ISSN 1392 – 1215 ELEKTRONIKA IR ELEKTROTECHNIKA. 2005. Nr. 6(62)

T 121 SIGNALŲ TECHNOLOGIJA

RKHS-Methods at Solving Some Radiophysics Problems

S. Chumachenko, V. I. Hahanov, O. V. Melnikova
AD Department, Kharkov National University of Radio Electronics,
Lenin Avenue, 14, Kharkov, Ukraine, 61166, phone: (380) 57- 70-21-326, e-mail: ri@kture.kharkov.ua, hahanov@kture.kharkov.ua

Introduction

Operating speed of digital logic devices depends on type of silicon: PLD, Gate Array or ASIC. FPGAs are the lowest in risk, low in project budget but have the highest cost per unit. Gate Arrays utilize less custom mask making than standard cell and stand in the middle from all of three and fallen from wide use today. Cell based ASICs have the highest performance and lowest cost per unit in case of mass production, but they also have the longest and most expensive design cycle. Also digital designs can be divided on CPU based systems on chip (SoC) and non CPU logic devices. CPU as universal processing unit can solve broad spectrum of various tasks from all areas of human activity. Nevertheless there are exist bottlenecks where CPU can't satisfy required performance. Usually it happens during implementation of mathematical tasks that require big number of iterations and hence big time expenses to obtain desired result with desired accuracy.

To increase efficiency of solving of computational tasks there are used mathematical co-processors. There implemented most efficient ways of computing equations, integrals, differential coefficients. It is obvious that after discovering of new methods of increasing computation accuracy and decreasing computation time it is necessary to re-implement mathematical co-processors or use new generation of IP-cores in PLD, Gate Array, ASIC designs. It is presented, easy to implement as IP-core, method of reduction of computation of certain types of series to exact function that is widely used during calculation of parameters of high radio frequency devices. Presented method decrease computation time of such tasks in tens and hundred times and its inaccuracy is equals to zero.

Statement of the problem

The investigation is based on fundamental works of Aronzajn [1], Razmahnin and Yakovlev [2]. Mathematical models based on Reproducing Kernel Hilbert Space methods are used in Wavelets Analysis, namely: at Pattern Recognition, Digital Data Processing, Image Compression, Computer Graphics; and also in Learning theory: for example, at Exact Incremental Learning, in Statistical Learning theory, in Regularization theory and Support Vector Machines [3-5]. In mentioned arias we have not deal with exact Series Summation because it isn't necessary for considered cases. We use sum and finite summation, not series. But there are areas of scientific study where exact series summation it is necessary.

For such problems Series Summation Method in RKHS - can be useful [6, 7]. We are going to point out these areas.

The purpose of the investigation is to originate a new Series Summation Method based on RKHS-theory and to demonstrate the new results which develop theoretical statements of Series Summation Method in RKHS.

The research problems are:

- Base statements of Series Summation in RKHS;
- New results for Series Summation, solving summatory and integral equations, proving integral identities.

Base theoretical statements and investigation essence

Reproducing Kernel Hilbert Space is a subspace of Hilbert space with Reproducing Kernel (RK). RK is a function Ker of two variables with two properties:

1) $\forall t \in T \quad Ker(s_0, t) \in T$;

2) $\forall f \in H$, $f(t) = < f(s), Ker(s,t) >$;

where <…> – inner (scalar) product can be represented as a series on selective values. There is an operator G, which transfers any function from Hilbert space L_2 into function from RKHS and leaves without change function from RKHS H . Thus there is an operator G which transfers any function from Hilbert space L_2 into function from RKHS and doesn't change any function from RKHS H .

For example, the functions with finite spectrum of cosine- and sine-transformations and also Hankel-transformation form RKHS. The basic research of expansion problem on selective values was executed by K. Shannon and V.A. Kotelnikov. There are statements determining particular cases RKHS. Thus, any function from RKHS can be represented as selective values expansion. If there is series where the common summand can be reduced to a standard form, – it means to extract reproducing kernel by equivalent transformations, – then for any series one can put in accordance a function from RKHS. In other words, a series can be summarized by known formulas.

Thus, the main idea of proposed method is to obtain and to use the following relation:

$$f(s) \;=\; \sum_k f(t_k)\, Ker(s,t_k)\,, \tag{1}$$

in right-hand side of this relation we can see a series on selective values of function $f(t)$; left-hand side represents value of function f in point s.

We use four base kinds of Reproducing Kernels, which originate four RKHS accordingly [2]:

1. RKHS H_{Ker_1} is a space of functions which have finite Fourier-transformations.

2. Space H_{Ker_2} contains a class of functions with finite Hankel-transformation.

3. Space H_{Ker_3} consists of functions with finite sine-transformations.

4. Space H_{Ker_4} has all functions those cosine-transformations are finite.

For these spaces there are four types of the Kernel Functions and Series on selective values accordingly.

Statement 1. Any function $f \in H_{Ker_1}$ in RKHS H_{Ker_1} is series expansion of selective values

$$f(s) = \sum_{k=-\infty}^{\infty} f(k)\frac{\sin\pi(s-k)}{\pi(s-k)}\,,\;\; -\infty < s < \infty\,. \tag{2}$$

Statement 2. Any function $f \in H_{Ker_2}$ in RKHS H_{Ker_2} is series expansion of selective values

$$f(s) = \sum_{k=1}^{\infty} f(t_k)\frac{2(st_k)^{1/2}}{\pi J_{\nu+1}(\pi t_k)}\frac{J_\nu(\pi s)}{(t_k^2 - s^2)}\,,\;\; 0 \le s < \infty\,, \tag{3}$$

where πt_k are positive zero points of cylindrical functions.

Statement 3. Any function $f \in H_{Ker_3}$ in RKHS H_{Ker_3} is series expansion of selective values

$$f(s) = \sum_{k=1}^{\infty} f(k)\frac{2k}{s+k}\frac{\sin\pi(s-k)}{\pi(s-k)}\,,\;\; 0 < s < \infty\,. \tag{4}$$

Statement 4. Any function $f \in H_{Ker_4}$ in RKHS H_{Ker_4} is series expansion of selective values

$$f(s) = f(0)\frac{\sin\pi s}{\pi s} + \sum_{k=1}^{\infty} f(k)\frac{2s}{s+k}\frac{\sin\pi(s-k)}{\pi(s-k)}\,,$$

$$0 < s < \infty \tag{5}$$

or $$f(s) = \sum_{k=0}^{\infty} f(k)\frac{\varepsilon_k s}{s+k}\frac{\sin\pi(s-k)}{\pi(s-k)}\,,\;\; 0 < s < \infty\,, \tag{6}$$

where ε_k – the Neumann number.

Based RKHS-theory the new approach to definition of series sum is proposed. It is called *Series Summation Method in RKHS.* It allows analytically to obtain an alternative representations for some kinds of series in the finite form.

Theoretical results

Series Summation. The new formulas for calculating the sum of series (including alternating) can be obtained by proving some theorems. The new mathematical results of the series summation theory having practical significance are proposed. Numerical ground for some examples is given.

There are following relation for alternating series in RKHS

$$\sum_{k=1}^{\infty}(-1)^k \frac{kF(k)}{a^2 - k^2} = \frac{\pi}{2\sin\pi a}F(a)\,, \tag{7}$$

where $F(x) \in H$, $a > 0$, $a \ne 1,2,\dots$.

The following examples show that (7) is true.

Example 1. To proof of the following formula truth:

$$\sum_{k=1}^{\infty}(-1)^k \frac{k\sin kx}{a^2 - k^2} = \frac{\pi}{2}\frac{\sin ax}{\sin\pi a}\,, -\pi < x < \pi\,, a > 0\,, a \ne 1,2,\dots\,, \tag{8}$$

the residues theory is used by Doetsch G. (1967). However application of (7) gives the same result:

$$\sum_{k=1}^{\infty}(-1)^k \frac{k\sin kx}{a^2 - k^2} = \left.\frac{\pi\sin kx}{2\sin\pi a}\right|_{k=a} = \frac{\pi\sin ax}{2\sin\pi a}\,,$$

$$a > 0\,, a \ne 1,2,\dots. \tag{9}$$

Also we can show correctness of (8) by numerically. On Fig. 1 there are two diagrams in the equal co-ordinates for parameter $a = 2{,}5$ were obtained by (8). Fig. 2 demonstrates the absolute uncertainty.

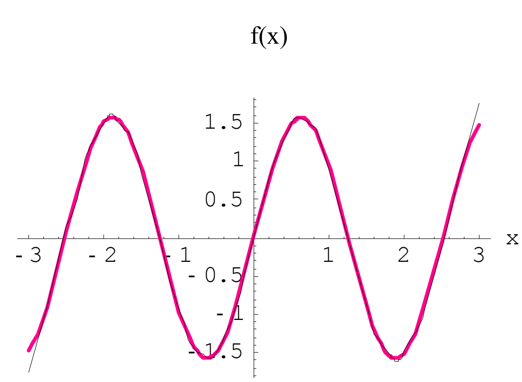

Fig. 1. Graphs of function from right-hand side of (8) (the bold curve) and left hand side (thin curve) of (8)

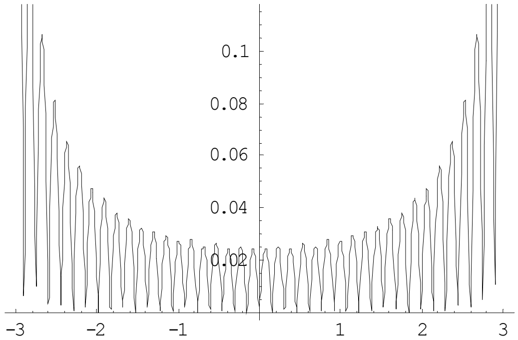

Fig. 2. Absolute uncertainty (error) by (8)

Example 2. To proof of the following identity truth the Laplace transformation is used by Watson G.N. (1952). However application of the formula (7) gives the same result but it is more simpler mathematical derivation:

$$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k}{k^2-a^2}J_{2n+1}(kx)=\sum_{k=1}^{\infty}\frac{(-1)^{k}k}{a^2-k^2}J_{2n+1}(kx)=$$

$$=\frac{\pi}{2}J_{2n+1}(ax)cosec(a\pi)\,,\ -\pi<x<\pi\,. \qquad (10)$$

We can proof the following relation for alternating series in RKHS:

$$\sum_{n=1}^{\infty}\frac{F(n)}{b^2-n^2}=\frac{\pi}{2b}ctg\pi bF(b)\,, \qquad (11)$$

where $F(x)\in H$, $0<b<\infty$, $b\neq 1,2,\ldots$. For example, $F(n)=e^{-n}$, then we can receive by the formula (11) the following result:

$$\sum_{n=1}^{\infty}\frac{e^{-n}}{b^2-n^2}=\frac{\pi}{2b}(ctg\pi b)e^{-b}\,,\ 1<b<\infty\,,\ b\neq 2,3,\ldots\,. \qquad (12)$$

Let's carry out a comparative analysis of results obtained by (12). On Fig. 3 there are two diagrams in the equal co-ordinates. The upper curve describes the function on the right side of (12), while the lower curve describes the function on the left side for parameter values $5<b<9$. The values of b parameter are given on abscissa axis, the function values are given on axis of ordinates. It is obvious that the curves are of the same class.

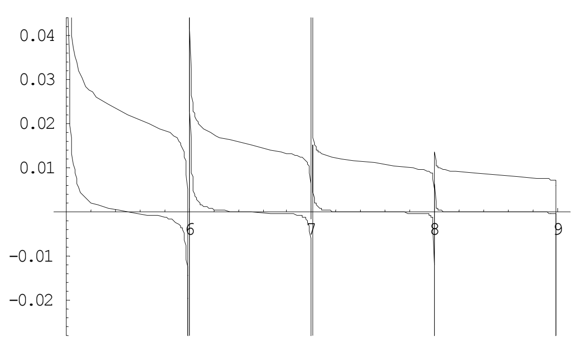

Fig. 3. Functional dependence defined by right and left sides of equation (9) $5<b<9$

One can see that absolute uncertainty (error) is equal to hundredth (Fig. 4).

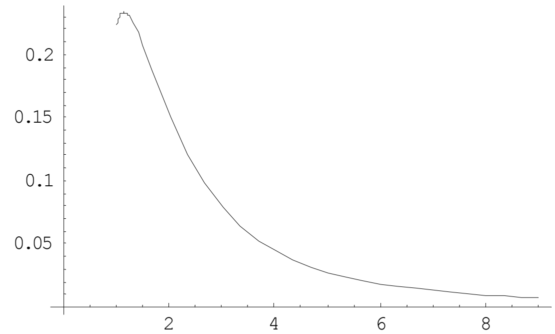

Fig. 4. Absolute uncertainty (error) $1<b<9$

Summation of Double-Series. We can show that the following formula is true:

$$\sum_{k=1}^{\infty}\frac{(-1)^k k}{y^2-k^2}\sum_{m=1}^{\infty}\frac{(-1)^m mF(m,k)}{x^2-m^2}=\frac{\pi^2F(x,y)}{4\sin(\pi x)\sin(\pi y)}\,,$$

$$0<x<\infty\,,\ 0<y<\infty\,,\ x,y\neq 1,2,3,\ldots\,. \qquad (13)$$

Solving summatory and integral equations. The solution of some summatory equations relatively unknown coefficients which define electromagnetic field in diffraction problems has been found by Series Summation Method in RKHS. The solution of the following integral equation which is interesting for some important electrodynamics problems has been found [7] by mentioned method: for the integral equation

$$\int_{-\infty}^{\infty}P(\mu)\exp(ikx\mu)d\mu=0\,. \qquad (14)$$

The function

$$P(\mu)=\frac{A}{\mu}J_{2ka+1}(ka\mu)\,, \qquad (15)$$

is its solution, where k,a – known values, A – constant.

We have deal with equation of kind (14) at solving some electrodynamics problems such as: diffraction electro-magnetic waves for plane conductive screen with slot; normal incidence of H-polarized wave on band of; diffraction electromagnetic wave on structure from finite quantity strips with different widths, which are non-equidistantly arranged.

Integral identities. For $k=1,2,3,\ldots$, $0<x<\infty$, $0<w<\infty$ the following integral identities are true

$$\int_0^{\infty}\frac{\sqrt{w}\sin\pi(w-k)}{\pi(w-k)(w+k)}dw\int_{C-i\infty}^{C+i\infty}tI_\nu(xt)K_\nu(wt)dt=\frac{i\sin\pi(x-k)}{\sqrt{x}(x^2-k^2)}\,,(16)$$

where $I_\nu(z)$, $K_\nu(z)$ are modified Bessel functions (it can be shown by transforming to integral representation of Meijer from Bateman H., Erdelyi A., 1953);

$$\frac{-2\sin\pi(x-k)}{(x^2-k^2)}=\int_0^\infty \frac{\sin\pi(w-k)}{(w^2-k^2)}\frac{dw}{w}\int_{-i\infty}^{i\infty} tJ_t(x)H_t^{(2)}(w)dt\,, \quad (17)$$

where $H_t^{(2)}(z)$ is Bessel function of 3 kind (it can be shown by transforming to integral representation of Kontorovich-Lebedev from Bateman H., Erdelyi A., 1953);

$$\frac{\sin\pi(x-k)}{\pi(x-k)(x+k)}=\int_0^\infty \frac{\sin\pi(w-k)}{\pi(w-k)}\frac{wdw}{w+k}\cdot$$

$$\cdot\int_0^\infty J_\nu(tx)J_\nu(wt)tdt \quad (18)$$

(it can be shown by transforming to integral representation of Hankel from Bateman H., Erdelyi A., 1953).

Concluding remarks

The obtained results allow to make following conclusions: 1) For summation of selected alternating series it is proposed to use Series Summation method in RKHS; 2) Advantages of this method consist of: application of equivalent transformations to the common member of a series, that enables to obtain the analytical solution for smaller quantity of steps; in absence of necessity to use the tables of integral transformations and to address to a complex integration; 3) New mathematical results can be used and can be implemented in Mathematics program products. The proposed method is effective one and perspective mathematical vehicle for solution of the actual and practical problems.

References

1. **Aronzajn A.** Theory of reproducing kernels // Trans. American Math. Soc., 1950. – V.68 – P. 337–404.
2. **Functions** with double orthogonality in radio electronics and optician / By edition M.K. Razmahnin and B.P. Yakovlev. – M.: Sov. Radio, 1971. – 256 p. (in Russian).
3. **Saitoh S.** Integral Transforms, Reproducing Kernels and Their Applications // Pitman Res. Notes in Math. Addison Wesley Longman, UK. – Series, 369 (1997).
4. **Laslo Mate.** Hilbert Space Methods in Science and Engineering, 1990. – 200 p.
5. **Sethu Vijayakumar, Hidemitsu Ogawa.** RKHS based Functional Analysis for Exact Incremental Learning / Neurocomputing: Special Issue on Theoretical analysis of real valued function classes. – Elsevier Science (1999). Vol.29. – No.1–3. – P.85–113.
6. **Chumachenko S.V., Gowher Malik, Khawar Parvez.** Reproducing Kernel Hilbert Space Methods FOR CAD Tools // Proc. EWDTW, 2004. Yalta-Alushta, September 23-26. – P. 247–250.
7. **Svetlana Chumachenko, Vladimir Hahanov.** Reproducing Transformations method for IP-core of summatory and integral equations solving // Proc. of DSD 2004 EUROMICRO SYMPOSIUM ON DIGITAL SYSTEM DESIGN: Architectures, Methods and Tools. Rennes – France. August 31 – September 3, 2004.

Pateikta spaudai 2005 05 17

S. Čiumačenko, V. Hahanov, O. Melnikova. Žilberto metodų taikymas sprendžiant kai kurias radiofizikos problemas // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2005. – Nr. 6(62). – P. 5–8.

Nagrinėjami Žilberto erdvių metodai su atgaminančiaisiais eilių sumavimo branduoliais. Juos naudojant galima gauti alternatyvias baigtinės formos eiles. Pasiūlyti nauji matematiniai eilių teorijos rezultatai, turintys praktinę vertę. Pateiktas kai kurių pavyzdžių skaitmeninis pagrindimas, pasiūlyti nauji teoriniai eilių sumavimo metodai. Il. 4, bibl.7 (anglų kalba; santraukos lietuvių, anglų ir rusų k.).

S. Chumachenko, V. Hahanov, O. Melnikova. RKHS-Methods at Solving Some Radiophysics Problems // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 6(62). – P. 5–8.

Reproducing Kernel Hilbert Space (RKHS) Methods for Series Summation that allow analytically to obtain alternative representations for series in the finite form are considered. The new mathematical results of the series theory having practical significance are proposed. Numerical ground for some examples is given. The paper demonstrates the new results which develop theoretical statements of Series Summation Method in RKHS. Ill. 4, bibl. 7 (in English; summaries in Lithuanian, English and Russian).

С. Чумаченко, В.И. Хаханов, О.В. Мельникова. Методы ГПВЯ при решении некоторых задач радиофизики // Электроника и электротехника. – Каунас: Технология, 2005. - № 6(62). – С. 5–8.

Рассматриваются методы гильбертовых пространств с воспроизводящими ядрами (ГПВЯ) для суммирования рядов, которые позволяют аналитически получать альтернативные представления рядов в конечной форме. Предлагаются новые математические результаты теории рядов, имеющие практическое применение. Дается численное обоснование некоторых примеров. Демонстрируются новые результаты, которые развивают теоретические положения метода суммирования рядов в ГПВЯ. Ил. 4, библ. 7 (на английском языке; рефераты на литовском, английском и русском яз.).