

## Dynamic Models of Controlled Linear Induction Drives

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### Introduction

A great number of companies produce linear motors of different construction. They advertise their production widely, for example, via internet. At present the typical applications of LIM are: sliding doors, aluminium can propulsion, mixer and stirrer drives, wire winding, baggage handling, pallet drives [1], flexible manufacturing systems, robotic systems, bogie drives, conveying systems, steel tube movement, revolving doors, sheet metal movement, linear accelerators, automated postal systems, ship test tank drive, extrusion pullers, multi-motor, in-track systems, low profile drives target movement, sewage distributors, slewing drives, crane drives, stage and curtain movement, scrap sorting and movement, research machines, turntable drives, automated warehousing, flat circular motors, theme park rides, personal, rapid transport systems.

The larger air gap of LIM determines the greater losses. The most expedient area to apply LIM is in the equipment with short time and intermediate operating mode [2-4] where a drive operates in dynamic mode. To solve dynamic problems of controlled linear electric drive usually is used model of LIM in stationary reference frame or in travelling with synchronous speed [5-7]. This method can be applied for small power and small speed LIM with assumption about symmetry of its magnetic and electric circuit and being energized by balance three phase supply voltages. In practice open magnetic circuit causes asymmetry in motor phase currents and model in transformed reference frame doesn't fit reality. The article discusses the model of a linear induction motor with Y connected windings without neutral wire made for instantaneous supply voltage. Investigation of operating characteristics can be useful for LIM applied in equipment with motoring and braking modes.

### Mathematical model of a linear induction motor with Y connected windings without neutral wire

In the induction motors with symmetrical windings supplied with balanced voltage system current does not flow in neutral wire. They can be modelled by the method described above even if they are connected without neutral wire [8].

If phase windings of a motor are non-symmetrical and connected without neutral wire or if they are symmetrical but supplied by unbalanced voltage system due to absence

of neutral wire they must be modelled considering supply by line voltages.

A scheme of a three-phase motor with Y connected windings without neutral wire is given in Fig. 1.

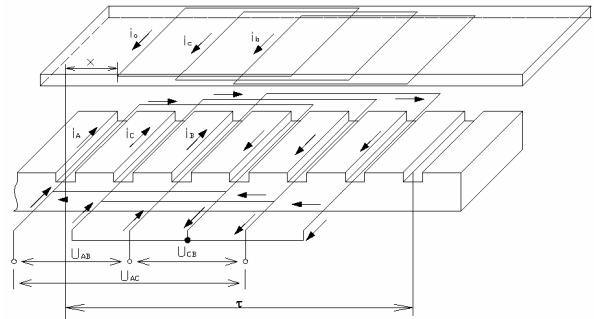


Fig. 1. Scheme of a three-phase LIM with Y connected windings without neutral wire

Equations for an inductor and secondary are:

$$\begin{cases} u_{AC} = R_A i_A + \frac{d\Psi_A}{dt} - R_C i_C - \frac{d\Psi_C}{dt}, \\ u_{BC} = R_B i_B + \frac{d\Psi_B}{dt} - R_C i_C - \frac{d\Psi_C}{dt}, \\ u_{ac} = R_a i_a + \frac{d\Psi_a}{dt} - R_c i_c - \frac{d\Psi_c}{dt}, \\ u_{bc} = R_b i_b + \frac{d\Psi_b}{dt} - R_c i_c - \frac{d\Psi_c}{dt}, \end{cases} \quad (1)$$

where  $u_{AC}$ ,  $u_{BC}$ ,  $u_{ac}$ ,  $u_{bc}$  are instantaneous values of line voltages of stator and rotor.

There are two independent currents in Y connection. By Ohm's law the third one is equal to the sum of these two currents taken with minus sign. Therefore, one current of equation (1) can be expressed as a sum others two. This statement is valid for flux linkages also. If two independent quantities are chosen, for example, currents of a and b windings, then current and flux linkages of phase c are calculated as:

$$\begin{cases} i_C = -(i_A + i_B), \quad \Psi_C = -(\Psi_A + \Psi_B), \\ i_c = -(i_a + i_b), \quad \Psi_c = -(\Psi_a + \Psi_b). \end{cases} \quad (2)$$

Substituting (2) to (1) yields:

$$\begin{cases} u_{AC} = (R_A + R_C)i_A + R_B i_B + \frac{d(2\Psi_A + \Psi_B)}{dt}, \\ u_{BC} = R_C i_A + (R_B + R_C)i_B + \frac{d(\Psi_A + 2\Psi_B)}{dt}, \\ u_{ac} = (R_a + R_c)i_a + R_c i_b + \frac{d(2\Psi_a + \Psi_b)}{dt}, \\ u_{bc} = R_c i_a + (R_b + R_c)i_b + \frac{d(\Psi_a + 2\Psi_b)}{dt}. \end{cases} \quad (3)$$

Flux linkages of windings are calculated according to formulas, presented in [7]. Substituting these formulas to (3) and considering (2) yields:

$$\begin{aligned} u_{AC} = & (R_A + R_C)i_A + R_B i_B + \\ & + 2(L_A - M_S)\frac{di_A}{dt} + (L_B - M_S)\frac{di_B}{dt} + \\ & + 3M_{SR} \left[ \cos\left(\frac{\pi}{\tau}x\right)\frac{di_a}{dt} + \cos\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right)\frac{di_b}{dt} \right] - \\ & - 3M_{SR} \frac{\pi}{\tau} \left[ i_a \sin\left(\frac{\pi}{\tau}x\right) + i_b \sin\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right) \right] \frac{dx}{dt}; \quad (4) \end{aligned}$$

$$\begin{aligned} u_{BC} = & R_C i_A + (R_B + R_C)i_B + \\ & + (L_A - M_S)\frac{di_A}{dt} + 2(L_B - M_S)\frac{di_B}{dt} + \\ & + 3M_{SR} \left[ \cos\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right)\frac{di_a}{dt} + \cos\left(\frac{\pi}{\tau}x\right)\frac{di_b}{dt} \right] - \\ & - 3M_{SR} \frac{\pi}{\tau} \left[ i_a \sin\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right) + i_b \sin\left(\frac{\pi}{\tau}x\right) \right] \frac{dx}{dt}; \quad (5) \end{aligned}$$

$$\begin{aligned} u_{ac} = & (R_a + R_c)i_a + R_c i_b + 3M_{SR} \cos\frac{\pi}{\tau}x \frac{di_A}{dt} + \\ & + 3M_{SR} \cos\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right)\frac{di_B}{dt} + \\ & + 2(L_a - M_R)\frac{di_a}{dt} + (L_b - M_R)\frac{di_b}{dt} - \\ & - 3M_{SR} \frac{\pi}{\tau} \left[ i_A \sin\frac{\pi}{\tau}x + i_B \sin\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right) \right] \frac{dx}{dt}; \quad (6) \end{aligned}$$

$$\begin{aligned} u_{bc} = & R_c i_a + (R_b + R_c)i_b + 3M_{SR} \cos\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right)\frac{di_A}{dt} + \\ & + 3M_{SR} \cos\frac{\pi}{\tau}x \frac{di_B}{dt} + \\ & + (L_a - M_R)\frac{di_a}{dt} + 2(L_b - M_R)\frac{di_b}{dt} - \\ & - 3M_{SR} \frac{\pi}{\tau} \left[ i_A \sin\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right) + i_B \sin\frac{\pi}{\tau}x \right] \frac{dx}{dt}. \quad (7) \end{aligned}$$

The system of equations (4) – (7) describes induction motor with Y connected winding without neutral wire. To solve it in Matlab, it is necessary to express them in the following way:

$$\mathbf{M}(t)\mathbf{x}' = \begin{pmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & 0 \\ B_1 & B_2 & B_3 & B_4 & B_5 & 0 \\ C_1 & C_2 & C_3 & C_4 & C_5 & 0 \\ D_1 & D_2 & D_3 & D_4 & D_5 & 0 \\ 0 & 0 & 0 & 0 & \frac{\tau}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & m \end{pmatrix} \begin{pmatrix} di_A/dt \\ di_B/dt \\ di_a/dt \\ di_b/dt \\ dv/dt \\ dv^2/d^2t \end{pmatrix}, \quad (8)$$

where the following notations are used:

$$\begin{aligned} A_1 = & 2(L_A - M_S); B_2 = 2(L_B - M_S); \\ C_3 = & 2(L_a - M_R); D_4 = 2(L_b - M_R); \\ A_2 = & L_B - M_S; B_1 = L_A - M_S; \\ C_4 = & L_b - M_R; D_3 = L_a - M_R; \\ A_3 = & B_4 = C_1 = D_2 = 3M_{SR} \cos\frac{\pi}{\tau}x; \\ D_1 = & A_4 = 3M_{SR} \cos\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right); \\ C_2 = & B_3 = 3M_{SR} \cos\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right); \\ A_5 = & -3M_{SR} \left[ i_a \sin\left(\frac{\pi}{\tau}x\right) + i_b \sin\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right) \right]; \\ B_5 = & -3M_{SR} \left[ i_a \sin\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right) + i_b \sin\frac{\pi}{\tau}x \right]; \\ C_5 = & -3M_{SR} \left[ i_A \sin\frac{\pi}{\tau}x + i_B \sin\left(\frac{\pi}{\tau}x - \frac{\pi}{3}\right) \right]; \\ D_5 = & -3M_{SR} \left[ i_A \sin\left(\frac{\pi}{\tau}x + \frac{\pi}{3}\right) + i_B \sin\frac{\pi}{\tau}x \right]. \end{aligned}$$

The left-hand side of the equation is expressed as:

$$\mathbf{F}(t, \mathbf{x}) = \begin{pmatrix} U_{AC} - (R_A + R_C)i_A - R_C i_B \\ U_{BC} - R_C i_A - (R_B + R_C)i_B \\ U_{ac} - (R_a + R_c)i_a - R_c i_b \\ U_{cb} - R_c i_a - (R_b + R_c)i_b \\ v \\ F_{em} - F_{st} \end{pmatrix}. \quad (9)$$

Electromagnetic force is calculated from the expression:

$$\begin{aligned} F_{em} = & \frac{\pi}{\tau} \left\{ -pM_{SR} \left[ (i_A i_a + i_B i_b + i_C i_c) \sin\frac{\pi}{\tau}x \right] - \right. \\ & - pM_{SR} \left[ (i_A i_b + i_B i_c + i_C i_a) \sin\left(\frac{\pi}{\tau}x + 120^\circ\right) \right] - \\ & \left. - pM_{SR} \left[ (i_A i_c + i_B i_a + i_C i_b) \sin\left(\frac{\pi}{\tau}x - 120^\circ\right) \right] \right\}. \quad (10) \end{aligned}$$

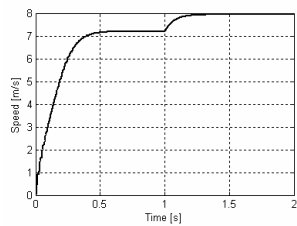
## Motor operation mode of linear induction drive

Motoring is the main operation of all LIM. Induction motor operates at motor mode at slip  $0 < s < 1$ . At this time motor takes power from motor supply source and transforms it to mechanical power.

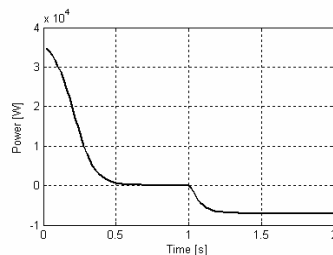
Fig.2 illustrates the speed rising process at starting LIM while time  $t < 1$  s. Speed of the secondary by exponential curve approaches to synchronous. Fig. 3 shows the average power transferred from the motor supply source to the motor. Average power is calculated as instantaneous power per one period, i.e. for each period of supply voltage is applied formula:

$$P = \frac{1}{T} \int_i^{i+T} (i_A u_{AC} + i_B u_{BC}) dt . \quad (11)$$

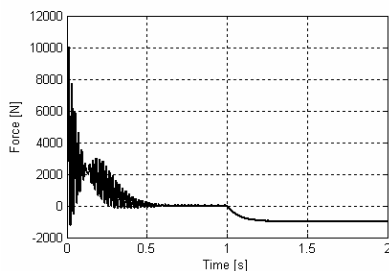
During starting process power consumed from motor supply source some tens times exceeds rated. At this time energy is used to establish magnetic field [8]. When the secondary of the motor accelerates up and gets its steady-state value, the power is used to cover power losses in windings. The iron losses in this model are neglected.



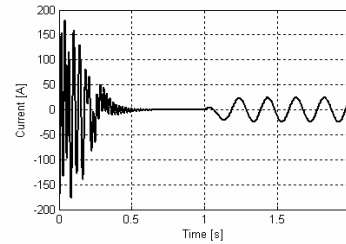
**Fig. 2.** Speed curve at starting ( $t < 1$ ) and regenerative braking mode ( $t > 1$ )



**Fig. 3.** Dependences of average power at motor operation mode ( $t < 1$ ) and regenerative braking mode ( $t > 1$ )



**Fig. 4.** Dependences of developed force at motor operating mode ( $t < 1$ ) and regenerative braking mode ( $t > 1$ )



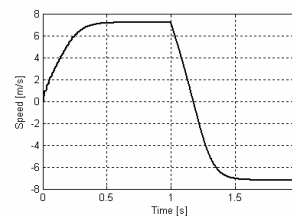
**Fig. 5.** Current of the secondary at starting ( $t < 1$ ) and regenerative braking mode ( $t > 1$ )

## Regenerative braking mode of linear induction drive

Induction motor operates in the regenerative braking mode if the speed of secondary exceeds synchronous. If the external force of working machine impresses the secondary and accelerates that it is possible to fit this condition. Graphs of the LIM variables at a regenerative braking mode are presented in Fig. 2-5 at time instant  $t > 1$ . This situation has been modelled with setting up an external force pointing in direction with movement of secondary and accelerating the secondary. This situation happens in lifting mechanisms, when the load falls down and accelerates the motor. The curve of speed shown in Fig. 2 (at  $t > 1$ ) exponentially rises and gets a new steady-state value established by F-v curve. Fig. 3 shows transient of average power which at  $t > 1$  is negative. This corresponds to power being returned back to motor supply source during regenerative braking. The power can be returned back to motor supply source if it exceeds power losses in the motor. Dependence of developed electromagnetic force is presented in Fig. 4 at  $t > 1$ . The process is similar to that of power. Developed electromagnetic force is negative and exponentially approaches to steady-state value. This negative value shows opposite direction of force and speed due to regenerative braking. In the Fig. 5 is shown current of the secondary. As the speed of the secondary is greater than synchronous, the currents flow in the secondary at regenerative braking mode.

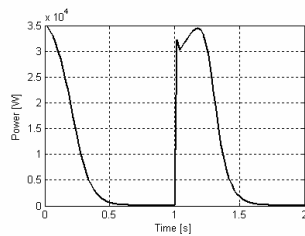
## Reverse-current braking of the linear induction drive

Dependences of LIM variables at starting and reversing are shown in Fig. 6-9. During a time period  $t < 1$  the starting process takes part and during  $t > 1$  – reverse process at a changed phases sequence. The reverse-current braking, as shown in Fig. 6, is characterized with high efficiency. The speed of the secondary decreases to zero during a  $1/3$  of speed rise time.



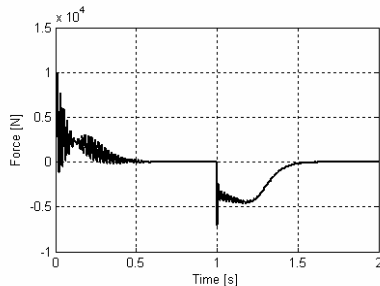
**Fig. 6.** Speed curve at starting ( $t < 1$ ) and reverse-current braking mode ( $t > 1$ )

Fig. 7 presents the average power. The reverse of LIM can be considered as heavy duty due to large amount of consumed power which reaches its maximum value at the first period of supply voltage at reversing.



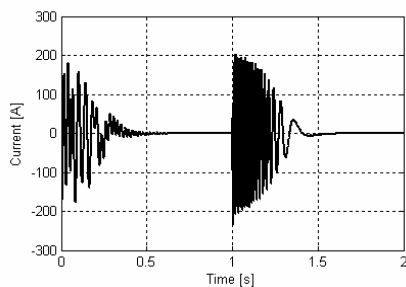
**Fig. 7.** Dependences of average power at motor operation mode ( $t < 1$ ) and reverse-current braking mode ( $t > 1$ )

The graph of electromagnetic force is presented in Fig. 8. During reverse it changes direction and reaches its maximum value at the first supply voltage period when the secondary reaches synchronous speed and developed force approaches to zero.



**Fig. 8.** Dependences of developed force at motor operation mode ( $t < 1$ ) and reverse-current braking mode ( $t > 1$ )

The phase current of the secondary is shown in Fig. 9. Note, that reverse current exceeds the starting that. In the beginning of reverse a frequency of secondary current is two times greater than supply voltage frequency.



**Fig. 9.** Current of the secondary at starting ( $t < 1$ ) and reverse-current braking mode ( $t > 1$ )

## Conclusions

1. Developed mathematical and computer model of linear induction motor with Y connected windings gives

possibility to consider magnetic and electric non-symmetry of LIM.

2. Designed computer model of LIM gives possibility to investigate dynamic processes of motor variables in motor operation mode, regenerative braking and reverse-current braking mode.

3. Developed model facilitates investigation of dynamic processes of linear induction drive at different operation modes.

4. The greatest developed electromagnetic force at reverse braking doesn't exceed that at starting of LIM.

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Nagrinėjamas sudarytas tiesiaieigės asinchroninės pavaros matematinis modelis, kai jo apvijos sujungtos žvaigžde be nulinio laido. Sudarytasis modelis leidžia tirti elektros pavarų su tiesiaiegiais asinchroniniais varikliais pereinamuosius procesus, varikliui veikiant variklio arba rekuperacinio bei priešinio jungimo stabdymo būdu, esant vienodiems ar skirtingiems fazinių apvijų parametrų, variklį maitinant simetrine ar nesimetrine įtampa. Išvesta formulė variklio sukuriamai jėgai apskaičiuoti. Skaičiavimo rezultatai: variklio sukuriama jėga, greitis, antrinio elemento kiekvienos fazinės apvijos srovės ir kiti parametrai gaunami grafišką forma. Grafikai rodo, kad rekuperacinio stabdymo metu energija yra grąžinama į tinklą. Il. 9, bibl. 8 (anglų kalba; santraukos lietuvių, anglų ir rusų k.)

**R. Rinkevičienė, A. Petrovas. Dynamic Models of Controlled Linear Induction Drives // Electronics and Electrical Engineering. – Kaunas: Technologija, 2005. – No. 5(61). – P. 23–27.**

The article deals with elaborated mathematical model of linear induction motor with Y connected inductor windings. Developed model gives possibility to investigate transients of electric drives with linear induction motors at balanced and unbalanced supply voltage and at equal or distinct parameters of phase windings during motor operation mode, regenerative braking mode and reverse braking mode. Derived formula to calculate developed force is presented. Carried out program to solve the model. Calculated results: force developed by linear induction motor, speed, currents of phase windings of inductor and secondary and other parameters are obtained in form of graphs. Graphs show that energy is returned back to motor supply source during regenerative braking. Il.9, bibl. 8 (in English; summaries in Lithuanian, English and Russian).

**Р. Ринкявичене, А. Петровас. Динамические модели управляемых линейных асинхронных электроприводов // Электроника и электротехника. – Каунас: Технология, 2005.– № 5(61). – С. 23–27.**

Рассматривается математическая модель линейного асинхронного электропривода и на этой основе разработанная программа для исследования динамических режимов линейного асинхронного электропривода. Разработанная программа позволяет исследовать переходные процессы двигательного режима, рекуперационного режима торможения и торможения противотоком электропривода с линейными двигателями при соединении их обмоток в звезду при симметричных и не симметричных напряжениях питания, при одинаковых или разных обмоточных параметрах фазных обмоток. Выведена формула для расчёта развиваемой двигателем силы. Результаты расчёта: усилие, развиваемое двигателем, токи всех обмоток индуктора и вторичного элемента и другие параметры получают в графической форме. Ил. 9, библи. 8 (на английском языке; рефераты на литовском, английском и русском яз.)