruševičius G. Analysis of cardiosignals cohesion based on Hankel matrix. Electronics and Electrical Engineering 2008, nr. 8(88). ISSN 1392-1215 p. 55-58.
- Kriščiukaitis A., Tamošiūnas M., Macas A., Bakšytė G., Braždžionytė J. Complex analysis of 24h simultaneous ECG and chest impedance signal recordings // Proceedings of International Conference. – Kaunas. – 2004. – P. 49–52.
- Navickas Z., Bikulčienė L. Expressions of solutions of ordinary differential equations by standard functions // Proceedings of the 10th International Conference Mathematical Modeling and Analysis 2005 and 2nd International Conference Computational Methods in Applied Mathematics, June 1-5, 2005, Trakai, Lithuania. – ISBN 9986-05-924-0. – Vilnius. – 2005. – P. 143–150.
- Piskorski J., Guzik P., Geometry of the Poincaré plot of RR intervals and its asymmetry in healthy adults // Physiol. Meas. – March 2007. – Vol. 28, No. 3. – P. 287–300.
- Saul J. P., Albrecht P., Berger R. D., et al. Analysis of long term heart rate variability: Methods, 1/f scaling and implications // Comp. in Cardiology. – 1987. – No. 14. – P. 419–422.
- Salerno D. M. Zanetti J. Seismocardiography: A new technique for recording cardiac vibration: concept, method, and initial observation // Journal of Cardiovascular Technology. – 1990. – No. 9. – P. 111–118.
- Silke B., Hanratty C. G., Riddell J. G. Heart rate variability effects of badrenoreceptor agonists (Xameterol, Prenalterol and Sulbutamol) assessed nonlinearly with scatter plots and sequence methods // J. Cardiovasc. Pharmac. – 1991. – No. 33. – P. 859–867.
- Siniša S. Ilic Detection of the left bundle branch block in continuous wavelet transform of ECG signal. Electronics and Electrical Engineering 2007, nr. 2(74). ISSN 1392-1215 p. 33-36.
- Schwarz G. Estimating the dimension of a model // Annals of statistics. – 1978. – No. 6. – P. 461–464.

19. **Ulbikas J., Čenys A., Žemaitytė D., et al.** Searching for the low dimensional noise in heart rate data // *Noise in Physical Systems and 1/f Fluctuations: Proceedings of the 13<sup>th</sup> International Conference* May 29 – June 3, 1995, Palanga. – Editors Bereikis V., Katilius R. – Singapore, New Jersey, London, Hong-Kong: World Scientific, 1995. P. 662.
20. **Vanoreckas G., Martinkenas A., Paskeviciute J., Podlipskyte A., Zemaityte, D.** Prediction of maintenance of sinus rhythm using heart rate variability characteristics after conversion in patients with atrial fibrillation // *Computers in cardiology*. September 25-28, 2005, Lyon, France. – 2005. – ISSN 0276-6547. – Vol. 32. – P. 77–80.
21. **Vainoras A., Gargasas L., Jurkonienė R., Jurkonis V., Jaruševičius G., Berškienė K., Navickas Z.** Analysis of electric cardiac signals - methods and application results. *Electronics and Electrical Engineering* 2008, nr. 5(85). ISSN 1392-1215 p. 81-84.
22. **Zemaitytė D., Vanoreckas G., Ozeraitis E. et al.** Heart rate Poincare plots and their hemodynamic correlates: discrimination between sinus and ectopic rhythms // *Biomedicine*. – December 2001. – Vol. 1, No. 2. – P. 80–89.
23. **Zemaityte D., Varoneckas G., Duoneliene I., Dauksys R.** Changes in the Cardiovascular function in healthy subjects and patients with ischemic heart disease during sleep // *Human Physiology*. – 2000. – Vol. 26, No. 6. – P. 692–702. – Translated from: *Fiziologiya Cheloveka*. – 2000. – Vol. 26, No. 6. – P. 50–61.

Received 2009 02 15

**G. Kersulyte, Z. Navickas, A. Vainoras, L. Gargasas. Calculation of the Hankel Matrix Ranks of Electric and Haemodynamic Processes in the Heart // *Electronics and Electrical Engineering*. – Kaunas: Technologija, 2009. – No. 3(91). – P. 43–48.**

Efficient diagnosis of cardiovascular system diseases could be gained by development of the new technologies for cardio signal analysis. The aim of the work was to adapt the analysis of the ranks evaluating the RR intervals from three synchronously recorded cardio signals – electrocardiogram (ECG), impedance cardiogram (ICG) and seismocardiogram (SCG). The interpersonal and intrapersonal analysis of ECG, ICG and SCG is performed basing on the ranks of Hankel matrix. The relatively different information is evaluated from the analysis of ranks based on Poincare plots. The results show that expressing cardio signals with Hankel matrix could be useful for development of diagnostic technologies, because ranks counted in each RR interval make possibilities for efficient classification of “healthy” and “sick” persons groups. Il. 18, bibl. 19 (in English; summaries in English, Russian and Lithuanian).

**Г. Кершулите, З. Навицкас, А. Вайнорас, Л. Гаргасас. Вычисление рангов матрицы Ханкеля для электрических и гемодинамических процессов сердца // *Электроника и электротехника – Каунас: Технология*. 2009. – № 3(91) – P. 43–48.**

Одним из способов решения проблем эффективной диагностики заболеваний сердечно-сосудистой системы является разработка новых технологий для анализа электрических сигналов сердца. Основная цель работы заключалась в том, чтобы приспособить анализ рангов для оценки и сопоставлении трех синхронно зарегистрированных электрических сигналов сердца – электрокардиограммы (ЭКГ), импеданс кардиограммы (ИКГ), сейсмокардиограммы (СКГ) при их разделении на отрезки, равные длительности интервала RR. При помощи рангов было исследовано интерперсональная и интраперсональная связь ЭКГ, ИКГ и СКГ сигналов. Для этой цели использован анализ Поинкаре кривых по рангам и в итоге получена полезная диагностическая информация. Полученные результаты позволяют сделать вывод, что выражение электрических сигналов сердца рангом матрицы Ханкеля может быть полезным для развития диагностических технологий, потому что такое описание интервала RR позволяет эффективно классифицировать группы практически здоровых лиц и больных с патологией сердечно-сосудистой системы. Ил. 18, библи. 19 (на английском языке; рефераты на английском, русском и литовском яз.).

**G. Keršulytė, Z. Navickas, A. Vainoras, L. Gargasas. Hankelio matricių rangų skaičiavimas elektriniams ir hemodinaminiamis širdies procesams // *Elektronika ir elektrotechnika*. – Kaunas: Technologija, 2009. – Nr. 3(91). – P. 43–48.**

Vienas iš efektyvių širdies ligų diagnostikos problemų sprendimo būdų – kurti naujas kardiosignalų analizės technologijas. Darbo tikslas buvo pritaikyti rangų analizę įvertinant bei lyginant tris sinchroniškai užregistruotus širdies elektrinius signalus – elektrokardiogramą (EKG), impedanskardiogramą (IKG) bei seismokardiogramą (SKG), suskaidant juos RR intervalo ilgio atkarpomis. Darbe pagal rangus buvo tiriamas EKG, IKG ir SKG interpersonalinis ir intrapersonalinis sąryšis. Sinchroniškai užregistruotų EKG, IKG ir SKG signalų interpersonalinėms ir intrapersonalinėms charakteristikoms įvertinti taikyta Poincare kreivių analizė pagal rangus leido gauti diagnostikai naudingos informacijos. Gauti rezultatai rodo, kad širdies signalų išreiškimas Hankelio matricos rangų gali padėti tobulinti diagnostines technologijas, nes toks RR intervalų aprašymas įgalina efektyviai klasifikuoti „sveikų“ ir „ligonių“ grupes. Il. 18, bibl. 19 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).

DOI: 10.5755/j02.eie.10498